The Challenge of the Crustal Gravity Field

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1. Introduction

Two different methods have traditionally been used separately or together for determination of the geoid, namely 1) the *astrogeodetic levelling* method and 2) the *gravimetric* method. The former is based on the use of astrogeodetic deflections of the vertical as observables, which can be interpreted as horizontal gradients of the geoid undulation field, while the latter is based on the use mean gravity anomalies of surface blocks which should cover the whole surface of the Earth.
The advent of the artificial satellites has presented us with new methods to model the geoid. One of them is based on the use of ellipsoidal heights determined from GPS-observations. Namely, when confronting the ellipsoidal heigth H^* with the orthometric height H known from the precise levelling, the geoid undulation N is obtained simply by taking the difference $N = H^* - H$. This method leads to an extremely accurate determination of the geoid, provided naturally that sufficient number of accurate levelling points are available.
Details of the geoid can extensively be explored also by means of deep seismic sounding (DSS). This is possible because the data obtained from DSS can be used to construct a 3d-velocity structure model for the crust in the area to be studied. The velocity model can further be converted to a 3d-density model using the empirical relationship that holds between seismic velocities and crustal mass densities. Undulations of the geoid can then be estimated from the 3d-density model as shown by Wang, 1998 (also in Kakkuri and Wang, 1998).

2. Deep seismic sounding method

Deep seismic sounding and ocean drilling have revealed that the Earth's crust is not homogeneous but has a layered structure in the continental as well as in the oceanic areas. The vertical structure of thick continental crust is, however, more complicated than that of oceanic crust, and, in addition, in the continents the structure of ancient shield areas differ from that of younger basins. A three-layered crustal structure is observed in most parts of the shield areas, characterized by P-wave velocities of 6.0 - 6.5, 6.5 - 6.9 and 7.0 - 7.3 km/s, respectively. More complicated structures exist in quite a few places, mostly in the vicinity of the transition zones from continental crust to oceanic crust. The generalized structure of the basins is four-layered, a thick sediment cover being in the top and three igneous layers below.
Oceanic crust is only 5 - 10 km thick. Its top part consists of a layer of sediments that increases in thickness away from the oceanic ridges. The igneous oceanic basement consists of a thin (~0.5 km) upper layer of superposed basaltic lava flows underlain by a complex of basaltic intrusions, the sheeted dike complex. Below this the oceanic crust consists of gabbroic rocks (Lowrie, 1997).
The velocity at which compressional seismic P-waves travel through homogeneous materials can be expressed in the form

$$v_p = \sqrt{\frac{k + \frac{4}{3}n}{\rho}} \tag{1}$$

where ρ is the density, k is the bulk modulus and n is the shear modulus of the material. It can be seen that the velocity of P-waves depends on the elastic constants and the density of the material.

Thus, when the elastic parameters are known, the density can be calculated from the observed velocity. Unfortunately, as the elastic parameters are poorly known for materials inside the Earth, Eq. 1 is not applicable as such. For practical applications, it can be replaced by a linear relation known as Birch's law

$$v_p = a(\overline{m}) + b\rho \qquad (2)$$

where a depends on the mean atomic weight $\overline{m}$ only, and b is a constant. For plutonic and metamorphic rocks, which are the main types of rocks in the shield areas, the mean atomic weight plays an insignificant role and can be safely neglected from the density-velocity relation (Gebrande, 1982). The following linear relations represent the shield areas (Chroston and Brooks 1989, Lebedev *et al.* 1977):

For upper crust (v_p = 6.0, 6.5 km/s) $v_p = 2.538\rho - 0.568 \pm 0.256$ km/s (2a)

For mid-crust (v_p = 6.5, 6.9 km/s) $v_p = 3.184\rho - 2.580 \pm 0.122$ km/s (2b)

For lower crust (v_p = 6.8, 7.3 km/s) $v_p = 2.717\rho - 1.250 \pm 0.120$ km/s (2c)

Using the above relations we can estimate the velocity-density relations as follows:

Table 1. Density-velocity relations for plutonic and metamorphic rocks.

v_p (km/s)	ρ (g/cm³)
6.0	2.58 ± 0.11
6.4	2.80 ± 0.11
6.8	3.06 ± 0.05
7.3	3.15 ± 0.05

The velocities of seismic waves are generally found to be greater in igneous and crystalline rocks than in sedimentary ones (Parasnis, 1972). In the sedimentary rocks they tend to increase with depth of burial and geological age, and the application of Birch's law to sedimentary rocks is therefore questionable. Density data from drilling holes should be used instead of DSS-data in that case.

3. Mathematical modelling

Gravitational potential of a body can be written in the spherical coordinate system as follows (e.g. Heiskanen & Moritz 1967)

$$V(r,\theta,\lambda) = G\int \frac{\rho(r',\theta',\lambda')}{\sqrt{r^2 + r'^2 - 2rr'\cos\psi}}\, r'^2 \sin\theta'\, dr'\, d\theta'\, d\lambda' \qquad (3)$$

where ψ is the angle between the vector $\overline{OQ}$ of the point $Q(r',\theta,\lambda')$ and the vector $\overline{OP}$ of the point $P(r,\theta,\lambda)$ as shown in Fig. 1, $\rho(r',\theta,\lambda')$ is the density of a mass element at point $Q(r',\theta',\lambda')$, and G is the Newtonian gravitational constant. In addition,

$$\cos\psi = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\lambda - \lambda') \qquad (4)$$

The potential field of the crust can be constructed by slicing the crust into small spherical elements that take the form of a spherical prism and are filled with homogeneous masses, Fig. 2. The potential field of the whole crust is then the summation of potentials of the spherical prisms.

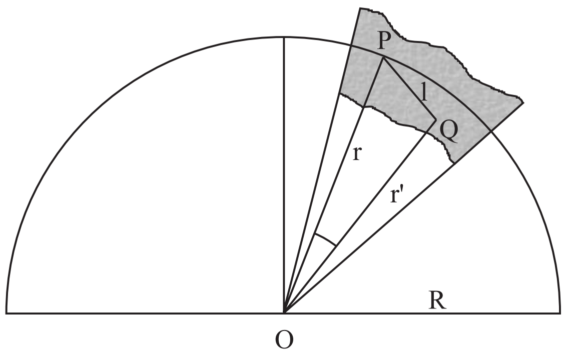

Fig. 1: The shaded area represents the whole crust from the surface down to The Moho.

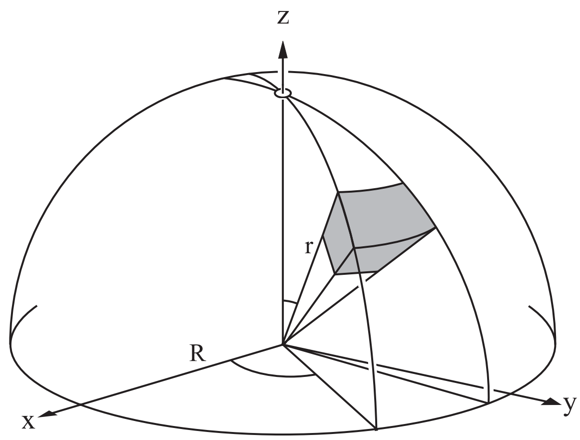

Fig. 2: A finite element of a mass body in a spherical prism form.

In order to evaluate Eq. 3 on the geoid, it is convenient to expand it into series of spherical harmonics. The expansion is to be performed separately for a case in which a mass element is above the reference sphere, i.e. for $\boldsymbol{r'} > \boldsymbol{R} = \boldsymbol{r}$, and separately for a mass element located below the reference sphere, i.e. for $\boldsymbol{r'} < \boldsymbol{R} = \boldsymbol{r}$.

The former, $r' > R = r$, is the case for most parts of the continental topographic masses. In this case Eq. 3 is given as follows:

$$V(r,\theta,\lambda) = G\rho \int d\lambda' \, d\theta' \sin\theta' \int dr' \, r' \sum_{l} \left(\frac{r}{r'}\right)^{l} P_l(\cos\psi) \tag{5}$$

where $P_l(\cos\psi)$ is the Legendre polynomial of degree *l*. Finally, according to Wang (1998), we have:

$$V = G\rho \sum_{l=0}^{\infty} H_l \sum_{m=-1}^{l} \overline{Y}_l^{m}(\theta,\lambda) \int d\lambda' \, d\theta' \sin\theta' \, \overline{Y}_l^{m}(\theta',\lambda') \tag{6}$$

where

$$\overline{Y}_l^{m} = \begin{cases} \overline{P}_l^{m}(\cos\theta)\cos m\lambda, m \geq 0 \\ \overline{P}^{|m|} \cos(\theta)\sin|m|\lambda, m < 0 \end{cases}$$

with $\overline{P}_l^{m}(\cos\theta)$ being the fully normalized associated Legendre function, and

$$H_l = \frac{R^2}{2l+1}\left(\frac{h_2 - h_1}{R} + \frac{1-l}{2}\frac{h_2^2 - h_1^2}{R^2} + \frac{(1-l)(-l)}{6}\frac{h_2^3 - h_1^3}{R^3} + \ldots\right)$$

where

$$\left.\begin{array}{l} h_1 = r_1 - R \\ h_2 = r_2 - R \end{array}\right\} \qquad \text{with } r_1 < r_2 .$$

The latter, $r' < R = r$, is the case where masses are located below the geoid as in most parts of the Earth's crust. For derivation of the useful formulas, Eq. 3 is at first re-written as follows:

$$V(r,\theta,\lambda) = G\rho \int \frac{1}{\sqrt{1+\left(\frac{r'}{r}\right)^2 - 2\left(\frac{r'}{r}\right)\cos\psi}} \frac{r'^2}{r} \sin\theta' \, dr' \, d\theta' \, d\lambda' \tag{7}$$

and then developed into series as follows (Wang 1998):

$$V = G\rho \sum_{l=0}^{\infty} D_l \sum_{m=-l}^{l} \overline{Y}_l^m(\theta,\lambda) \int d\lambda' \, d\theta \sin\theta' \, \overline{Y}_l^m(\theta',\lambda'') \tag{8}$$

with

$$D_l = \frac{R^2}{2l+1}\left(\frac{D_2 - D_1}{R} - \frac{l+2}{2}\frac{D_2^2 - D_1^2}{R^2} + \frac{(l+2)(l+1)}{6}\frac{D_2^3 - D_1^3}{R^3} - \ldots\right)$$

where

$$\left.\begin{array}{l} D_1 = R - r_2 \\ D_2 = R - r_1 \end{array}\right\} \qquad \text{with } D_1 < D_2 ;\ D \text{ being positive downwards.}$$

In order to investigate the contribution of the crust on the geoid, the geoidal undulation N caused by density anomalies in the crust is to be calculated. This is obtained from the well-known Bruns formula $N = T/\gamma$, where T is the disturbing potential on the geoid and γ is the normal gravity. The disturbing potential is the difference of the actual potential of the crust from the normal potential field. In order to calculate the normal potential field, the crust is to be divided into three homogeneous layers of equal thickness, Fig. 3. The depth of such a layer is the volume weighted mean depth of the corresponding layer of the actual crust, and its density is equal with the mean density of the actual layer.

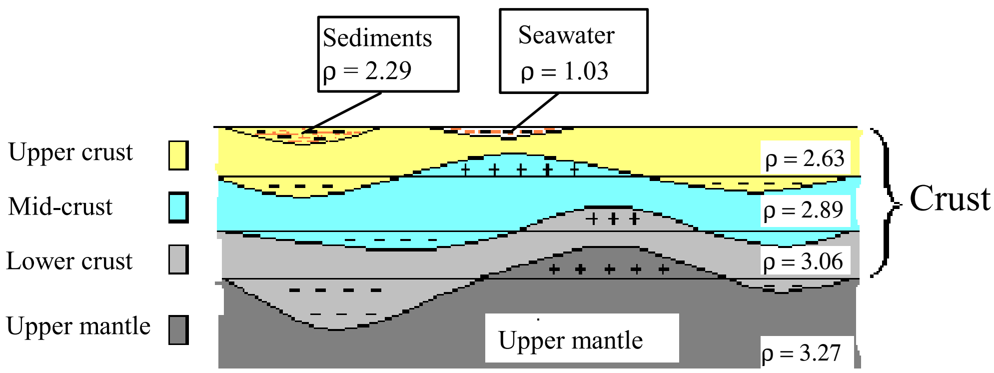

Fig.3: Mass models used for estimating the geoidal undulations from the crust. Straight lines show the boundaries of the normal (reference) mass model and curved lines those of the seismic (empirical) mass model. Positive and negative signs show the areas of mass surplus and mass deficiency, respectively.

4. Discussion

The deep seismic sounding method described was tested in Finland by Wang (1998) for estimating the contribution of the crust on the Fennoscandian gravimetric geoid. The work was the first contribution towards the solution of the problems related to this method. Influence of the layered structure of the crust on the geoid was found to be mainly due to the variation of the geometric shape of crustal layers. Variation of density inside the layers played a secondary role but was not insignificant. Accuracy obtained was found to be sufficient for the geophysical interpretation of the undulations of the Fennoscandian gravimetric geoid.

In the same way, the layered structure of the whole continental crust can be determined with the DSS for geophysical interpretation of the anomalies of the continental gravity field. To carry this out and to solve the problems related to the DSS method is a challenge to the geodesists and geophysicists in the next millenium.

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