A solution of Stokes′ problem for the ellipsoidal Earth by means of Green′s function

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A solution of Stokes′ boundary problem for the ellipsoidal (spheroidal) Earth accounting for terms of the order of the Earth's flattening can be obtained in spheroidal coordinates *u, v, w*, which are connected with cartesian ones by formulas

$$x=c\sin u\cos v\,\mathrm{ch}w,\; y=c\sin u\sin v\,\mathrm{ch}w,\; z=c\cos u\,\mathrm{sh}w,$$

Lamé′s coefficients being

$$h_u=h_w=c(\mathrm{ch}^2w-\sin^2u)^{1/2},\; h_v=c\sin u\,\mathrm{ch}w.$$

On the Earth′s spheroid we have

$$h_u=h_v=a\,(1-\tfrac{c^2}{a^2}\sin^2 u)^{1/2},\;\; h_v=a\sin u,$$

where $c=(a^2-b^2)^{1/2}$ and *a*, *b* are the semi-axes of the spheroid. Green′s function of the Stokes′ problem for the ellipsoidal Earth can be established by means of Green′s formula

$$4\pi T=\int_s (T\tfrac{\partial}{\partial\nu}\cdot\tfrac{1}{r_{Tds}}-\tfrac{1}{r_{Tds}}\cdot\tfrac{\partial T}{\partial\nu})dS, \tag{1}$$

T being the disturbing potential, ν the external normal to the ellipsoidal surface *S*, r_{Tds} being the distance between the fixed point out of the spheroid and the element *dS*. The Green′s function can be found as a sum $\frac{1}{r_{Tds}}+\varphi,$ the function φ being a harmonic function out of the spheroid. The derivative $\frac{\partial T}{\partial\nu}$ can be excluded from the formula (1) by means of the boundary condition

$$\tfrac{\partial T}{\partial\nu}-\tfrac{T}{\gamma}\cdot\tfrac{\partial\gamma}{\partial\nu}=-(g-\gamma),$$

g being the gravity, γ its normal value. Then

$$T=\tfrac{1}{4\pi}\int_S\left[T\tfrac{\partial}{\partial\nu}(\tfrac{1}{r_{Tds}}+\varphi)-\tfrac{1}{\gamma}\cdot\tfrac{\partial\gamma}{\partial\nu}(\tfrac{1}{r_{Tds}}+\varphi)\;\right]+(\tfrac{1}{r_{Tds}}+\varphi)(g-\gamma)\Big]dS,$$

the unknown part φ of the Green′s function should satisfy the boundary condition

$$\tfrac{\partial\varphi}{\partial\nu}-\tfrac{\varphi}{\gamma}\cdot\tfrac{\partial\gamma}{\partial\nu}+\tfrac{\partial}{\partial\nu}\;\tfrac{1}{r_{T_{Tds}}}-\tfrac{1}{\gamma}\;\tfrac{\partial\gamma}{\partial\nu}\cdot\tfrac{1}{r_{Tdss}}=0. \tag{2}$$

This approach is suggested by Ostach (1969). It can be written

$$4\pi\varphi = \int_s \left[\varphi \frac{\partial}{\partial \nu}\frac{1}{r_{\varphi ds}} - \frac{1}{r_{\varphi ds}} \cdot \frac{\partial \varphi}{\partial \nu}\right] dS \qquad (3)$$

The derivative $\frac{\partial \varphi}{\partial \nu}$ can be excluded from (3) by means of the boundary condition (2) and an integral equation for function φ will be obtained

$$4\pi\varphi - \int_s \varphi\left[\frac{\partial}{\partial \nu}\frac{1}{r_{\varphi ds}} - \frac{1}{\gamma}\cdot\frac{\partial \gamma}{\partial \nu}\cdot\frac{1}{r_{\varphi ds}}\right] dS = \int_s \frac{1}{r_{\varphi ds}}\left[\frac{\partial}{\partial \nu}\frac{1}{r_{Tds}} - \frac{1}{r_{Tds}}\cdot\frac{1}{\gamma}\cdot\frac{\partial \gamma}{\partial \nu}\right] dS$$

or

$$4\pi\varphi - \int_s \varphi\left[\frac{\partial}{\partial w}\frac{1}{r_{\varphi ds}} - \frac{1}{\gamma}\cdot\frac{\partial \gamma}{\partial \nu}\cdot\frac{1}{r_{\varphi ds}}\right] h_\nu du d\nu = \int_s \frac{1}{r_{\varphi ds}}\left[\frac{\partial}{\partial w}\cdot\frac{1}{r_{Tds}} - \frac{1}{r_{Tds}}\cdot\frac{1}{\gamma}\cdot\frac{\partial \gamma}{\partial \nu}\right] h_\nu du dv. \qquad (4)$$

As it is known

$$\frac{1}{r_{\varphi ds}} = \frac{i}{c}\sum_{n=o}^{\infty} (2n+1)\Big[P_n(\cos u)P_n(\cos u_\varphi)Q_n(ishw_\varphi)P_n(ishw) +$$

$$+2\sum_{m=1}^{\infty} (-1)^m \left(\frac{(n-m)!}{(n+m)!}\right)^2 P_{nm}(\cos u)P_{nm}(\cos u_\varphi)Q_{nm}(ishw_\varphi)P_{nm}(ishw)\cos m(\nu-\nu_\varphi)\Big].$$

We have for the current point of the surface S

$$\frac{\partial}{\partial w}\frac{1}{r_{\varphi ds}} = \frac{i}{c}\sum_{n=0}^{\infty} (2n+1)\Big[P_n(\cos u)\, P_n(\cos u_\varphi)Q_n(ishw_\varphi)\frac{dP_n(ishw)}{dw} +$$

$$+2\sum_{m=1}^{n} (-1)^m \left(\frac{(n-m)}{(n+m)}\right)^2 P_{nm}(\cos u)P_{nm}(\cos u_\varphi)Q_{nm}(ishw_\varphi)\frac{dPnm(ishw)}{dw}\cos m(\nu-\nu_\varphi)\Big],$$

where

$$\left(\frac{dP_{nm}(ishw)}{dw}\right)_{shw=\frac{b}{c}} = -\frac{ci}{a}\left[(n-m+1)P_{n+1,m}(i\tfrac{b}{c}) - (n+1)i\tfrac{b}{c}P_{nm}(i\tfrac{b}{c})\right].$$

It can be obtained by means of known formulas (Gradshtein, Ryzhik 1965)

$$Q_{nm}(i\tfrac{b}{c})\left(\frac{dP_{nm}(ishw)}{dw}\right)_{shw=\frac{b}{c}} \approx -(-1)^m \frac{ci}{a}\left(\frac{a}{b}\right)^{2m}\frac{(n+m)!}{(n-m)!}\frac{n}{2n+1} \times$$

$$\times\left[1+\frac{c^2}{b^2}\left(\frac{(n-m)(n^2-mn+n+2m)}{2n(2n-1)} - \frac{(n+m+1)(n+m+2)}{2(2n+3)}\right)\right]$$

and

$$Q_{nm}(i\tfrac{b}{c})P_{nm}(i\tfrac{b}{c}) \approx (-1)^m \frac{c}{ib}\left(\frac{a}{b}\right)^{2m}\frac{1}{2n+1}\frac{(n+m)!}{(n-m)!}\times$$

$$\times\left[1+\frac{c^2}{b^2}\left(\frac{(n-m-1)(n-m)}{2(2n-1)} - \frac{(n+m+2)(n+m+1)}{2(2n+3)}\right)\right].$$

It can be written with the assumed accuracy

$$\gamma = \frac{GM}{b^2}\left(1-\frac{1}{2}\frac{c^2}{b^2}-\frac{3}{2}p-\frac{1}{2}\frac{c^2}{b^2}\cos^2 u+\frac{5}{2}p\cos^2 u\right),$$

$$\frac{\partial\gamma}{\partial w} = -\frac{2GM}{b^2}\left(1-\frac{1}{2}\frac{c^2}{b^2}-\frac{c^2}{b^2}\cos^2 u-\frac{1}{2}p+\frac{5}{2}p\cos^2 u\right),$$

$$\frac{1}{\gamma}\cdot\frac{\partial\gamma}{\partial w} = -2\left(1-\frac{1}{2}\frac{c^2}{b^2}\cos^2 u+p\right),$$

where $p = \frac{\omega^2 b^3}{GM}$. The function φ for the spheroidal Earth can be represented by an expansion

$$\varphi = \frac{i}{c}\sum_{n=0}^{\infty} (2n+1)\Big[A_n P_n(\cos u_T)P_n(\cos u)Q_n(ishw_T)P_n(ishw) +$$
$$+2\sum_{m=1}^{n} (-1)^m (\tfrac{(n-m)!}{(n+m)!})^2 (\tfrac{b}{a})^{4m} A_{nm}P_{nm}(\cos u_T)P_{nm}(\cos u)\times$$
$$\times Q_{nm}(ishw_T)P_{nm}(ishw)\cos m(\nu_T - \nu)\ \Big]$$

or

$$\varphi = \frac{1}{b}\sum_{n=0}^{\infty} \Big[A_n P_n(\cos u_T)P_n(\cos u)\Big[1+\tfrac{c^2}{b^2}(\tfrac{n(n-1)}{2(2n-1)} - \tfrac{(n+1)(n+2)}{2(2n+3)})\ \Big]+$$
$$+2\sum_{m=1}^{n} A_{nm}\tfrac{(n-m)!}{(n+m)!}(\tfrac{b}{a})^{2m}P_{nm}(\cos u_T)P_{nm}(\cos u)\times \tag{5}$$
$$\times\Big[1+\tfrac{c^2}{b^2}(\tfrac{(n-m-1)(n-m)}{2(2n-1)} - \tfrac{(n+m+1)(n+m+2)}{2(2n+3)})\ \Big]\cos m(\nu_T - \nu)\Big].$$

It will be possible to express coefficients A_{nm} after substitution of formula (5) into the equation (4) and collecting terms with $P_{nm}(\cos u_t)\ P_{nm}(\cos u_\varphi)$. Since

$$\int_0^{2\pi} \cos m(\nu-\nu_T)\cos m(\nu-\nu_\varphi)dv = \pi\cos m(\nu_T - v_\varphi),$$
$$\int_s \varphi\frac{\partial}{\partial w}\frac{1}{r_{\varphi ds}}h_\nu dud\nu = \frac{4\pi}{b}\sum_{n=0}^{\infty} \Big\{\frac{A_n n}{2n+1}P_n(\cos u_T)P_n(\cos u_\varphi)\Big[1+\tfrac{c^2}{b^2}(\tfrac{n^2}{2n-1} - \tfrac{(n+1)(n+2)}{2n+3})\ \Big]+$$
$$+2\sum_{m=1}^{n} \frac{A_{mn}n}{2n+1}P_{nm}(\cos u_T)P_{nm}(\cos u_\varphi)\tfrac{(n-m)!}{(n+m)!}\Big[1+\tfrac{c^2}{b^2}(\tfrac{(n-m)(n^2-mn+m)}{n(2n-1)} - \tfrac{(n+m+1)(n+m+2)}{2n+3})\ \Big]\times$$
$$\times\cos m(\nu_T - \nu_\varphi)\ \Big\}.$$

The next integral can be expressed by means of the known relation

$$cos^2 u\, P_{nm}(\cos u) = \tfrac{(n+m)(n+m-1)}{(2n-1)(2n+1)}P_{n-2,m}(\cos u) + \tfrac{2n^2-2m^2+2n-1}{(2n-1)(2n+3)}P_{nm}(\cos u) +$$
$$+\tfrac{(n-m+1)(n-m+2)}{(2n+1)(2n+3)}P_{n+2,m}(\cos u).$$

Then

$$\int_s \frac{\varphi}{r_{\varphi ds}}\cdot\frac{1}{\gamma}\cdot\frac{\partial\gamma}{\partial w}h_\nu dudv = -2(1+p)\int_s \frac{\varphi}{r_{\varphi ds}}h_\nu dud\nu + \frac{c^2}{b^2}\int_s \frac{\varphi}{r_{\varphi ds}}h_\nu \cos^2 u\, dudv =$$
$$= -\frac{8\pi a}{b^2}(1+p)\sum_{n=0}^{\infty} \Big\{\frac{A_n}{2n+1}P_n(\cos u_T)P_n(\cos u_\varphi)\Big[1+\tfrac{c^2}{b^2}(\tfrac{n(n-1)}{2n-1} - \tfrac{(n+1)(n+2)}{(2n+3)})\ \Big]+$$
$$+2\sum_{m=1}^{n} \frac{A_{nm}}{2n+1}\cdot\frac{(n-m)!}{(n+m)!}P_{nm}(\cos u_T)P_{nm}(\cos u_\varphi)\times$$
$$\times\Big[1+\tfrac{c^2}{b^2}(\tfrac{(n-m-1)(n-m)}{2n-1} - \tfrac{(n+m+1)(n+m+2)}{2n+3})\ \Big]\cos m(\nu_T - v_\varphi)\ \Big\}+$$

$$+\frac{4\pi}{b}\cdot\frac{c^2}{b^2}\sum_{n=0}^{\infty}\left\{\frac{A_n}{2n+1}P_n(\cos u_T)P_n(\cos u_\varphi)E_n+\right.$$

$$\left.+2\sum_{m=1}^{n}\frac{Anm}{2n+1}\cdot\frac{(n-m)!}{(n+m)!}P_{nm}(\cos u_T)P_{nm}(\cos u_\varphi)E_{nm}\cos m(\nu_T-\nu_\varphi)\right\},$$

where

$$E_{nm}=\frac{(n+m+2)(n+m+1)}{(2n+3)(2n+5)}+\frac{2n^2-2m^2+2n-1}{(2n-1)(2n+3)}+\frac{(n-m-1)(n-m)}{(2n-3)(2n-1)},E_n=(E_{nm})_{m=0}$$

We get also

$$a\int_s\frac{1}{r_{\varphi ds}}\cdot\frac{\partial}{\partial w}\frac{1}{r_{Tds}}\sin u du d\nu=\frac{4\pi}{b}\sum_{n=0}^{\infty}\frac{n}{2n+1}\{P_n(\cos u_T)P_n(\cos u_\varphi)\times$$

$$\times\left[1+\frac{c^2}{b^2}\left(\frac{n^2}{2n-1}-\frac{(n+1)(n+2)}{2n+3}\right)\right]+2\sum_{m=1}^{n}\frac{(n-m)!}{(n+m)!}P_{nm}(\cos u_T)P_{nm}(\cos u_\varphi)\left(\frac{a}{b}\right)^{4m}\times$$

$$\times\left[1+\frac{c^2}{b^2}\left(\frac{(n-m)(n^2-nm+m)}{n(2n-1)}-\frac{(n+m+1)(n+m+2)}{2n+3}\right)\right]\cos m(\nu_T-\nu_\varphi)\},$$

$$a\int_s\frac{1}{r_{\varphi ds}}\cdot\frac{1}{r_{Tds}}\cdot\frac{1}{\gamma}\cdot\frac{\partial\gamma}{\partial\nu}\sin u du d\nu=-2a(1+p)\int_s\frac{1}{r_{\varphi ds}}\cdot\frac{1}{r_{Tds}}\sin u du dv+$$

$$+a\frac{c^2}{b^2}\int_s\frac{1}{r_{\varphi ds}}\cdot\frac{1}{r_{Tds}}\cos^2 u\sin u\,du d\,v=$$

$$=-\frac{8\pi a}{b^2}(1+p)\sum_{n=0}^{\infty}\frac{1}{2n+1}\left\{P_n(\cos u_T)P_n(\cos u_\varphi)\left[1+\frac{c^2}{b^2}\left(\frac{n(n-1)}{2n-1}-\frac{(n+1)(n+2)}{2n+3}\right)\right]+\right.$$

$$+2\sum_{m=1}^{n}\frac{(n-m)!}{(n+m)!}P_{nm}(\cos u_T)P_{nm}(\cos u_\varphi)\left(\frac{a}{b}\right)^{4m}\times$$

$$\left.\times\left[1+\frac{c^2}{b^2}\left(\frac{(n-m-1)(n-m)}{2n-1}-\frac{(n+m+1)(n+m+2)}{2n+3}\right)\right]\cos m(\nu_T-\nu_\varphi)\right\}+$$

$$+\frac{4\pi}{b}\cdot\frac{c^2}{b^2}\sum_{n=0}^{\infty}\frac{1}{2n+1}\left\{E_nP_n(\cos u_T)P_n(\cos u_\varphi)+\right.$$

$$\left.+2\sum_{m=1}^{n}E_{nm}\frac{(n-m)!}{(n+m)!}P_{nm}(\cos u_T)P_{nm}(\cos u_\varphi)\cos m(\nu_T-\nu_\varphi)\right\}.$$

We get

$$K_{nm}A_{nm}=L_{nm}$$

where

$$K_{nm} = (n-1) - 2(\tfrac{a}{b}(1+p)-1) + (2n+1)\ ((\tfrac{\mathrm{b}}{\mathrm{a}})^{2m} - 1) +$$
$$+\tfrac{c^2}{b^2}\Big\{(2n-3)(\tfrac{(n-m-1)(n-m)}{2(2n-1)} - \tfrac{(n+m+1)(n+m+2)}{(2n+3)}) +$$
$$+(\tfrac{(n+m+1)(n+m+2)}{2n+3} - \tfrac{(n-m)(n^2-mn+m)}{n(2n-1)}) + E_{nm}\ \Big\},$$
$$L_{nm} = n + 2 - 2(1-(1+p)(\tfrac{a}{b})^{4m+1} + n((\tfrac{a}{b})^{4m} - 1) +$$
$$+\tfrac{c^2}{b^2}\Big\{n\ (\tfrac{(n-m)(n^2-mn+m)}{n(2n-1)} - \tfrac{(n+m+1)(n+m+2)}{2n+3}) + 2\ (\tfrac{(\mathrm{n-m-1})(\mathrm{n-m})}{2n-1} - \tfrac{(n+m+1)(n+m+2)}{2n+3}) - E_{nm}\ \Big\}.$$

The coefficient A_{n} cannot be determined by n=1. It can be expressed

$$A_{nm} = \tfrac{L_{nm}}{K_{nm}} = \tfrac{n+2}{n-1}\Big\{1 + 2p\tfrac{2n+1}{(n-1)(n+2)} + \tfrac{c^2}{b^2}\Big\{\tfrac{4mn^2+7mn+2n-2m+1}{(n-1)(n+2)} +$$
$$+\tfrac{(n-m)(n^2-mn+m)(2n+1)}{(n-1)(n+2)(2n-1)} - \tfrac{(n-m-1)(n-m)(2n^2-3n-2)}{2(n-1)(n+2)(2n-1)} +$$
$$-\tfrac{(n+m+1)(n+m+2)(2n+1)}{2(n-1)(2n+3)} - \tfrac{(2n+1)}{(n-1)(n+2)}E_{nm}\ \Big\}.$$

By excluding differences of great numbers we obtain

$$\varphi = \tfrac{1}{b}\sum_{\substack{n=0\\ n\neq 1}}^{\infty}\ \tfrac{n+2}{n-1}\Big\{1 + \tfrac{2n+1}{(n-1)(n+2)}(2p - \tfrac{c^2}{b^2}E_n) +$$
$$-\tfrac{1}{2}\tfrac{c^2}{b^2}\Big\{1 - \tfrac{4n+2}{(n-1)(n+2)} - \tfrac{3n^2+43n-22}{4(2n^3+n^2-5n+2)} - \tfrac{27n+45}{4(2n^2+n-3)}\ \Big\}P_n(\cos u_T)P_n(\cos u_\varphi)\ \Big\} +$$
$$+\tfrac{2}{b}\sum_{\substack{n=0\\ n\neq 1}}^{\infty}\ \tfrac{(n+2)}{(n-1)}\sum_{m=1}^{n}\ \Big\{1 + \tfrac{2n+1}{(n-1)(n+2)}(2p - \tfrac{c^2}{b^2}E_{nm}) +$$
$$-\tfrac{1}{2}\tfrac{c^2}{b^2}\Big\{1 - \tfrac{4n^2+6mn+2n+12m}{n(n^2+n-2)} - \tfrac{2m^2n^2+3n^3-24mn^2+22m^2n-36mn+43n^2-12m^2-22n+24m}{4n(2n^3+n^2-5n+2)} +$$
$$+\tfrac{6m^2n-27n^2-24mn-18m^2-45n-36m}{4n(2n^2+n-3)}\ \Big\}\tfrac{(n-m)!}{(n+m)!}P_{nm}(\cos u_T)P_{nm}(\cos u_\varphi)\cos m(\nu_T - \nu_\varphi)\ \Big\}.$$

Since

$$\tfrac{1}{r} = \tfrac{1}{b}\sum_{n=0}^{\infty}\ \Big\{\ P_n(\cos\psi) - \tfrac{1}{2}\tfrac{c^2}{b^2}(1 + \tfrac{1}{4(2n-1)} - \tfrac{1}{4(2n+3)})P_n(\cos u_T)P_n(\cos u_\varphi) +$$
$$-\tfrac{c^2}{b^2}\sum_{m=1}^{n}\ \tfrac{(n-m)!}{(n+m)!}P_{nm}(\cos u_T)P_{nm}(\cos u_\varphi)(1 - \tfrac{2m^2-n}{4n(2n-1)} - \tfrac{6m^2+n}{4n(2n+3)})\cos m(\nu_T - \nu_\varphi)\ \Big\}.$$

Green's function S_{sph} for Stokes' problem of the spheroidal Earth can be written

$$S_{\mathrm{sph}} = S(\cos\psi) - 1 + \tfrac{1}{b}\sum_{\substack{n=0\\ n\neq 1}}^{\infty}\ \Big[\ \tfrac{n+2}{n-1}\Big\{\tfrac{2n+1}{(n-1)(n+2)}(2p - \tfrac{c^2}{b^2}E_n) - \tfrac{1}{2}\tfrac{c^2}{b^2}\Big\{\ 1 - \tfrac{4n+2}{(n-1)(n+2)} +$$
$$-\tfrac{3n^2+43n-22}{4(2n^3+n^2-5n+2)} - \tfrac{27n+45}{4(2n^2+n-3)}\ \Big\}\ \Big\} - \tfrac{1}{2}\tfrac{c^2}{b^2}(1 + \tfrac{1}{4(2n-1)} - \tfrac{1}{4(2n+3)})\ \Big]P_n(\cos u_T)P_n(\cos u_\varphi) +$$

$$+\frac{2}{b}\sum_{\substack{n=0\\n\neq 1}}^{\infty}\sum_{m=1}^{n}\left\{\frac{n+2}{n-1}\left[\frac{2n+1}{(n-1)(n+2)}\left(2p-\frac{c^2}{b^2}E_{nm}\right)-\frac{1}{2}\frac{c^2}{b^2}\left[1-\frac{4n^2+6mn+2n+12m}{n(n^2+n-2)}+\right.\right.\right.$$

$$-\frac{2m^2n^2+3n^3-24mn^2+22m^2n-36mn+43n^2-12m^2-22n+24m}{4n(2n^3+n^2-5n+2)}+$$

$$\left.\left.+\frac{6m^2n-27n^2-24mn-18m^2-45n-36m}{4n(2n^2+n-3)}\right]\right]+$$

$$\left.-\frac{1}{2}\frac{c^2}{b^2}\left[1+\frac{n-2m^2}{4n(2n-1)}-\frac{n+6m^2}{4n(2n+3)}\right]\right\}\frac{(n-m)!}{(n+m)!}P_{nm}(\cos u_T)P_{nm}(\cos u_\varphi)\cos m(\nu_T-\nu_\varphi),$$

where $$S(\cos\psi)=\frac{1}{b}\sum_{n=2}^{\infty}\frac{2n+1}{n-1}P_n(\cos\psi).$$

If the gravity anomalies are expressed by

$$g-\gamma=\sum_{n=0}^{\infty}A_nP_n(\cos u)+\sum_{n=2}^{\infty}\sum_{m=1}^{\infty}\begin{matrix}A_{nm}\cos m\nu\\B_{nm}\sin m\nu\end{matrix}P_{nm}(\cos u),$$

their influence upon the disturbing potential is

$$\delta T=b\sum_{\substack{n=0\\n\neq 1}}^{\infty}\sum_{m=1}^{n}\begin{matrix}A_{nm}\cos m\nu\\B_{nm}\sin m\nu\end{matrix}\left[\frac{n+2}{n-1}\left\{\frac{1}{(n-1)(n+2)}\left(2p-\frac{c^2}{b^2}E_{nm}\right)-\frac{1}{2(2n+1)}\frac{c^2}{b^2}\left\{1-\frac{4n^2+6mn+2n+12m}{n(n^2+n-2)}+\right.\right.\right.$$

$$\left.\left.-\frac{2m^2n^2+3n^3-24mn^2+22m^2n-36mn+43n^2-12m^2-22n+24m}{4n(2n^3+n^2-5n+2)}+\frac{6m^2n-27n^2-24mn-18m^2-45n-36m}{4n(2n^2+n-3)}\right\}\right\}+$$

$$\left.-\frac{1}{2(2n+1)}\cdot\frac{c^2}{b^2}\left(1+\frac{n-2m^2}{4n(2n-1)}-\frac{n+6m^2}{4n(2n+3)}\right)\right]P_n(\cos u_T).$$

The terms of this expression decrease as 1: n. Some useful indications can be found at Thông and Grafarend (1989).

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