Triple frequency GPS for precise positioning

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Abstract

In this paper we dwell upon the possibility to determine the GPS phase ambiguities from double difference triple frequency GPS phase and code data more or less instantaneously. We take advantage of the well known fact, that the widelane ambiguity is easily fixed from such observables. It is shown that this holds also for a third signal of carrier wavelength (λ_3) in the range 14.3 cm $\leq \lambda_3 \leq$ 30.0 cm. At the limits λ_3 = 14.3 cm (2100.6 MHz) and λ_3 = 30.0 cm (997.1 MHz) the base ambiguities are easily fixed as soon as the widelane ambiguities have been determined. For other choices of λ_3 the method is less optimal.

Recently the US Department of Defence announced that GPS satellites launched after December 2004 will be equipped with three civil GPS signals, where L_1 and L_2 are the same as today's signals, and the third signal will operate in the frequency 1176.45 MHz (λ_3=25.44 cm). This design will allow rapid precise position over long baselines with significant ionospher biases.

Key-words: Ambiguity, GPS, triple frequency.

1 Introduction

Precise positioning with the Global Positioning System is related with reliable fixing of the signal phase ambiguities. The success in real time positioning is dependent on the fast ambiguity fixing. Various methods have been developed for fast and reliable ambiguity estimation over short baselines, but for long baselines the needed time to fix ambiguities increases drastically due to the influence of various systematic effects, in particular the ionosphere bias. However, one well-known exception is the widelane ambiguity, which can be quickly determined also for long baselines from a linear combination of phase and code data. Sjöberg (1996) and (1997, 1998, 1999) took advantage of this fact to solve for the base ambiguities. It turned out (Sjöberg,1998, 1999 and Almgren, 1998) that this method works very well for short baselines, but mainly the ionosphere effect restricts its success for long baselines.

Recently the US Department of Defence (DoD) has announced, that it plans to introduce a second civil frequency identical with the current L_2 frequency, and later, after December 2004, a third civil frequency at 1176.45 MHz (λ_3=25.44 cm) is planned to operate on all new GPS satellites.

The goal of this paper is to take advantage of all three signals for fast phase ambiguity resolution for short as well as long baselines. First we speculate on the optimum choice of the third frequency with respect to accurate and reliable ambiguity resolution. Second, we compare the result with the proposed frequency.

2 Ambiguity estimation for dual frequency data

Consider the following dual frequency phase and code observation equations for receiver-to-satellite ranges (Sjöberg 1996, 1997, 1998)

$$\left.\begin{array}{l} \tilde{l}_1 = \tilde{\phi}_1\lambda_1 = u + \lambda_1 N_1 - \frac{\mu}{f_1^2} + \epsilon_{11} \\ \tilde{l}_2 = \tilde{\phi}_2\lambda_2 = u + \lambda_2 N_2 - \frac{\mu}{f_2^2} + \epsilon_{12} \\ \tilde{l}_3 = \tilde{R}_1 = u + \frac{\mu}{f_1^2} + \epsilon_{21} \\ \tilde{l}_4 = \tilde{R}_2 = u + \frac{\mu}{f_2^2} + \epsilon_{22}, \end{array}\right\} \tag{1}$$

where $\tilde{l}_1$ and $\tilde{l}_2$ (with phase $\tilde{\phi}_1$ and $\tilde{\phi}_2$) are the phase observables scaled by their wavelengths λ_1 and λ_2, and $\tilde{l}_3$ and $\tilde{l}_4$ are the code observables. ϵ_{11}, ϵ_{12}, ϵ_{21} and ϵ_{22} are random observation errors. The unknowns are $u = \rho + c\Delta\delta$, which is the sum of the satellite-to-receiver range (ρ) and the product of velocity of light (c) and receiver and satellite clock bias difference ($\Delta\delta$). Furthermore μ is the ionosphere bias and N_1 and N_2 are the phase ambiguities on L_1 and L_2, respectively, with the known frequencies f_1 and f_2. Usually we will consider these equations for double differenced data, i.e. for pairs of receivers and satellites. Obviously eqn. (1) contains 4 independent equations and 4 unknowns, and, at least in principle, it can be directly solved for N_1 and N_2. The problem is, however, that these estimates are too poor to be useful (Sjöberg ibid.). On the contrary the widelane ambiguity can be accurately determined by

$$\hat{N}_w = \hat{N}_1 - \hat{N}_2 = \frac{\tilde{l}_1}{\lambda_1} - \frac{\tilde{l}_2}{\lambda_2} - \frac{f_1 - f_2}{f_1 + f_2}\left(\frac{\tilde{l}_3}{\lambda_1} + \frac{\tilde{l}_4}{\lambda_2}\right) \tag{2}$$

Subsequently $\hat{N}_w$ is independent of baseline length, ionosphere bias (and the time variable satellite-to-receiver range), and in most cases it can quickly be fixed to its correct integer value.

Having fixed N_w we may form an observation equation for the base ambiguity N_1:

$$N_1 = N_w + \hat{N}_2 - \epsilon_2, \tag{3}$$

where $\hat{N}_2$ is the primary estimate of N_2 from eqn. (1), and ϵ_2 is its random error. Another equation of N_1 is given by

$$N_1 = \hat{N}_1 - \epsilon_1, \tag{4}$$

i.e. by its primary estimate $\hat{N}_1$ from eqn. (1) with error ϵ_1. The errors ϵ_1 and ϵ_2 are very significantly correlated. Denoting the covariance matrix between the above two equations Q the least squares solution for N_1 becomes

$$\hat{\hat{N}}_1 = \left(e^T Q^{-1} e\right)^{-1} e^T Q^{-1} \begin{pmatrix} \hat{N}_1 \\ \hat{N}_2 + N_w \end{pmatrix}, \tag{5}$$

where

$$e^T = (1, 1).$$

It turns out (Sjöberg 1996 and 1998) that the base ambiguity is well determined by this method for short baselines, where the ionosphere bias (μ) can be omitted from the model (1). This conclusion is confirmed by the numerical analyses of Almgren (1998). On the other hand the result is very pessimistic for long baselines, due to the fact that the ionosphere bias prevent the nice error reduction of the joint solution as was the case for short baselines.

The above solution (5) could also be obtained more directly from the original equations (1) after the substitution of the second equation by

$$\tilde{l}_2' = \tilde{l}_2 + \lambda_2 N_w = u + \lambda_2 N_1 - \frac{\mu}{f_2^2} + \epsilon_{12}. \tag{6}$$

This means that we have reduced the number of unknowns to three by using the known widelane ambiguity. The least squares solution of the revised eqn. (1) is the same as in (5).

3 Ambiguity estimation for triple frequency data

Assuming that there are three independent GPS signals L_1, L_2 and L_3 with carrier wavelengths λ_1, λ_2 and λ_3 and frequencies f_1, f_2 and f_3, we can form 6 independent observation equations similar to the system (1):

$$\left.\begin{aligned}
\tilde{l}_1 &= u + \lambda_1 N_1 - \tfrac{\mu}{f_1^2} + \epsilon_{11}\\
\tilde{l}_2 &= u + \lambda_2 N_2 - \tfrac{\mu}{f_2^2} + \epsilon_{12}\\
\tilde{l}_3 &= u + \lambda_3 N_3 - \tfrac{\mu}{f_3^2} + \epsilon_{13}\\
\tilde{l}_4 &= u + \tfrac{\mu}{f_1^2} + \epsilon_{21}\\
\tilde{l}_5 &= u + \tfrac{\mu}{f_2^2} + \epsilon_{22}\\
\tilde{l}_6 &= u + \tfrac{\mu}{f_3^2} + \epsilon_{23}.
\end{aligned}\right\} \tag{7}$$

From these observables we may estimate the following widelane ambiguities

$$N_{w12} = N_1 - N_2 = \frac{\tilde{l}_1}{\lambda_1} - \frac{\tilde{l}_2}{\lambda_2} - \frac{f_1 - f_2}{f_1 + f_2}\left(\frac{\tilde{l}_4}{\lambda_1} + \frac{\tilde{l}_5}{\lambda_2}\right) \tag{8}$$

and

$$N_{w13} = N_1 - N_3 = \frac{\tilde{l}_1}{\lambda_1} - \frac{\tilde{l}_3}{\lambda_3} - \frac{f_1 - f_3}{f_1 + f_3}\left(\frac{\tilde{l}_4}{\lambda_1} + \frac{\tilde{l}_6}{\lambda_3}\right). \tag{9}$$

For f_3 chosen rather close to f_1 the small factors $(f_1 - f_2)/(f_1 + f_2)$ and $(f_1 - f_3)/(f_1 + f_3)$ efficiently reduce the noise of the code observables $\tilde{l}_4$, $\tilde{l}_5$ and $\tilde{l}_6$. This explains the resulting low noise in the widelane ambiguity.

Let us now insert eqs. (8) and (9) into the second and third equations of (7). This yields the revised system of equations (in matrix form and altered order).

$$\begin{pmatrix} 1 & 1 & 0\\ 1 & \nu^2 & 0\\ 1 & \alpha^2 & 0\\ 1 & -1 & 1\\ 1 & -\nu^2 & \nu\\ 1 & -\alpha^2 & \alpha \end{pmatrix} \begin{pmatrix} u\\ \mu/f_1^2\\ \lambda_1 N_1 \end{pmatrix} = \begin{pmatrix} \tilde{l}_4 - \epsilon_{21}\\ \tilde{l}_5 - \epsilon_{22}\\ \tilde{l}_6 - \epsilon_{23}\\ \tilde{l}_1 - \epsilon_{11}\\ \tilde{l}_2 + \lambda_2 N_{w12} - \epsilon_{12}\\ \tilde{l}_3 + \lambda_3 N_{w13} - \epsilon_{13} \end{pmatrix}, \tag{10}$$

where

$$\nu = f_1/f_2 = \lambda_2/\lambda_1$$

and

$$\alpha = f_1/f_3 = \lambda_3/\lambda_1.$$

The system (10) is over determined with 3 redundancies. However, the first 3 observations, from the pseudoranges, have much lower accuracy than the last three ones of phase observables. (The ratio of the standard errors is of the order 100/1.) Subsequently, the code observables adds very little to the least squares solution of the system (10). Neglecting these equations we are left with the system

$$AX = L - \epsilon,$$

where

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -\nu^2 & \nu \\ 1 & -\alpha^2 & \alpha \end{pmatrix} \qquad X = \begin{pmatrix} u \\ \mu/f_1^2 \\ \lambda_1 N_1 \end{pmatrix} \tag{11}$$

and $L - \epsilon$ is the vector of the last three equations of (10). The system (10) corresponds to the normal equations

$$A^T A X = A^T L \tag{12}$$

with the unique solution

$$\hat{X} = \left(A^T A\right)^{-1} A^T L \tag{13}$$

and the covariance matrix of $\hat{X}$

$$Q_{\hat{X}\hat{X}} = \sigma_0^2 \left(A^T A\right)^{-1}, \tag{14}$$

where σ_0^2 is the variance of unit weight. In this study we are particularly interested in the standard error of the estimated base ambiguity N_1, which is included in (14). More precisely it reads

$$\sigma_{\hat{N}_1} = \frac{\sigma_0}{\lambda_1} \left(A^T A\right)_{33}^{-\frac{1}{2}}. \tag{15}$$

¿From the matrix A given by (11) one easily obtains

$$\sigma_{\hat{N}_1} = \frac{\sigma_0}{\lambda_1} \sqrt{\frac{c}{d}}, \tag{16}$$

where

$$c = 2\left(1 + \nu^4 + \alpha^4 - \alpha^2 - \nu^2 - \alpha^2\nu^2\right)$$

and

$$\begin{aligned} d = {} & 3\left(1 + \nu^4 + \alpha^4\right)\left(1 + \alpha^2 + \nu^2\right) + 2\left(1 + \nu + \alpha\right)\left(1 + \alpha^2 + \nu^2\right)\left(1 + \alpha^3 + \nu^3\right) \\ & - \left(1 + \nu + \alpha\right)^2\left(1 + \alpha^4 + \nu^4\right) - 3\left(1 + \alpha^3 + \nu^3\right)^2 - \left(1 + \alpha^2 + \nu^2\right)^3. \end{aligned}$$

For modern geodetic GPS receivers the carrier phase observable noise (σ_0) can be set to 3 mm. The L_1 carrier wavelength (λ_1) is 19.0 cm and $\nu = f_1/f_2 = \lambda_2/\lambda_1 = 24.4/19.0 = 1.28421$. Also $f_1 = 1575.42$ MHz and $f_2 = 1227.60$ MHz. For these constants $\sigma_{\hat{N}_1}$ is given as a function of $\alpha(= f_1/f_3 = \lambda_3/\lambda_1)$ in Fig 1 (dashed curve). It shows that the standard error increases dramatically for $\alpha = 1 (f_1 = f_3)$ and $\alpha = \nu (f_2 = f_3)$. For, say, $\alpha < 0.8$ and $\alpha > 1.5 \sigma_{\hat{N}_1}$ is well below unity. The figure shows also (solid line) the standard error of the widelane ambiguity determined from the L_1 and L_3 signals given by the (approximate) formula

$$\sigma_{\hat{N}_{w13}} = \frac{\sigma_R}{\lambda_1} \frac{|1-\alpha|}{1+\alpha} \left(1 + \frac{1}{\alpha}\right) \tag{17}$$

with α_R set to 30.0 cm. This curve has a minimum (0) for $\alpha = 1$. For α close to 0.75 and 1.58 both $\sigma_{\hat{N}_1}$ and $\sigma_{\hat{N}_{w13}}$ are close to 0.6 as presented in Table 1. Obviously, the optimum choice of α can be found for these values.

The L_3 signal proposed by DoD of 1146.45 MHz (λ_3=25.44 cm) to some extent fulfills the above demands. For this frequency α becomes 1.339, yielding the standard errors of N_{w13} and N_1 less than 0.5 and about 2.5, respectively. (Cf. Fig. 1.)

Table 1: Optimum choices of L_3 and corresponding standard errors of N_1 and N_{w13}

α	λ_3(cm)	f_3	$\sigma_{\hat{N}_1}$	$\sigma_{\hat{N}_{w13}}$
0.75	14.3	2100.6 MHz	0.56	0.53
1.58	30.0	997.1 MHz	0.59	0.58

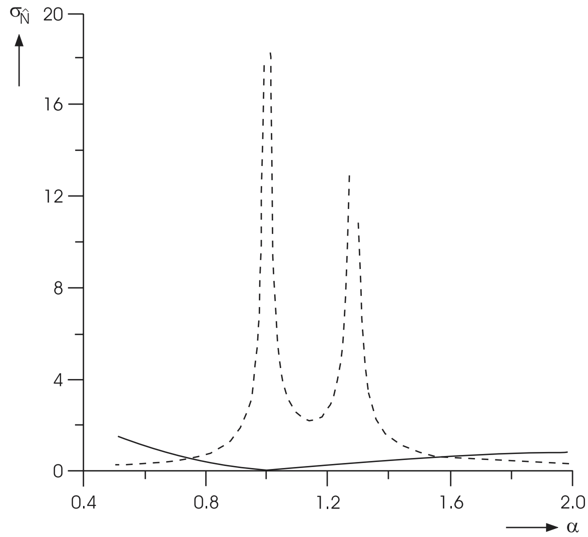

Figure 1: The figure shows the standard errors of the base ambiguity N_1 (dashed line) and the widelane ambiguity N_{w13} (solid line) as functions of α. Constants: $\sigma_R = 30.0$ cm, $\sigma_0 = 3$ mm, $\lambda_1 = 19.0$ cm.

4 Conclusions

We conclude that our method to resolve GPS phase ambiguities from double difference phase and code data with three GPS signals will be optimal for a third signal of frequency of about 997 MHz or 2101 MHz, at which frequencies both the widelane and base phase ambiguities are quickly resolved. Obviously this method solves the problem caused by the ionosphere bias for long baselines. The L_3 frequency (1146.5 MHz) proposed by DoD will be a good tool in solving this problem.

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