Geodetic science and the tools of the trade

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Abstract:

When asked to contribute to this Festschrift for my doctoral opponent and old friend Erik W. Grafarend, I ended up writing once again a philosophical little text, in which I consider the roles of mathematics, and of information technology, in geodesy and by extension in science in general.
I believe that science, to remain true to itself, should adhere to *conceptual transparency*, i.e., in everything the scientist does, he/she should strive to understand, and if necessary control, every step of the process that is relevant to producing the final scientific result. One such essential tool is mathematics; and happily, with his physics background, Prof. Grafarend has always been sharply aware of its importance and has wielded the tool with great flair.
In using mathematics as well as information technology, one great danger is "garbage-in, garbage-out". Throughout his professional career Prof. Grafarend has stressed the importance of *understanding what is going on*; posing the "*why?*" question, conceptual clarity, not automated garbage processing. The present contribution is written in this spirit.

1 Science and understanding

One of the things that distinguishes science from other fields of human endeavour, is an almost pathological preoccupation with the question "*why?*", and with ways to answer it. Sometimes this inclination borders on the ridiculous; where for the vast majority of people it is quite sufficient to have a common sense confirmation that something is so or "just works", without any deep understanding of what makes this so, this will never do for a scientist.
The remarkable thing is, that one can actually lead a full and rewarding life without ever asking the *why* question. Nevertheless, the fact that this is so is to a large extent the achievement of a fairly small number of exceptional individuals that knew to ask this question, at the right time, in the right context. All of our science and technology, and our prosperity and good life, is built upon this.
As experience has shown, knowing what you're doing, understanding what you're doing and why, can be worth gold and is a hard requirement if you want to do something properly. Industry and government have learned this lesson; that's why in all developed countries, research and development are rightly considered activities vital to society.
As scientists, we have a duty to remain loyal to this core element of our tradition. Understanding matters.
In science, we can distinguish three stages of activity:

1. Observation, measurement, data collection

2. Data processing, analysis, theory building

3. Dissemination of results, processed data, publication.

The requirement of knowing what you are doing, of *conceptual transparency*, should be applied to all three of these stages. A scientist should have a fundamental understanding and working knowledge of all the tools of his or her trade.

2 The role of mathematics

One important tool in the exact sciences, of course, is mathematics; mathematically formulating a scientific finding or argument gives it an authoritativeness that is much envied by practitioners of those fields of scientific study where it is inappropriate, or not feasible, to use it. But where it is both appropriate and feasible, the use of the language of mathematics conveys a tractability to the argument which allows anyone with a command of this tool to absorb the argument on a level of detail and exactness that cannot be achieved without it. And surprisingly but fortunately, the physical universe seems to be very friendly to mathematical description.

2.1 Scaling laws in biology

As an illustration of the use of mathematics in science, I want to refer to a presentation given by Geoffrey West at the recent 22nd Conference on Mathematical Geophysics in Cambridge UK [6]. He presented his ideas on the scaling laws of river systems, starting from the scaling laws that he had found in biology, e.g., that the metabolic rate is proportional to body weight to the power 3/4 for a very wide range of organisms. According to his studies, the physical mechanism producing this law is simply the supply of some vital resource (food, oxygen, river water) to/from all the nooks and crannies of the system sustained, dissipating minimum energy in transport.

2.2 Scaling laws for GPS network densifications

I decided to try if it would be possible to similarly model the propagation of the resource "co-ordinate precision" through a cascade of geodetic GPS network densifications to the end points established, using minimal total station occupation time.
We cover an area S with GPS stations in a hierarchical arrangement; the hierarchy level we call ω, $\omega = 1, 2, \ldots, \Omega$. Then, when the "branching number" for a given level ω is n_ω, starting from a single level-zero station, the total number of stations is:

$$N = \prod_{\omega=1}^{\Omega} n_\omega .$$

The mean distance between stations up to a level ω, assuming, e.g., a rectangular arrangement of stations and leaving off any irrelevant constant factors:

$$l_\omega = \sqrt{S} \left[\prod_{\omega'=1}^{\omega} n_{\omega'} \right]^{-\frac{1}{2}} .$$

We postulate the measurement variance to depend on both baseline length and occupation time in the following way:

$$\sigma_\omega^2 = C\, l_\omega^p\, t_\omega^q ,$$

where t_ω is occupation time and C a proportionality constant. For the assumption of temporally uncorrelated noise, $q = -1$. For comparison, is known that for levelling networks, $p = +1$, which might also apply to GPS networks (Bernese rule-of-thumb, [3]).

We compute the total measurement variance (assuming uncorrelated hierarchy levels ω) as

$$\sigma^2 = \sum_{\omega=1}^{\Omega} \sigma_\omega^2 = \sum_{\omega=1}^{\Omega} C\, l_\omega^p\, t_\omega^q =$$
$$= \sum_{\omega=1}^{\Omega} C S^{\frac{p}{2}} \left[\prod_{\omega'=1}^{\omega} n_{\omega'}\right]^{-\frac{p}{2}} t_\omega^q\,. \tag{1}$$

Now, defining

$$K \equiv C S^{\frac{p}{2}}$$

and

$$N_\omega \equiv \prod_{\omega'=1}^{\omega} n_{\omega'}\,,$$

we may write for the total measurement receiver time:

$$\tau = \sum_{\omega=1}^{\Omega} \left[\prod_{\omega'=1}^{\omega} n_{\omega'}\right] t_\omega = \sum_{\omega=1}^{\Omega} \tau_\omega\,, \tag{2}$$

with

$$\tau_\omega \equiv \left[\prod_{\omega'=1}^{\omega} n_{\omega'}\right] t_\omega = N_\omega\, t_\omega.$$

Substitute this into Eq. (1):

$$\sigma^2 = \sum_{\omega=1}^{\Omega} K\, N_\omega^{-\frac{p}{2}} \cdot N_\omega^{-q} \tau_\omega^q =$$
$$= \sum_{\omega=1}^{\Omega} K N_\omega^{-\frac{p}{2}-q} \cdot \tau_\omega^q\,.$$

Now *optimize*, i.e., minimize, σ^2, given τ by Eq. (2). Lagrange multipliers: find the point where the gradient of $\sigma^2(\tau_1, \tau_2, \ldots, \tau_\Omega)$ stands perpendicular to the surface $\sum \tau_\omega = \tau$. I.e., demand:

$$\frac{\partial\left(\sigma^2\right)}{\partial \tau_\omega} = qKN_\omega^{-\left(\frac{p}{2}+q\right)} \tau_\omega^{q-1} = \lambda, \quad \omega = 1, 2, \ldots, \Omega.$$

It follows that

$$\tau_\omega^{q-1} = \frac{\lambda}{qK} N_\omega^{\left(\frac{p}{2}+q\right)} \quad \Rightarrow \quad \tau_\omega = \kappa N_\omega^{\left(\frac{\frac{p}{2}+q}{q-1}\right)}\,,$$

where

$$\kappa \equiv \left[\frac{\lambda}{qK}\right]^{\frac{1}{q-1}}$$

is another constant. So:

$$\tau_\omega = \kappa N_\omega^{\frac{\frac{p}{2}+q}{q-1}}, \tag{3}$$

with the above definition of N_ω, and

$$t_\omega = N_\omega^{-1} \tau_\omega = \kappa N_\omega^{\frac{\frac{p}{2}+q}{q-1}-1}\,. \tag{4}$$

τ_ω is the *total instrument-time* of a certain stage ω in the network densification. t_ω is the duration of *one measurement* at stage ω. N_ω is the total number of stations up to stage ω.

2.3 Example

Let $p = +1$, $q = -1$, $n_\omega = 16$, so $N_\omega = 16^\omega$. Then, using Eqs. (3) and (4):

$$\frac{\frac{p}{2}+q}{q-1} = \frac{-\frac{1}{2}}{-2} = \frac{1}{4}$$

$$\frac{\frac{p}{2}+q}{q-1} - 1 = \frac{-\frac{1}{2}}{-2} - 1 = -\frac{3}{4}.$$

And we find:

ω	$\tau_\omega = \kappa \cdot 16^{\omega/4}$	$t_\omega = \kappa \cdot 16^{-3\omega/4}$
1	2κ	$\kappa/8$
2	4κ	$\kappa/64$
3	8κ	$\kappa/512$
4	16κ	$\kappa/4096$
$\tau =$	30κ	

Applying this to the Finnish permanent GPS network FinnRef™, and the Finnish EUREF densification campaigns, yields:

Level	ω	N_ω	n_ω	$\tau_\omega^{\text{theor}}$	t_ω^{theor}	t_ω^{pract}	l_ω
Permanent network	1	12	12	1.8κ	0.15κ	"∞"	200 km
EUREF densification	2	100	8	3.0κ	0.03κ	48^{h}	70 km
"Project 400"	3	400	4	4.5κ	0.01κ	4^{h}	35 km
Total receiver-time				9.3κ			

We see that in practice, the time spent on the second densification phase (Project 400) was quite a bit shorter than theory requires. This was for practical reasons: limited resources available and the time constraints of moving from point to point within a working day.
Of course also, the above assumed p and q values are probably not quite realistic. More likely, p lies between $+1$ and $+2$, while q is almost certainly larger than -1 in the occupation time range considered.

2.4 Discussion

The scaling law derived above under severely simplifying assumptions of course formulates in an unconventional way existing, conventional wisdom that geodesists have always been aware of, occasionally formulated explicitly, and the validity of which which the GPS age has in no way diminished:

> A network should be built up hierarchically, moving from the large to the small scale — the "from large to small" paradigm.

This has relevance for the discussion on the possibility of short-circuiting the hierarchical process by providing precise geodetic positions without the intermediation of network layers, even in real time by the RTK (real time kinematic) positioning technique. In some countries, nation wide services are being set up based on the existing geodetic permanent GPS infrastructures.
It would appear from the above that the logistics of such a set-up may be unnecessarily heavy and better – both optimally accurate and affordable – results are almost always guaranteed using the simplest, most cost effective approach of setting up your own local reference receiver and radio link within the area of study.
Our Finnish EUREF densification project will provide a good starting point for this sensible practice.

3 The role of information technology

And now for something entirely different! We recall our division of the scientific process into three phases, cf. Section 1. Modern information technology has relevance to all three of these phases. In the following, we will shortly consider the issue mainly from the point of view of GPS geodesy, with which the author is most familiar.

3.1 Data collection

Data is collected by GPS receivers in the field, permanently mounted in fixed locations, and transmitted to a data collection centre at regular intervals. In Finland, the Finnish Geodetic Institute operates twelve such receivers, which send their collected data at 24 hour intervals by modem and dial-up phone line to the data centre in Masala, close to Helsinki. This permanent network is multi-purpose, serving, e.g., the study of Earth crustal motions, and providing a basis for the establishment and maintenance of a precise national mapping datum.

Many other countries around the world have such networks, which only differ in the technical details of their operation.

These data collection systems, which operate largely autonomously, have of course to be *reliable*. Running this kind of service reliably is a seriously nontrivial operation. From our own experience, it is highly desirable to be in control of all the elements of the system. The fact that in our current system most of the software running it, including the operating system (!) of the download server, is closed commercial software of which we do not have the source code – a shortage of *conceptual transparency* –, has on occasion complicated our task.

3.2 Analysis

For scientific analysis of GPS data, in case one is interested in the highest precision and in results that hold water scientifically, one should preferably use GPS processing software that is documented by its source code. There are several such packages circulating in the GPS geodetic community; we use the Bernese software package, produced by the Astronomical Institute of the University of Berne. This kind of software is typically written by users who are themselves scientists and publish in the scientific literature.

Having the source is important even if one does not have the time or the inclination to read it. Some people do, and any discrepancy between documented and actual behaviour does not remain invisible for long. As an illustration, one of our researchers (Matti Ollikainen, personal comm.) found a bug in the tropospheric mapping function code.

Of course using commercial processing software, which hides the true complexity of GPS processing from the user, takes its own, presumably intelligent, policy decisions and offers superior ease of use, is a good solution in many production situations. But it isn't necessarily the best if you want your scientific results to meet certain standards of tractability.

3.3 Visualization, presentation and publication

It should not be forgotten that also publishing results, for colleagues and possibly the public at large to read, is an important part of the scientific process. Valid results are significantly more useful if published well, and may even go unread if published badly.

Besides the issues of producing well written language and of publishing in the proper place, there is the important issue of the format of publication.

One thing information technology has done is make publication in electronic form not just possible, but easy. As a minimum one can post abstracts of paper publications on the World Wide Web. This circumstance rewards the use of authoring systems that allow for export of documents alternatively in Web or paper print form, or for easy conversion between formats. Besides the up-and-coming XML format, the traditionally used scientific authoring tools fortunately do these things well, better in fact than currently popular word processors.

3.3.1 Web publication

Information technology enables publishing on the World Wide Web using hypertext mark-up language. As readers familiar with the www know, there is a vast amount of poor format practice going on there. Web sites that are chaotically organized, cluttered, loud, containing hundreds of graphics where text would do fine, not offering textual alternative representations, using tables for formatting non-tabular material, and, last but not least, non-standard, browser-dependent features. Such sites are a nightmare to access with a plain text browser (Lynx) or, e.g., an auditory browser used by people of poor eyesight (cf. [2]).
The principles for a good scientific Web publication are no different from those for a good general Web information page: *don't do* any of the above. Much of the messiness of many www pages appears to be due to the tools used to create them, usually visual mark-up editors that give you full control of what the page will look like (well, in Browser X anyway), but *not of what goes into it.* While visually interactive (WYSIWYG) editors have their place in website design, they all too often illustrate the saying "What You See Is *All* You Get".
It is wise to learn a little HTML syntax, look at the pages produced by your favourite package in a text editor, and clean them up if necessary. And keep it simple. And in the near future, consider XML, the Extensible Mark-up Language, which Gecko, Netscape's new browser engine, will render in a standards-compliant way. Gecko exists also for Internet Explorer, as a plug-in. XML and its document type definitions are a superset of HTML, offering much improved facilities for producing content-structured documents.

3.3.2 Paper publication

Information technology has, perhaps unexpectedly, great relevance for paper publication as well. Excellent tools existed already a decade before personal computers became ubiquitous and word processing software useable by untrained personnel permanently lowered our quality expectations from computer typesetting, cf. [5]. A good looking paper publication is a joy to the eye and easy to read and the likelihood of the message coming across is greatest if the visual structure supports the logic of discourse. And of course, errors tend to distract and irritate the reader, planting doubts as to the reliability of the writer as well.
To produce a good looking paper, one uses a professional typesetting software system with well designed fonts. To easily produce a well structured, error free document having unity of form and function in the visual layout, one uses a professional document processing system. Both feature sets are offered by the LaTeX software [4]. True, it requires the discipline of learning its use, but – we scientists are supposed to be good at learning new things, it comes with the territory. And nowadays there are excellent, easy to use visual word processing tools available for LaTeX [1], removing the learning treshold and facilitating interoperation with non-scientists.
To produce a correct reference listing, one uses BibTeX, which extracts references used from a bibliographic database. There is no excuse for scientists not to use BibTeX; if the database is error free, the reference list will be too, guaranteed. And again, excellent visual tools exist for managing this database.

3.4 Real time systems

One more effect of IT on the scientific process may be the "telescoping" of the three phases listed in Section 1 into one continuous process. Data is collected, analysed in real time, and the results of the analysis visualised and presented to the end user also in real time.
Such systems are already in use in some places; GPS data collected and analysed in real time allows the presentation of animations of, e.g. crustal motion, ionospheric total electron contents,

or troposheric total water vapour contents on a publically accessible web page. We must expect such practices to become more widespread, as the underlying technology becomes more affordable, reliable and easy to use. But it is important to realize that the requirement of knowing what you're doing, of conceptual transparency, continues to apply to all the elements of such a system. There is no royal road to scientific knowledge, and "data" is not "understanding".

4 Conclusions

We leave those as an exercise for the reader.

References

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