Reproducing Estimators via Least Squares: An Optimal Alternative to the Helmert Transformation

— dedicated to Erik Grafarend on his 60th birthday —

Burkhard Schaffrin

Abstract

In the tradition of several joint papers by the honoree and myself, including E. Grafarend and B. Schaffrin (1974, 1976, 1988) and E. Grafarend, B. Schaffrin, and E. Knickmeyer (1982), I took up the subject again in B. Schaffrin (1984, 1989), N. Tamim and B. Schaffrin (1995) and B. Schaffrin and J. Cothren (1998), now with the emphasis on network densification methods which are rigorous, but which leave tie points unchanged along with their variances and covariances. Such a requirement led to the notion of "reproducing estimators" as first presented by B. Schaffrin (1997).

It has been known for a long time that the so-called "dynamic adjustment" of geodetic networks can be interpreted as least-squares collocation, according to H. J. Buiten (1978), which would not yield estimates with the "reproducing property." Also the "free net adjustment" followed by a Helmert transformation would not per se guarantee identical tie point coordinate estimates unless we "forget" about the deviations whose norm we had minimized, but usually not to zero. So the only "reproducing" alternative seemed to be given by the formulas of a classical least-squares adjustment with fixed constraint even though the tie points are "fiducial" in their nature, having typically a full dispersion matrix. Therefore, in the formula for the corresponding variance-covariance matrix we find an additional term which vanishes as soon as the "fiducial" points become real "fixed" points; see, for instance, B. Schaffrin (1984, 1989). This apparently has been the method, applied by JPL over the years, when they talked about "fiducial network strategies" as in S. M. Lichten et al. (1989), for instance, without spelling out exactly what they did.
In any case, as could have been expected, the latter one was shown not to be the optimal estimator with the "reproducing property" by B. Schaffrin (1997). Instead, the optimum turned out to be the "forgetful dynamic adjustment" where every densification point is found conventionally through least-squares collocation, but the tie points remain unchanged. Consequently, a free adjustment followed by a "forgetful Helmert transformation" must be considered inferior and may no longer be carried out routinely; see also K. R. Koch (1983) and F. W. O. Aduol (1993). Here we try to employ least-squares methodology to generate that "reproducing estimator" which, as we already know, will be optimal in this class.

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