GLONASS Carrier Phases

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ABSTRACT

Processing of GLONASS carrier phase observations differs from that of GPS. These differences are briefly reviewed. Presently GLONASS does not contain selective availability (SA). Simply graphing between-satellite differences reveals parts of SA that is implemented on GPS satellite signals.
The single difference and double difference carrier phase solutions are analyzed in terms of their suitability for baseline determination with GLONASS carrier phases. The single difference and double difference receiver bias terms for phases, labeled SDRB and DDRB respectively, are introduced. The DDRB is numerically verified from observations.
The double difference fixed solution depends on the initial receiver (rover) coordinates. The single difference solution does not have such a dependency. For a test data set the coordinates estimated from both solutions agree within one millimeter even though the initial coordinates were in error by 1.7 m. The double difference ambiguities were fixed using the LAMBDA technique. Using both GPS and GLONASS carrier phases, the ambiguities could be fixed correctly at all epochs, including the first one, with L1 phases only.

INTRODUCTION

There is a strong interest for including GLONASS satellites in any GPS positioning solution. It is well known that additional satellites and frequencies strengthen the solution. The benefits of additional GLONASS satellites are especially noticeable when attempting OTF (On-The-Fly) ambiguity resolution. The fact that GLONASS satellites transmit at different frequencies has attracted much attention, primarily by individuals interested in precise positioning, for example Raby and Dale (1993), Leick et al. (1995), Rossbach and Hein (1996), Hall et al. (1997), and Kozlov and Thachenk (1997).
We will revisit the topic of ambiguity fixing with GLONASS carrier phases and pay attention to frequency-dependent receiver errors. A well-known strength of double differencing GPS carrier phase observations is that the receiver channel bias cancels. This bias is the same for each satellite observed at the same receiver, but differs between receivers. In case of a hybrid GPS/GLONASS receiver the biases for GPS and GLONASS differ. They do not cancel when double differencing the phase observations from satellites of both systems. In fact, the GLONASS channel biases might even exhibit a small variation as a function of temperature and cable length (Dodsen et al. 1999). These variations are not discussed in this paper.
There are several other aspects of the GLONASS system that have been discussed widely in the literature. For example Bykhanov (1999) discusses the GLONASS time system. The differences between the PZ-90 and WGS-84 reference coordinate system have been studied for many years. Russian scientists reported some of their work in Bazlov et al. (1999). Many questions regarding the implications of the different timing and coordinate reference systems for GLONASS and GPS will be answered by the international IGEX campaign (Pascal, 1999). Finally the GLONASS broadcast ephemeris parameterization differs from that of GPS (Stewart and Tsakiri, 1998).

The data sets for this contribution were observed with R100 receivers manufactured by 3S Navigation of Irvine, California, in connection with a general study to asses GLONASS observations (Leick et al. 1998). The pseudoranges and carrier phases were recorded for GPS (L1 only) and GLONASS (L1 & L2). The receivers were located on the roof of the 3S Navigation offices at Irvine.

Data set A consists of several 1-2 week long observation series made with the same receiver at a recording interval of 5 minutes. The Data set was used primarily to compute UREs for GLONASS. The results are reported elsewhere.
Data set B was recorded on June 12, 1998 using a 10 s recording interval. Two receivers operated independently, i.e. they were not connected to an atomic clock.
We follow the RINEX conventions for naming the satellites. For example, G15 and R15 denote the GPS satellite PRN 15 and GLONASS satellite with almanac number 15 respectively.

SA FROM BETWEEN SATELLITE DIFFERENCES

Between satellite differences (BSD) do not depend on receiver clock errors. Their variation over time reveals, among other things, the satellite clock errors. Because there is no selective availability (SA) implemented on GLONASS satellites, the GLO-GPS differences will be affected by the SA dither on GPS. Figure 1 shows several L1 BSD carrier phases with respect to the GLONASS satellite R17. The dither of the GPS clocks is clearly visible from the dashed lines. The GLO-GLO pairs follow a more or less flat line around zero (solid lines).

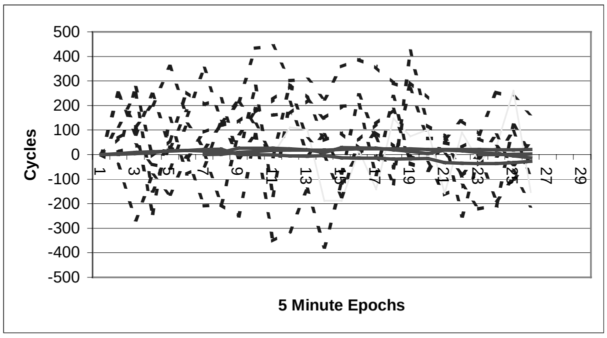

Figure 1: Between Satellite Differences (2 hours from Data set A on DOY 068)

ESTIMATING BASELINES FROM SINGLE DIFFERENCES

An advantage of the single difference formulation is that the signals from GPS and GLONASS satellites are not differenced explicitly. In the context of *single difference solutions*, the terminology *fixed solution* refers to the fact that GPS/GPS and GLO/GLO *double difference ambiguities* have been constrained to integers. For such fixed solutions the adjusted single difference ambiguities are still non-integers.
The mathematical model for carrier phase as applied to short baselines is written as

$$\varphi^{q}_{km,1,GPS} = \frac{f_1}{c}\rho^{q}_{km} + N^{q}_{km,1} + d_{km,1,GPS} - f_1\, dt_{km} \tag{1}$$

$$\varphi^{s}_{km,1,GLO} = \frac{f_1^{\,s}}{c}\rho^{s}_{km} + N^{s}_{km,1} + d_{km,1,GLO} - f_1^{\,s} dt_{km} \tag{2}$$

The superscripts $q = 1\ldots S_{GPS}$ and $s = 1\ldots S_{GLO}$ identify the satellites. The symbols $d_{km,1,GPS}$ and $d_{km,1,GLO}$ denote single difference receiver biases (SDRB) for the respective systems. The model assumes only one bias term per satellite system, i.e. it does not include frequency dependent terms for GLONASS that may result from temperature variation and other sources.

We combine the single difference ambiguity and the SDRB into a new parameter ξ as follows

$$\xi^{q}_{km,1,GPS} \equiv N^{q}_{km,1,GPS} + d_{km,1,GPS} \tag{3}$$

$$\xi^{s}_{km,1,GLO} \equiv N^{s}_{km,1,GLO} + d_{km,1,GLO} \tag{4}$$

The unknown receiver (rover) coordinates, the parameters ξ, and the receiver clock differences dt_{km} can now be estimated every epoch using Kalman filtering. The outcome of the ith epoch is the estimated parameter vector denoted by $X_i(+)$ and its covariance matrix $P_i(+)$. The parameter vector includes the epoch estimates $\xi^{q}_{km,1,GPS}(+)$ and $\xi^{s}_{km,1,GLO}(+)$.
Next, we transform the single difference estimates to double differences. Let p or r denote the GPS or GLONASS base satellite respectively, then the transformation is given by

$$\begin{bmatrix} \varphi^{pq}_{km,GPS}(+) \\ \varphi^{rs}_{km,GLO}(+) \end{bmatrix} = D \begin{bmatrix} \xi^{q}_{km,GPS}(+) \\ \xi^{s}_{km,GLO}(+) \end{bmatrix} \tag{5}$$

$$\Sigma_i = DC_iD^T \tag{6}$$

The matrix D has $(S_{GPS} + S_{GLO} - 2)$ rows and $(S_{GPS} + S_{GLO})$ columns. The matrix C_i is a submatrix of $P_i(+)$. Equation (6) follows from variance-covariance propagation. The symbol Σ_i denotes the covariance matrix of the double difference ambiguities at epoch i.
The transformation (5) generates only GPS/GPS or GLO/GLO pairs of double differences. These do not depend on the SDRB; the respective ambiguities are conceptually integers. It is now possible to attempt to determine the integer double differences ambiguities using a technique such as LAMBDA (Teunissen (1993). The input is the real-valued double difference ambiguities, $\left(\varphi^{pq}_{km,GPS}, \varphi^{rs}_{km,GLO}\right)$ of (5), and the covariance matrix, Σ, of (6). The outcome is a set of integers $\left(\Psi^{pq}_{km,GPS}, \Psi^{rs}_{km,GLO}\right)$. As a last step the epoch Kalman filter solution can be constrained to these integer values. The result is a single difference epoch solution with fixed double difference ambiguities.
The various steps discussed above are repeated for each epoch. Let's denote the updated ξ –parameters by $\hat{\xi}^{q}_{km,1,GPS}$ and $\hat{\xi}^{s}_{km,1,GLO}$. These are the values obtained after the double difference ambiguities have been constrained to integers. The fractional part for the GPS/GLO differences

$$\Delta\xi^{ps}_{km,1} = \hat{\xi}^{p}_{km,1,GPS} - \hat{\xi}^{s}_{km,1,GLO} \tag{7}$$

is the estimated double difference receiver bias (DDRB). This bias is expected to be constant with time and estimates the difference $d_{km,1,GPS} - d_{km,1,GLO}$.
For the sake of completeness let it be stated that the transformation (5) can also be directly implemented in (1) and (2).

Numerical Results: We used L1 pseudoranges and carrier phases of Data Sets B to investigate (7) as a function of time. All ambiguities could be correctly fixed for all epochs, including even the first one. Here we do not address the conditions under which it is possible to fix ambiguities at single epochs or for short intervals. Teunissen et al. (1998) provide an interesting contribution regarding the reliability of ambiguity resolution in such cases.
Figure 2 shows the DDRB differences (7) for the GPS-GPS and GPS-GLO the float solutions, i.e. the double difference ambiguity constraints are not yet imposed. The differences are taken with respect to satellite G5. It is readily seen that, after convergence of the Kalman filter, the GPS-GPS differences are located around zero. A variation of the order of a couple of hundredths of a cycle is seen, although the theoretical value is zero since all GPS satellites transmit on the same frequency.
The mixed GPS-GLO differences are offset by about 0.35 cycles and differ among each other by several hundredths of a cycle as well. Since this variation is of the same size as the one observed for

the GPS-GPS differences, it seems that this data set does not allow one to make the definitive statement about the dependency of the SDRB on the various GLONASS frequencies.

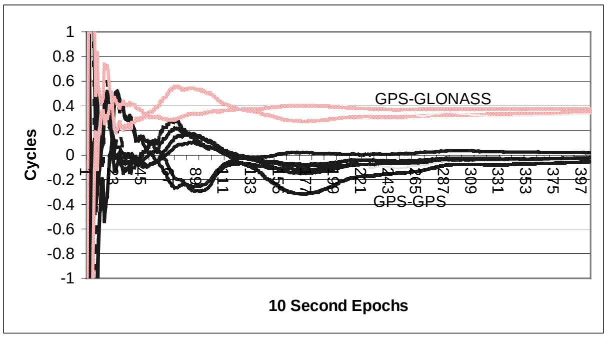

Figure 2: DDRB differences with Float Double Difference Ambiguities (Data set B)

Figure 3 shows the estimated DDRB differences (7) for the fixed solution, i.e. the double difference GPS-GPS and GLO-GLO integer ambiguities have been fixed. The initial variation prior to convergence of the Kalman filter is not present in this figure because the double difference ambiguities could be fixed at all epochs. Because all double difference ambiguities could be fixed, the figure shows identical graphs for each GLONASS satellite. The DDRB differences seem to vary by a couple of hundredths of a cycle over time. The cause for this variation must still be investigated. Figure 3 seems to suggest that is it permissible to constrain the DDRB differences (7) to a constant.

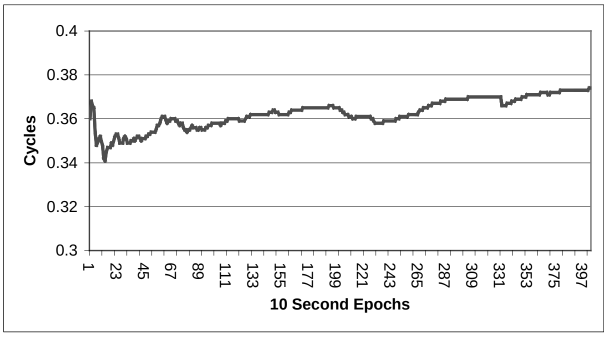

Figure 3: DDRB differences between GPS and GLONASS with fixed GPS/GPS and GLO/GLO Double Difference Integer Ambiguities.

Figure 4 shows the estimated length of the baseline for the float solutions, and the respective plus and minus standard deviations. The straight line at 1.751 m is the length estimated from the fixed solution. It is readily seen that the float and fixed solutions converge and that the fixed solution provides the correct position even at the first epoch. The standard deviations for the double difference ambiguities (not shown in the figure) are in the range of millimeter, whereas those for the single difference ambiguities (same fixed solution) and the receiver clock difference are about 1 cycle and 0.001 μs respectively. Successfully fixing the double difference ambiguities does not imply that the single difference ambiguities can be fixed as well (due to the correlation between ξ and dt_{km}).

The receiver clocks drifted about 440 μs. If we exclude the GLONASS observations, several epochs are needed to fix the ambiguities correctly.

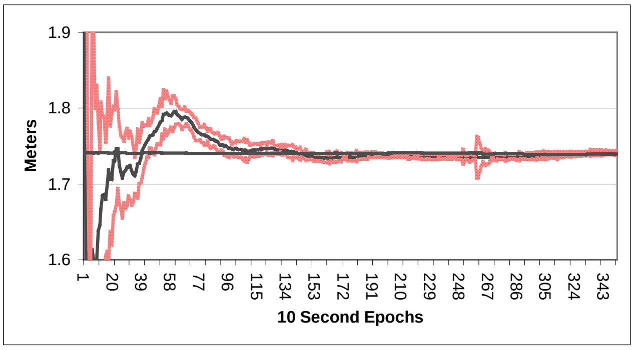

Figure 4: Epoch Position Solutions for Data set B

DRAWBACKS OF DOUBLE DIFFERENCING

Conventional double differencing for GLONASS observations gives

$$\begin{aligned}\varphi^{rs}_{km,1,GPS,GLO} &\equiv \varphi^{r}_{km,1} - \varphi^{s}_{km,1} \\ &= \frac{f_1^r}{c}\rho^r_{km} - \frac{f_1^s}{c}\rho^s_{km} + N^{rs}_{km,1} - \left(f_1^r - f_1^s\right)dt_{km}\end{aligned} \tag{8}$$

The double differences depend on the receiver clock error and the frequencies. Figure 5 displays this dependency; the O-C values were computed using known coordinates for the stations and then translated to zero at the first epoch. The dependency on the frequency can readily be seen from the figure; the reference satellite is G5 (1575.42 MHz). The GPS-GPS differences graphically coincide with the horizontal axis and are not visible in this figure.

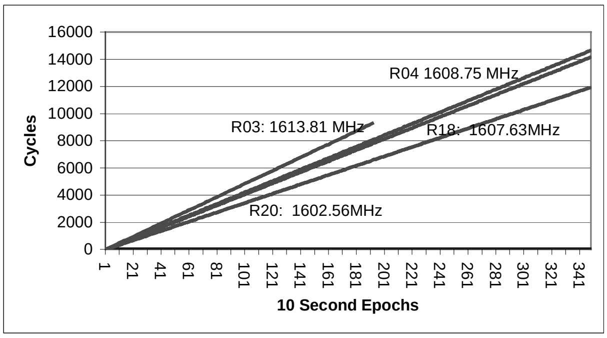

Figure 5: Double difference O-C values for known baseline (Data set B)

Scaling the carrier phases to distance, or to a mean GLONASS frequency, or to f_1^r or f_1^s for the (r,s)pair eliminates the receiver clock term but introduces a linear combination of single difference ambiguities whose coefficients are non-integer. The transformed double difference

$$\begin{aligned}\tilde{\varphi}^{rs}_{km,1,GLO} &\equiv \frac{f_1^s}{f_1^r}\varphi^{r}_{km,1,GLO} - \varphi^{s}_{km,1,GLO} \\ &= \frac{f_1^s}{c}\rho^{rs}_{km} + \tilde{N}^{rs}_{km} + \frac{f_1^s}{f_1^r}N^{r}_{km,1,0} + \eta^{rs}\end{aligned} \tag{9}$$

$$\eta^{rs} = \left(\frac{f_1^s - f_1^r}{f_1^r}\right)dN^r_{km,1} \le 0.01 dN^r_{km,1} \tag{10}$$

contains an integer term $\tilde{N}_{km}^{rs}$ and a small term η^{rs}. The symbol $N_{km,1,0}^{r}$ represents an integer approximation of the single difference ambiguity $N_{km,1}^{r}$ which can be derived from pseudoranges. The size of the small η-term depends on the quality of the initial estimate of $N_{km,1,0}^{r}$, and constitutes a model error when neglected in the fixed solution. This limitation does not apply to the float solution.
Figure 6 shows the double difference residuals G5 - GLO for the batch least-squares implementation of (9) using Data set B. The double difference ambiguities GPS/GPS and GLO/GLO are fixed. The top set of lines is based on approximate coordinates which were in error by about 1.7 m, thus $dN_{km,1}^{r}$ is correspondingly large. Using approximate coordinates that are even less accurate, one would eventually recognize a frequency dependency within this band of lines. The accurate coordinates were used for the bottom set of lines, thus $dN_{km,1}^{r}$ is correspondingly small. The model error (10) causes the shift between both sets of lines. The model error falsifies the position estimate even when the ambiguities are formerly fixed. The bottom set of lines can be directly compared with Figure 3 for the single difference solution. Again, the DDRB differences could be modeled by a constant.

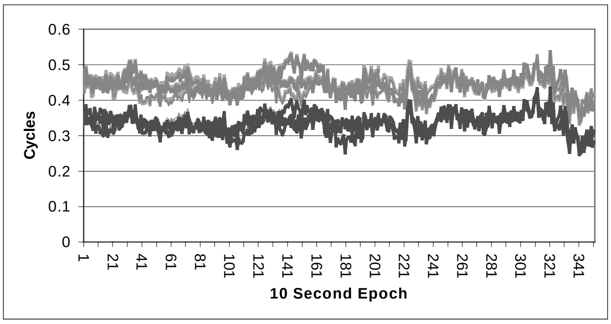

Figure 6: DDRB from Double difference Solution

SUMMARY

The model error that occurs in GLONASS double difference fixed solutions does conceptually not occur with GPS observations.
When processing GLONASS carrier phase observations, caution should be exercised. Ambiguity search might identify the wrong integers and, as such, introduce a bias in the fixed solution. For double differencing to work correctly one must have good a priori knowledge of single difference ambiguities which, in turn, are derived from pseudoranges. Since the accuracy of pseudoranges are potentially effected by multipath, one might be inclined to favor the single difference formulation and fix the propagated double difference ambiguities.

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BIOGRAPHY

Dr. Alfred Leick is a professor at the University of Maine, Department of Spatial information Science and Engineering. He is author of the book *GPS Satellite Surveying*. He spent the academic year 1985-86 at the University of Stuttgart collaborating with Prof. Grafarend while being supported by the Alexander von Humboldt foundation. He revisited the Geodetic Institute in August 1999, being supported again by the Humboldt foundation.