Datum Accuracy and its Dependence on Network Geometry

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ABSTRACT

Free networks are still very popular in geodesy, surveying and mapping due mainly to their unique property of being independent of errors in external data (fixed higher order control).
The results of free net adjustment and their superior quality serve as an authentic reflection (undistorted mirror image) of the measurements' quality. The above property is particularly important today when we employ GPS measurements for densification of conventional control, where the later is usually of a relatively lower quality. Notwithstanding the optimal properties of free networks (including GPS networks as well), we examine in this paper the relationship between network geometry and the positional accuracy of its points. In particular we study the "accuracy as a function of geometry" equation as related to datum definition of a network by free net adjustment constraints. The paper presents new concepts and describes relevant parameters that are capable of defining and quantifying the datum accuracy of a free network. Following the development of conceptual and theoretical tools, the paper presents results of numerical experiments carried out with a schematic 2-D (horizontal) GPS network.
The results lead to a somewhat surprising (although intuitively acceptable) conclusion: "Positional accuracy of points in a free network depends heavily on the spatial distribution of its datum-definition points". The above dependence on geometry is in excess to the well-known and trivial property of networks where positional accuracy of a point is inversely proportional to its distance from the mass center of the datum-points' sub-net.

1. Introduction. Datum definition in free networks.

This paper deals with the initial stages in the complex process of establishing a geodetic control network. According to Grafarend (1974), at the "zero-order-design" stage the datum of the network has to be defined. In case of a densification net, as part of the hierarchical structure, datum is defined by the "higher order" control points i.e. their respective coordinates which remain fixed throughout the adjustment of the measurements. In this case there is little or no room for design variations. All the higher order points that can be accessed are used. It is wise to look for possible inconsistencies and hope, at the same time, that none would be found.

In case of an independent, "free" or "floating" network, datum is defined by the selection of a subset of its points. The selection of those datum-points is subjective and depends on a number of criteria such as, for example, the destination of the network, the specific type of measurements, the geotechnical properties of the ground, monumentation characteristics and many more. In the following sections we investigate the geometry of the datum-points subset and its possible effect on point-position accuracy in the network.

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We should point out that in this paper the overall geometry of the network is taken for granted. There are hardly any useful analytical tools that have been developed over the years to solve satisfactorily Grafarend's (1974) "first-order-design" problem of a control network. Even if such tools were available, their practical application is in most cases problematic due to the inevitable constraints imposed by other trivial but also rather important considerations such as topography, logistics, cost, speed etc.

Let us have a network of k points in s-dimensional space (s=1,2,3). We seek an estimate of the k.s coordinates of those k points including their respective accuracies (the k.s by k.s covariance matrix of the coordinates). We note that the coordinates and their accuracies are subjective quantities as they depend heavily on the particular reference system (datum) which has been selected. The selection of a particular datum is accomplished as follows:
Out of the total of k network points, m points ($m \leq k$) are selected and declared as a datum-base subset. A datum definition weight matrix P_m is formed which elaborates the above selection by assigning to each of the m datum-points an appropriate weight. In most cases P_m is a diagonal matrix with at least d non-zero (usually positive) elements. The number d is the so-called datum defect of the free network. It depends on s (dimensionality) and on the specific type of the measurements that are being used for constructing the network. As an example: for GPS vectors "measured" in 3-D the datum defect is d=3 (origin of the reference system); for GPS vectors which have been reduced to horizontal distances (s=2) the datum defect in 2-D is d=3 (origin-2 and orientation -1 of the reference system). The approximate coordinates of the datum points are, in general, inconsistent with the correct relative positions. In spite of this inconsistency the approximate coordinates of the m datum-points together with the P_m matrix serve as a data basis for datum definition of the network [See Papo&Perelmuter, 1981].

The mathematical model of the n measurements is linearized at the approximate values of the coordinates (vector X^o) and produces the set of n observation equations:

$$L + V = A \, . \, X \tag{1.1}$$

where

X is the vector (k.s) of coordinate corrections.
A is the matrix (n by k.s) of partial derivatives.
L is the vector (n) of differences between observed and approximate measurements.

The basis of null space of A [See Papo, 1987] known also as the Helmert's transformation matrix E is a k.s by d matrix with the following important property.

$$A.E = 0 \tag{1.2}$$

Datum based on the selected m subset is defined by subjecting the vector X in (1.1) to the following linear constraints [See Papo&Perelmuter, 1981; Papo, 1989].

$$E^T P_m X = 0 = E_m^T \; X \tag{1.3}$$

where

$$E_m = P_m \, . \, E$$

We will see later that (1.3) is derived from seeking a minimum of the following quadratic form:

$$X^T P_m X = \min. \tag{1.4}$$

But first we minimize the $V^T PV$ quadratic from in order to create the set of normal equations:

$$NX = U \tag{1.5}$$

The above set is singular due to the need for datum definition. It can be solved only if we apply on X at least d independent linear constraints. If we apply (1.3), for example, to (1.5) we will obtain a particular solution denoted as X_m. There is an infinite number of sets of d constraints which when applied to (1.5) will produce that many different solution vectors X, all of which satisfy the normal equation (1.5).

Let us consider one particular branch of such solutions, which is characterized by the contents of the P_m weight matrix. Out of the multitude of P_m matrices (and respective datums) there is one particular solution which is unique in that the P_m matrix is equal to the identity matrix. We will denote that unique solution as X_k and its datum - the k-datum. As can be seen, in this case m=k where each of the k points in the network contributes to datum definition on an even basis as P_k= I - the identity matrix of size k.s by k.s. Meissl in (1969) and also Koch in (1987) have shown that the X_k solution compared to all the rest is optimal and unique in the sense that the trace of the k.s by k.s covariance matrix is a minimum. In addition, it can be shown [See Koch, 1987] that $Q_{k,}$ the unscalled (σ_o^2) covariance matrix of X_k is the pseudo-inverse of N, known also as the Moore-Penrose inverse of N ($Q_k = N^+$). According to Meissl (1969) the above optimal X_k solution is the only one which reflects faithfully the accuracy of the measurements where all the other (external) sources of error have been effectively filtered out.

According to Wolf (1977) any two X vectors, which pertain to two different datums - X_m and X_k, for example - are related through the following simple equation:

$$X_m = X_k + E\, t_{km} \qquad (1.6)$$

where t_{km} is a d vector containing transformation parameters from the k - to the m - datums.
From (1.6) under the minimum condition (1.4) we derive a solution for t_{km} as follows [See Wolf, 1977 and Papo, 1987]:

$$t_{km} = -\left(E_m^T E\right)^{-1} E_m^T X_k = -H_m X_k \qquad (1.7)$$

which when substituted in (1.6) results in yet another form of (1.6)

$$X_m = (I - EH_m)\, X_k = R_m\, X_k \qquad (1.6')$$

Equation (1.6') when multiplied from the left by E_m^T results in equation (1.3) - the d linear constraints which define the specific m - datum. Equations (1.7), (1.6') and the covariance matrices of t_{km} and X_m will be investigated in the following two sections to reveal the dependence of point-accuracy on datum definition in a free network.

The unscalled covariance matrices of X_m and t_{km} are evaluated as follows:

$$Q_m = R_m\, Q_k\, R_m^T \qquad (1.8)$$

$$Q_{tm} = H_m\, Q_k\, H_m^T \qquad (1.9)$$

where as mentioned above $Q_k = N^+$ is the pseudoinverse of N.

We note [See Meissl, 1969] that the R_m matrix is idempotent which means that:

$$Q_m = R_m\, Q_m\, R_m^T \qquad (1.8')$$

Another interesting property of the R_m-type matrices is that the result of a chain product of different R_m matrices is equal to the leftmost R_m matrix:

$$R_{mi} = R_{mi} \,.\, R_{mj} \,....\, R_{mz} \tag{1.10}$$

2. The datum accuracy concept.

The relative positions of network points in an adjusted free network under minimal (d) constraints are invariant and form a perfectly rigid structure [See Meissl, 1969, Koch 1987]. That means that no matter which particular datum is selected for the free network, point-coordinates will indeed change from one selection to the next, but not the adjusted distances or spatial angles between the points. In contrast to the above invariance, point - accuracies (the covariance matrix of point coordinates) depend heavily on the specific datum that was selected to display point positions in terms of their respective coordinates.

It seems reasonable to adopt Meissl's approach in his Annex F in (1969) and to declare the k-datum (P_k= I) (based evenly on all the points in the network) as the best of all possible datums and use it as a "starting point" to arrive at any specific datum. The supremacy of the k-datum is due mainly to its minimum trace property and also to its uniqueness. In spite of its optimal and unique properties, however, the application of the k-datum (P_k= I) in practice is a complete nonsense. It is hard to find even a single case in which we would like to base the datum of a network on the approximate coordinates of all of its points.

The line of reasoning, which was adopted in this paper, is to study the effect of datum transformations from the unique but meaningless k-datum to any particular m-datum (P_m). In particular we shall pay attention to the geometry of the m-datum points and to its effect on positional accuracy of the network points. Here we have also a unique opportunity to "estimate" the nonestimable datum parameters of a network including the respective covariance matrix. As a matter of fact we will estimate the datum-transformation parameters t_{km}: from the unique but meaningless k-datum to any specific (P_m) m-datum. That means that we will analyze the Q_{tm} matrix as a function of the geometry of the m datum points.

"Geometry" is understood here as the size and distribution (within the network) of the m-subset of datum points. From elementary combinatorial analysis we can have the discrete number ${}_kC_m$ of different m-sets taken from a population of k points:

$${}_kC_m = \frac{k!}{m!(k-m)!} \tag{2.1}$$

There are cases, mainly in deformation analysis, where we are interested is investigating the consistency in the behavior (motion, rotation and strain) of a specific g-subset (a block of g points) of the network. The displacement field (or the velocity field) of the g-subset can be partitioned into rigid motion of the block and homogeneous strain (global components) and residual displacements of the g points (individual components). In such a case it would be important to reveal the connection between geometry of the datum (the m-datum) and the covariance matrix of the global parameters (rigid motion and homogeneous strain) of the g-subset. For simplicity we make the following reasonable assumptions:

(a) there have been only two observational sessions at epochs t_0 and $t_{1;}$

(b) in both sessions datum has been defined by identical m-subsets (identical approximate coordinates and identical P_m matrices);

(c) in processing the second session the approximate coordinates of the g-subset points have been set equal to the results of the adjustment of the first session.

Under the above three assumptions the estimated coordinate corrections X as a result of the adjustment of the second session would represent the displacement field of the g-subset. Partitioning of the displacement field is done as follows [See Meissl, 1969 and Papo, 1989]:

$$X_g = V_g + E_g \, t_g \tag{2.2}$$

where V_g is the vector of residual displacements;
t_g is a vector of rigid motion and homogeneous strain parameters and E_g is the extended Helmert matrix [See Papo, 1986].
The t_g parameters are estimated so as to minimize the following quadratic from:

$$V_g^T \, Q_{mg}^{-1} \, V_g = \min \tag{2.3}$$

where Q_{mg} is the covariance matrix of the g-subset coordinates (m-datum). Analysis similar to the one shown in the previous section results in:

$$t_g = \left(E_g^T \, Q_{mg}^{-1} \, E_g\right)^{-1} E_g^T \, Q_{mg}^{-1} \cdot X_g \tag{2.4}$$

$$Q_{tg} = \left(E_g^T \, Q_{mg}^{-1} \, E_g\right)^{-1}$$

An example of an E_g matrix in 3-D is shown in the following table [See Papo, 1986]:

rigid motion						homogeneous strain					
translation			rotation			scale variation			orthogonality		
1	0	0	0	z1	-y1	-x1	0	0	0	-z1	-y1
0	1	0	-z1	0	x1	0	-y1	0	-z1	0	-x1
0	0	1	y1	-x1	0	0	0	-z1	-y1	-x1	0
1	0	0	0	z2	-y2	-x2	0	0	0	-z2	-y2
…	…	…	…	…	…	…	…	…	…	…	…
0	0	1	yg	-xg	0	0	0	-zg	-yg	-xg	0

Note that xi, yi, zi (i=1,2....g) are the approximate coordinates of the points in the g-subset.

Thus we study the effects of geometries of two subsets in the network: the (datum) m-subset and the (deformation block) g-subset. The contents of the following covariance matrices, regarded as quantifiers of accuracy, will be studied:

- Q_{tm} - m-datum-parameter accuracy (t_m)
- Q_m - m-datum network point-position accuracy (X)
- Q_{tmg} - g-block global parameters accuracy (t_{mg})

All three matrices pertain to the same m-datum and respective P_m matrix.

3. Experiments with datum and block geometries

In order to demonstrate the utility of the proposed method of analysis we designed a schematic GPS network in 2-D, consisting of 19 points (See fig. 3.1). At this time (1999) and in spite of their undisputed 3-D capacity, GPS measurements are still much more effective in constructing a horizontal (2-D) rather than a spatial (3-D) network. Mainly for this reason we have chosen to postpone, for the time being, experiments with datum geometries in 3-D.
As shown in figure 3.2 the points in the network are located at distances of exactly 20 km. from each other and form together a perfectly symmetrical 60^o grid. The simulated vectors are of two typical lengths: 20.00 km. vectors such as 1-2, 5-9, etc. and 34.64 km. vectors such as 4-10, 15-17 etc. The simulated GPS vectors were adjusted in two different modes:
mode I: where a vector is represented by components: Δn (north-south) and Δe (east-west);
mode II: where each vector is regarded as a single horizontal distance Δs. ($\Delta s^2=(\Delta n)^2+(\Delta e)^2$).

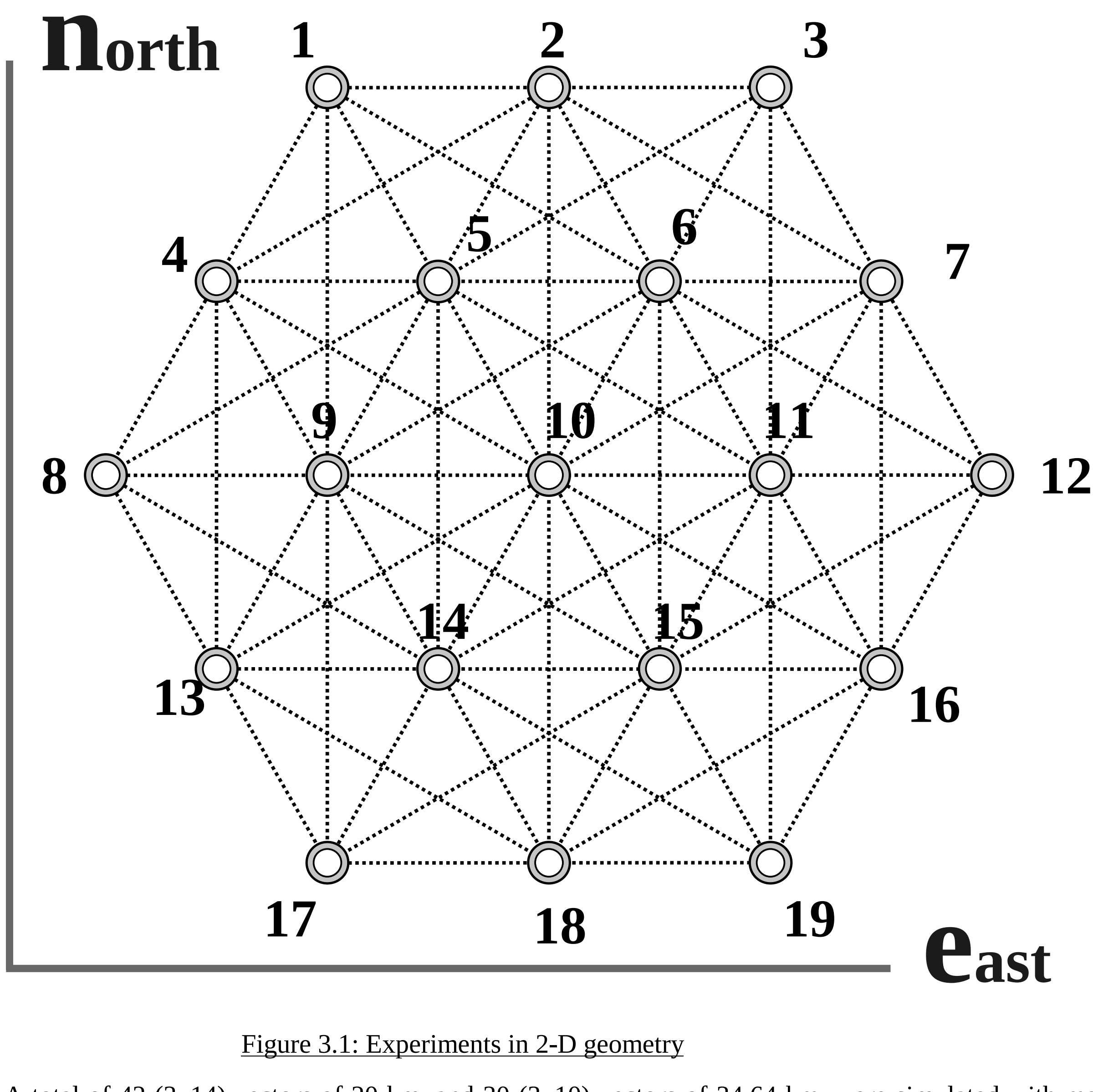

Figure 3.1: Experiments in 2-D geometry

A total of 42 (3x14) vectors of 20 km. and 30 (3x10) vectors of 34.64 km. were simulated with medium accuracy of about 1 ppm. In mode I a correlation of 70% was assumed between the two vector components. Zero correlation was assumed between any two different vectors. Weight matrices and partial derivatives were evaluated for each of the two measurement modes and finally two normal matrices were formed - one for mode I and one for mode II, each measuring 38 by 38. The datum defect for N_I is d_I=2 due to the need to define origin in “n” and in “e”. Note that orientation and scale are provided in mode I by the Δn and Δe measurements. The datum defect for N_{II} is d_{II}=3 due to the additional need to define orientation. Thus three Helmert datum transformation matrices are needed for the analysis as shown bellow. The values of ni and ei, in E_{II} and in E_g are the approximate coordinates of the respective network points. As for E_g , in this paper we limited the investigation of strain to homogeneous scale variation only. So the E_g matrix implies only four degrees of freedom as needed for partitioning the displacement field of a block in 2-D (See equation 2.2).

$$E_I^T = \begin{bmatrix} 1 & 0 & 1 & \ldots \\ 0 & 1 & 0 & \ldots \end{bmatrix} \quad ; \quad E_{II}^T = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & \ldots \\ 0 & 1 & 0 & 1 & 0 & \ldots \\ -e1 & n1 & -e2 & n2 & -e3 & \ldots \end{bmatrix} \begin{matrix} \text{origin for "n"} \\ \text{origin for "e"} \\ \text{orientation} \end{matrix}$$

$$E_g^T = \begin{bmatrix} 1 & 0 & 1 & 0 & \ldots \\ 0 & 1 & 0 & 1 & \ldots \\ -e1 & n1 & -e2 & n2 & \ldots \\ -n1 & -e1 & -n2 & -e2 & \ldots \end{bmatrix} \begin{matrix} \text{translation "n"} \\ \text{translation "e"} \\ \text{rotation of axes} \\ \text{variation of scale} \end{matrix}$$

Two datum configurations (geometries) were defined as follows (See fig. 3.2):

A: a “wide” datum consisting of points 1, 8, 9, 17;
B: a “narrow” datum consisting of points 4, 8, 9, 13

Two block geometries were defined for studying the virtual displacement field as follows:

C: a “narrow” block consisting of points 7, 11, 12, 16;
D: a “wide” block consisting of points 3, 11, 12, 19.

The units of the elements of the observation equations and the weight matrices were set so as to produce variance - covariance matrices (V-CM) with the following units:

V-CM of point coordinates "n" and "e" in [cm^2] (centimeters squared);
V-CM of datum-transformation/block partitioning parameters:

t(1) origin/translation in “n” [cm^2];
t(2) origin/translation in “e” [cm^2];
t(3) orientation/rotation of axes in [μrad^2] (microradians squared);
t(4) scale variation in [ppm^2] (parts per milion squared).

All the computations were performed with 12 significant digit accuracy on a PC using the APL*PLUS software. Results of the experiments are shown in the following tables:

Table 3.1: Standard deviations of point coordinates [cm^2].

mode I: Δn, Δe							mode II: Δs					
datum	F		A		B		F		A		B	
pt#	n	e	n	e	n	e	n	e	n	e	n	e
1	1.08	1.08	.96	.96	1.19	1.19	1.07	1.08	.94	.67	1.23	1.90
2	.93	.93	1.09	1.09	1.15	1.15	1.06	.84	1.35	1.15	1.82	1.83
3	1.08	1.08	1.31	1.31	1.35	1.35	1.07	1.08	1.72	1.52	2.57	2.04
4	.93	.93	.94	.94	.75	.75	.90	1.01	.93	1.11	.74	.56
5	.75	.75	.87	.87	.91	.91	.83	.77	.95	.96	1.25	1.04
6	.75	.75	.99	.99	1.05	1.05	.83	.77	1.31	1.04	2.01	1.28
7	.93	.93	1.21	1.21	1.25	1.25	.90	1.01	1.81	1.30	2.90	1.51
8	1.08	1.08	.88	.88	.78	.78	1.09	1.07	.93	.88	.69	.80
9	.75	.75	.65	.65	.64	.64	.75	.85	.69	.79	.61	.66
10	.67	.67	.89	.89	.91	.91	.76	.76	1.09	.95	1.52	.90
11	.75	.75	1.05	1.05	1.09	1.09	.75	.85	1.49	1.09	2.40	1.12
12	1.08	1.08	1.35	1.35	1.38	1.38	1.09	1.07	2.13	1.30	3.40	1.35
13	.93	.93	.94	.94	.75	.75	.90	1.01	.93	1.11	.74	.56
14	.75	.75	.87	.87	.91	.91	.83	.77	.95	.96	1.25	1.04
15	.75	.75	.99	.99	1.05	1.05	.83	.77	1.31	1.04	2.01	1.28
16	.93	.93	1.21	1.21	1.25	1.25	.90	1.01	1.81	1.30	2.90	1.51
17	1.08	1.08	.96	.96	1.19	1.19	1.07	1.08	.94	.67	1.23	1.90
18	.93	.93	1.09	1.09	1.15	1.15	1.06	.84	1.35	1.15	1.82	1.83
19	1.08	1.08	1.31	1.31	1.35	1.35	1.07	1.08	1.72	1.52	2.57	2.04

Note: F - minimum-trace datum; A - wide-geometry datum; B - narrow-geometry datum.

Table 3.2: Error ellipses : maj./min.semi-axes (a/b) [cm.], orientation (θ) [deg].

datum	F			A			B		
pt#	a	b	θ	a	b	θ	a	b	θ
1	1.09	1.07	60.0	.96	.65	13.6	1.93	1.19	-77.9
2	1.06	.84	.0	1.39	1.09	-23.8	2.29	1.20	-45.2
3	1.09	1.07	-60.0	1.90	1.29	-35.2	3.00	1.33	-35.2
4	1.06	.84	-60.0	1.16	.87	-65.0	.74	.56	-1.7
5	.85	.75	-30.0	.99	.92	-47.8	1.36	.89	-31.7
6	.85	.75	30.0	1.32	1.03	-11.5	2.13	1.06	-22.5
7	1.06	.84	60.0	1.83	1.27	-12.0	2.99	1.31	-16.0
8	1.09	1.07	.0	.93	.88	.0	.80	.69	90.0
9	.85	.75	90.0	.79	.69	90.0	.66	.61	90.0
10	.76	.76	8.2	1.09	.95	.0	1.52	.90	.0
11	.85	.75	90.0	1.49	1.09	.0	2.40	1.12	.0
12	1.09	1.07	.0	2.13	1.30	.0	3.40	1.35	.0
13	1.06	.84	60.0	1.16	.87	65.0	.74	.56	1.7
14	.85	.75	30.0	.99	.92	47.8	1.36	.89	31.7
15	.85	.75	-30.0	1.32	1.03	11.5	2.13	1.06	22.5
16	1.06	.84	-60.0	1.83	1.27	12.0	2.99	1.31	16.0
17	1.09	1.07	-60.0	.96	.65	-13.6	1.93	1.19	77.9
18	1.06	.84	.0	1.39	1.09	23.8	2.29	1.20	45.2
19	1.09	1.07	60.0	1.90	1.29	35.2	3.00	1.33	35.2

Table 3.2 applies to mode II only. The angle θ is between the major semi-axis "a" and "n".

Table 3.3: Variance-covariance matrices of rigid-motion and scale-variation parameters (t_{mg}).

mode I									
datum \ block		C				D			
A		1.2356	.5079	.1392	.0411	.9518	.4840	.0706	.0348
		.5079	1.2356	-.0411	-.1392	.4840	.9518	-.0348	-.0706
		.1392	-.0411	.0531	.0157	.0706	-.0348	.0316	.0156
		.0411	-.1392	.0157	.0531	.0348	-.0706	.0156	.0316
B		1.3275	.5747	.1388	.0410	1.0496	.5534	.0705	.0348
		.5747	1.3275	-.0410	-.1388	.5534	1.0496	-.0348	-.0705
		.1388	-.0410	.0531	.0157	.0705	-.0348	.0316	.0156
		.0410	-.1388	.0157	.0531	.0348	-.0705	.0156	.0316

mode II									
datum \ block		C				D			
A		2.7704	.0000	.3894	.0000	1.5765	.0000	.0050	.0000
		.0000	1.2234	.0000	-.1746	.0000	.9519	.0000	-.1156
		.3894	.0000	.2330	.0000	.0050	.0000	.1066	.0000
		.0000	-.1746	.0000	.0687	.0000	-.1156	.0000	.0433
B		3.9908	.0000	-.0320	.0000	2.9288	.0000	-.4129	.0000
		.0000	1.3512	.0000	-.1796	.0000	1.1045	.0000	-.1238
		-.0320	.0000	.3872	.0000	-.4129	.0000	.2460	.0000
		.0000	-.1796	.0000	.0687	.0000	-.1238	.0000	.0433

Units in the above table are: t(1), t(2) - [cm^2] ; t(3) - [μrad^2] ; t(4) - [ppm^2]

Table 3.4: Variance-covariance matrices of datum-transformation parameters (t_m).

datum		A			B			
mode I	t(1)	.25128		.17768	.32612	.23060		[cm^2]
	t(2)	.17768		.25128	.23060	.32612		[cm^2]
mode II	t(1)	.57675	.00000	-.13049	1.91514	.00000	-.54474	[cm^2]
	t(2)	.00000	.22493	.00000	.00000	.31342	.00000	[cm^2]
	t(3)	-.13049	.00000	.04310	-.54474	.00000	.17718	[μrad^2]

4. Conclusions. The importance of geometry.

Let us first take a closer look at tables 3.1 and 3.3. In those solutions under mode I (where GPS vectors are regarded as measurements which contain orientation information) the differences between results based on datum A (wide) and - on datum B (narrow) are hardly noticeable. Solutions obtained in mode II (see table 3.1 and 3.4) display clearly the effect of datum selection. The superiority of datum A (wide) over datum B (narrow) is highly significant. It is easy to see that the size and orientation of the error ellipses are a function of distance and di-

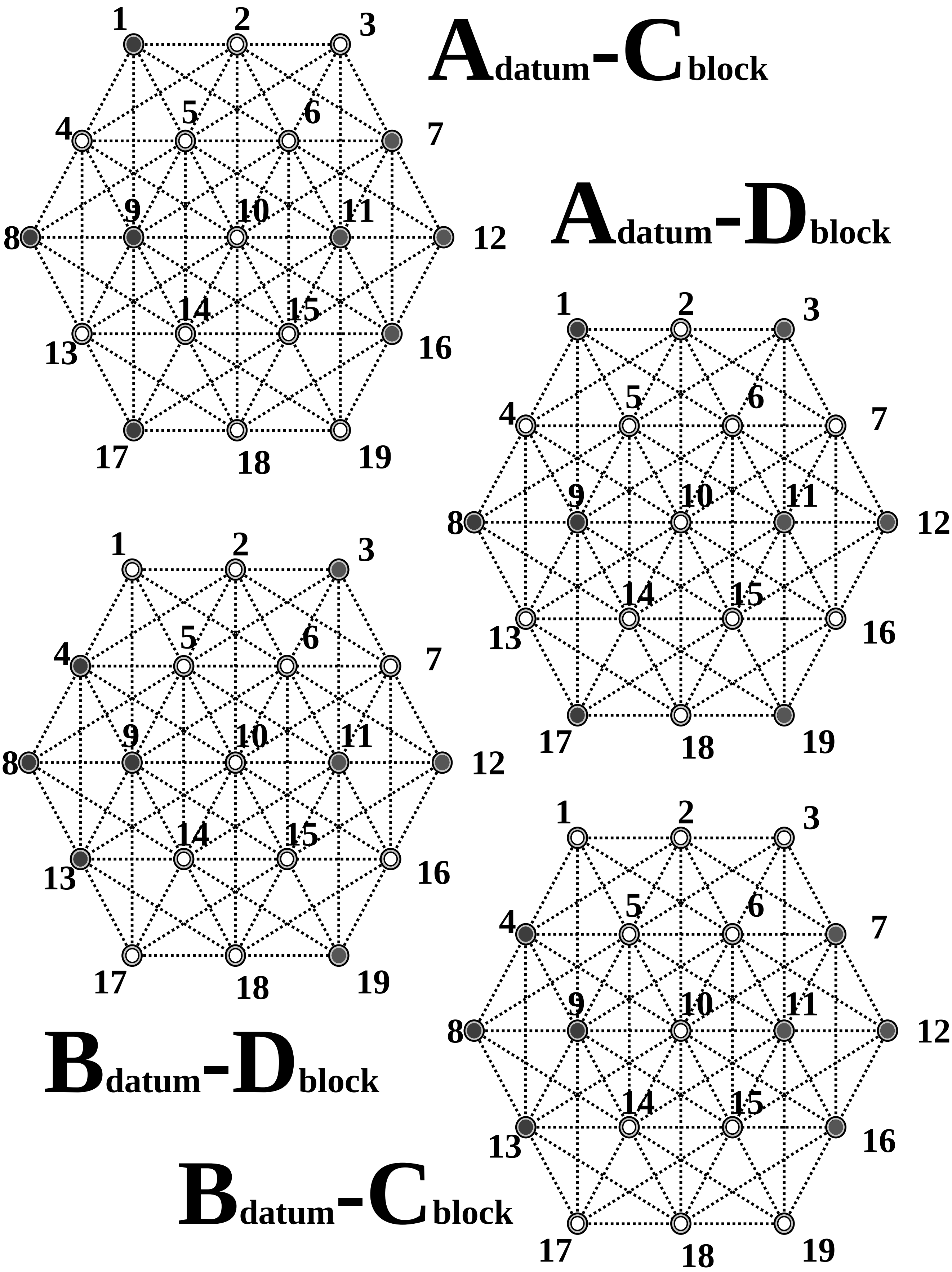

Figure 3.2: Four combinations of datum and block geometries

rection of the point relative to the “datum block” (table 3.2). The differences in “quality” of all three datum transformation parameters in mode II (see table 3.4) are indeed dramatic where variances pertaining to datum B (narrow) are 3 to 4 times larger as compared to those obtained under datum A (wide).

The effect of geometry on the quality of the parameters of block motion and strain (see table 3.3) is clearly demonstrated. The differences in mode I variances between B-C (narrow datum - narrow block) and the (wide datum-wide block) are of the order of no more than 30-40%. In mode II , however, the differences between B-C and A-D are distinctly larger. In particular it is worth noting the rotation parameter "t(3)", where due to the orientation “weakness” of datum B, the variance of the B-C t(3) parameter differs from that of A-D by almost 400%.

It is interesting to note in mode II the significant differences between the qualities of north-south t(1) and east-west t(2) origin/translation parameters. The span in the east-west direction of both the A as well as the B datum blocks is only 20 km., while the span in the north-south direction is 34.6 km. in datum B and 61 km. in datum A. The effect of the above contrast is felt consistently in the difference between variances of t(1) "n" and t(2) "e".

The results of our limited experiment as reported in this paper can be summarized as follows:

(a) The orientation content of the GPS measurements is extremely important and it made all the difference between mode I and mode II results. That is particularly important when constructing first order or "free" control networks. We should point out, however, that to ignore the geometry of the datum subset would be permissible only if the orientation content of the GPS measurements in terms of its consistency can be safely relied upon.

(b) Trilateration (mode II) as measured today by GPS is a very robust and accurate source of data. Yet, as in former "EDM times", it is notoriously week in orientation. To counteract that inherent weakness in orientation one has to make sure that the selected datum subset is characterized by a balanced and "good" geometry relative to the size and shape of the whole network.

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