Propagation Modelling of GPS Signals

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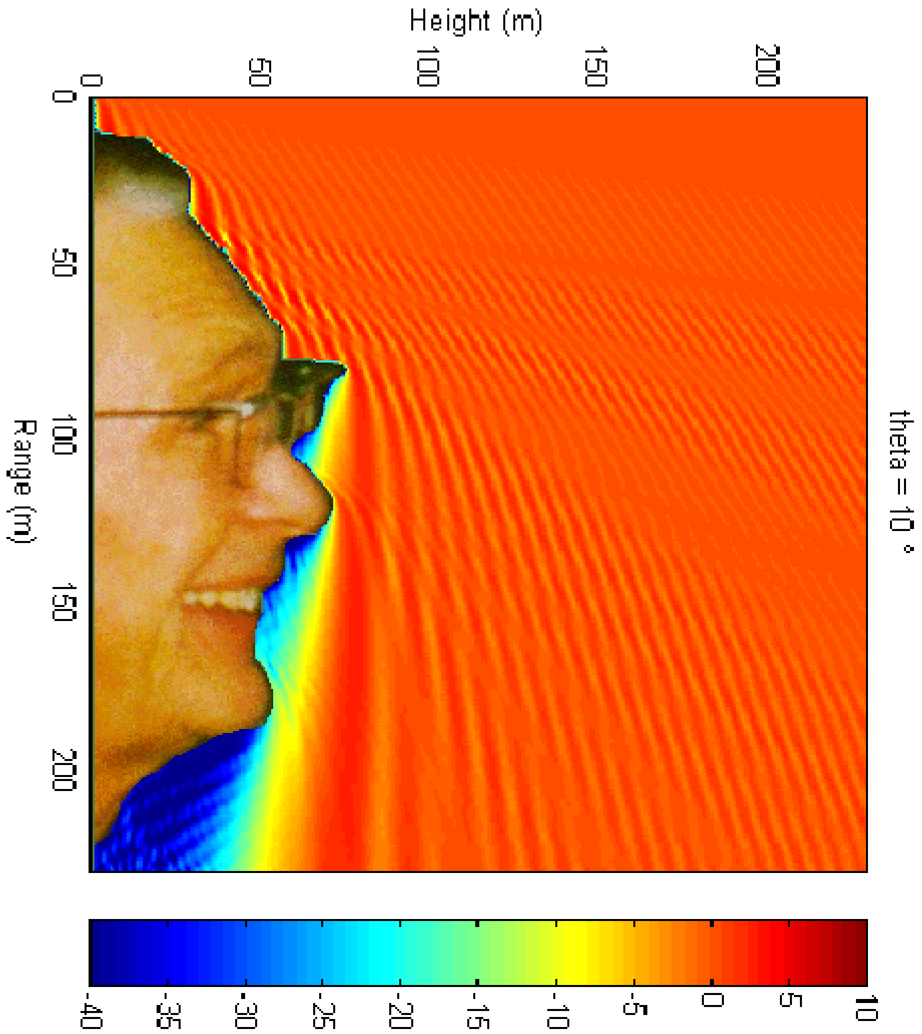

Simple Forward Propagation of 60 MHz Signal over Profile of Prof. Erik W. Grafarend

ABSTRACT

It has been previously shown [1] that the Parabolic Equation (PE) technique is well suited to solving GPS propagation problems over large domains. This paper presents the results of a modified model, which includes the effects of backscatter and provides time-domain representation of the propagated GPS signal. Results are presented for various difficult GPS propagation environments, including the determination of the Multipath Channel Impulse Response (MCIR) from the PE.

INTRODUCTION

The problem of signal multipath, in precision GPS applications, continues to remain largely unsolved. Traditionally two basic approaches have been taken to attempt to mitigate multipath effects in precision applications; special antenna designs; and specialised receiver architectures.

Special antenna designs, to mitigate multipath, are choke ring antennas and antenna ground planes. The fundamental aim of these designs is to physically limit the energy in unwanted signals, based on the direction of arrival at the antenna. A ground plane attenuates signals arriving at negative incident angles, whilst a choke ring antenna attenuates signals arriving below about 10°. For many environments these special antennas provide adequate performance, but for a truly ubiquitous sensor, it cannot be assumed that all multipath will arrive below 10° (see Figure 1). In the mining environment (or in any urban environment) a large variety of reflected signals will arrive from angles above 10° (sidewall reflections), thus reducing the effect of using such an antenna in these environments.

Another form of antenna mitigation useful for precision applications are beam-forming antennas. These antennas effectively form a narrow high-gain beam at the satellite signal of interest. Whilst these systems provide excellent multipath rejection in all environments, they are generally large and difficult to use in any practical system, since precise knowledge of the antenna attitude is required.

Special receiver architectures to minimise multipath effects essentially involve specialised correlator designs. Past developments in this area have included early-late slope (ELS)[3], narrow correlation[4], strobe correlation[5] and the Multipath Estimating Delay-Lock Loop MEDLL[6]. These approaches have all achieved some improvement in mitigation of the error effects of multipath. However, Weill[7] examined the theoretical limits for mitigation of code-phase multipath and found that one estimator for mitigating multipath can be claimed as optimal in a certain sense. Known as the minimum-mean-square error (MMSE) estimator, the multipath parameters are treated as random variables and the observed signal is used to construct a conditional probability density for the parameter values. In view of the disparity between the performance of the MMSE and what appears to be the state of the art, there seems to be room for more rigorous approaches.

The PE modelling technique is a linear time invariant model, however with the use of Fourier Time synthesis techniques, time dependence is able to be re-instated into the model. The resultant model is known as the PE-based Time Analysis (PETA) model.

The model is a wide-angle PE implementation optimised for GPS propagation. Multipath is characterised by amplitude, time delay, phase, and phase rate-of-change relative to the direct line-of-sight signal[2]. This model provides complete decomposition of the complex electromagnetic field components into these multipath parameters, through the simulation of the Multipath Channel Impulse Response. Results are presented for a variety of terrain features.

A discussion is made of the aims of this project whereby a complete environmental and receiver model is developed. This model encompasses the transmission of the signal from the satellite, its interaction with localised terrestrial terrain, and the manner in which the receiver correlators interpret the signal. This modelling strategy provides a tool that will assist in determining any relationships between multipath propagation behaviour and its effect in the receiver. These types of effects need to be determined before innovative mitigation techniques can be considered.

Without doubt, the progress in GPS multipath mitigation research has been significant and will continue. As the theoretical limits of correlation performance are approached there is justification for investigation of alternative receiver-based mitigation strategies. The PE propagation model, in conjunction with GPS receiver models, will form the basis for a comprehensive multipath analysis tool, necessary for extensive investigation of multipath mitigation techniques.

MULTIPATH MODELLING

The determination of propagation behaviour is important in the understanding of GPS multipath errors. The superposition of delayed replicas of the direct ranging signal leads to distortion of the received signal at the GPS antenna, and results in ranging errors of varying magnitude. In trying to develop an understanding of the impact these multipath signals have on the receiver, it is necessary to characterise the multipath signal. Multipath can be characterised by the three key parameters mentioned earlier; namely relative time delay, relative amplitude, and phase upon reflection. Together these parameters form the Multipath Channel Impulse Response (MCIR).

In this work it is shown how with the use of a PE propagation model and Fourier time-synthesis, the MCIR can be determined for various environments. Typical multipath propagation mechanisms are shown in Figure 1. To realistically model the propagation environment, we must not only deal with reflection, but also diffractive effects.

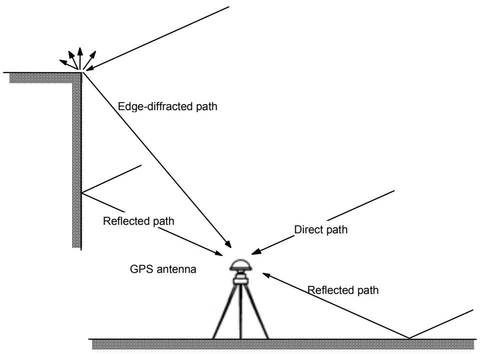

Figure 1 – Typical GPS antenna environment

The PE method is a full-wave solution to Maxwell's equations and hence provides the basis for the development of a multipath analysis tool.

PARABOLIC EQUATION MODELLING

The starting point for the development of an electromagnetic parabolic equation model is with the Helmholtz wave equation (1), for a field component, ψ, with assumed time dependence $e^{-j\omega t}$ [8].

$$\nabla^2\psi + k^2 n^2 \psi = 0 \tag{1}$$

where $k = \frac{2\pi}{\lambda}$ is the free space wave-number,

and $n = \frac{k}{k_0}$ is the refractive index.

We make use of cylindrical coordinates with assumed far-field invariance in azimuth, and remove the rapid phase variation through the reduced function u, with

$$\psi(x,z) \approx u(x,z)e^{jkx} \tag{2}$$

This yields the simplified elliptic equation,

$$\frac{\partial^2 u}{\partial r^2} + 2jk_0\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} + k_0^{\,2}\left(n^2 - 1\right)u = 0 \tag{3}$$

By defining the operators

$$P = \frac{\partial}{\partial r} \quad \text{and} \quad Q = \sqrt{n^2 + \frac{1}{k_0{}^2}\frac{\partial^2}{\partial z^2}}$$

we factorise equation (3) and select only the outgoing wave component. The result is a one-way, two-dimensional parabolic equation, which, for n equal to 1 (free-space propagation), is given by[9];

$$\frac{\partial u}{\partial r} + jk_0\left(1 - \sqrt{1 + \frac{1}{k_0{}^2}\frac{\partial^2}{\partial z^2}}\right)u = 0 \tag{4}$$

This equation is exact, within the limits imposed by the far-field approximation, and is evolutionary in range, allowing solution by an efficient Fourier transform based stepping technique[10]. The solution at a range-step (Δx) is given by

$$u(x + \Delta x, z) = F^{-1}\left[e^{jk\Delta x\left(\sqrt{1-\frac{p^2}{k^2}}-1\right)} F[u(x,z)] \right] \tag{5}$$

where p is the vertical wave-number and is related to k by $p = k\sin\theta$, with θ, the propagation angle relative to the horizontal. The *p-domain* defines the angular spectrum of the field, and together with the *z-domain*, they form a Fourier transform pair.

As can be seen from equation (5) we simply need to define some initial field condition at $x=0$, and march the solution out in range. For the case of GPS signal propagation, the field close to the Earth is essentially a uniform plane-wave. Thus we define our initial, or starting field condition, as a combination of incident and ground reflected Transverse Electric (TE) plane waves, and can write

$$E_i = E_0 e^{-jk(z\sin\theta - xcoz\theta)} + E_0 e^{-jk(z\sin\theta - xcoz\theta)}\Big|_{x=0} \tag{6}$$

Where E_0 is the incident field amplitude.

BOUNDARY-SHIFT TECHNIQUE FOR ARBITRARY TERRAIN

The boundary-shift technique[11], for handling arbitrary terrain within the PE code, involves the shifting of the field array (aperture) either up or down to account for the shift in the boundary position, and thus satisfy the boundary conditions. The field aperture immediately to the left of any obstructing terrain is stored then shifted down according to the height of the terrain element. The lower elements, those that would propagate into the terrain, are discarded and zeros inserted at the top of the array to maintain the correct number of elements. This modified field array is then propagated to the next array, with the Fourier-step technique. At negative terrain transitions, the reverse procedure is applied. The array is shifted up by the corresponding height, with the top elements discarded, and zeros inserted at the element positions where the field is obscured by the terrain. The result of the boundary shifting technique is simply a restructuring of the domain representation to that of a field propagating over a plane earth while accounting for diffractive effects over terrain.

IMPLEMENTATION OF BACKSCATTER

In the development of the PE, it was necessary to assume that the field was outgoing only. This one-way restriction can be lifted by using a store and forward method of back-propagating field components. The steps for implementation of a two-way PE model derived from a one-way model are as follows;

- The field is propagated with the one-way PE model in the forward (+x) direction
- The field components that will propagate into terrain (potential back-scatterers), are identified.
- These field values and indexes to their positions within the domain are stored for later use.

- The terrain profile and domain are mirrored vertically such that the one-way implementation can again be used without modifying the existing PE model code.
- The one-way PE is then used to propagate the stored field values that are added into the model as initial field conditions of the back-propagation.
- The field components of the forward and back implementations are then added to provide the resultant full field.

The use of this technique is justified by image theory, where the components at a vertical interface would travel to an image of the domain mirrored vertically about the vertical reflector. In addition, the method is complementary to the boundary-shift technique, where the down-shifted components normally discarded, are stored for use as the initial field values for a two-way PE implementation.

TIME-DOMAIN VIA FOURIER SYNTHESIS

The solution of the time-dependent field equation can be obtained by the Fourier transformation of the PE field solution[12], namely

$$u(x,z,t) = \int_{-\infty}^{\infty} S(f)\, u(x,z,f)\, e^{j2\pi ft}\, df \tag{7}$$

where *S(f)* is the spectrum of a source pulse and *u(x,z,f)* is the spatial transfer function derived from the PE modelling process. This integral is evaluated using Fast Fourier Transform (FFT) techniques at the spatial point of interest in the model domain, i.e. the antenna location. For this work we have chosen as our GPS time source, a *sinc* pulse of 1 nanosecond duration, modulated at the GPS *L1* frequency. The MCIR is the output of the PE Time analysis (PETA), and is given as a time series of delayed, and attenuated source pulses. The complex field in terms of the MCIR at a spatial point (*x,z*), is given by the addition of the decomposed plane waves

$$\psi_{PETA}(x,z) = e^{j2\pi ft_0} + \sum_{i=1}^{M} \alpha_i e^{j(2\pi ft_i + \phi_i)} \tag{8}$$

Here the first term represents the line-of-sight signal with a propagation time of t_0, from an arbitrary domain incident boundary at, $x=0$. The summation term represents, the M multipath signals, where α_i and t_i represent respectively, the i^{th} relative multipath amplitude and time of arrival. The phase term, ϕ_i, is the resultant phase shift due to the boundary reflection(s) for the i^{th} multipath signal. This equation can be normalised by assuming zero reference phase for the LOS signal. This normalisation is simply a change from absolute time delay, as presented by the PETA, to relative time delay, and is given by,

$$\psi_{PETA}(x,z) = 1 + \sum_{i=1}^{M} \alpha_i e^{j(2\pi f\tau_i + \phi_i)} \tag{9}$$

where τ_i is the time delay relative to the LOS signal.

DOMAIN REPRESENTATION AND PERFORMANCE

The propagation domain is represented by a two-dimensional plane that is specified by the azimuthal direction to the satellite, the maximum height, and the maximum range to be modelled. The antenna can be located at any point on the plane, above the terrain. Terrain information will be input from Digital Terrain Models (DTMs). The model domain is depicted in Figure 2.

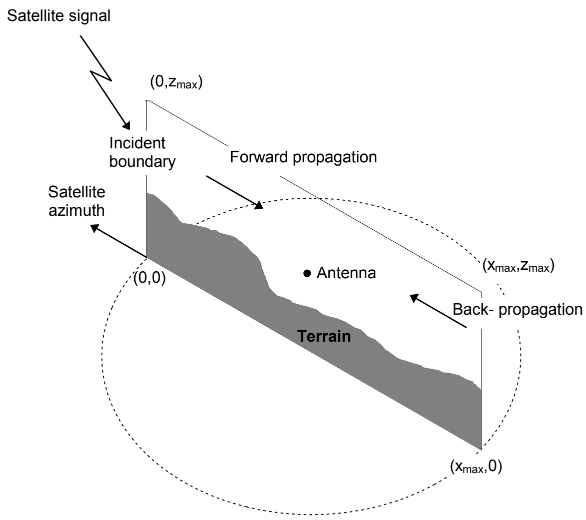

Figure 2 - Model Domain

The definitions of forward and back-propagation are relative to the directions specified in Figure 2.

Model simulation times for single frequency field values, with forward propagation only, is given by the proportionality

$$T_{PE}^{1-way} \propto k\theta A \tag{10}$$

where, k is the wavenumber, θ is the propagation angle, and A is the area of the domain plane. With inclusion of back-scatter this increases to

$$T_{PE}^{2-way} = (L+1)T_{PE}^{1-way} \tag{11}$$

for L back-scatterers. For the PETA the simulation time is

$$T_{PETA} = \frac{2\tau_{win}}{\tau_{pulse}} T_{PE}^{2-way} \tag{12}$$

where τ_{win} is the width of the time analysis window, and τ_{pulse} is the source pulse width.

GPS PROPAGATION RESULTS

Having established the basis for the modelling technique, results for several multipath environments are presented. These modelling results are based on the L1 GPS frequency of 1.575 GHz.

Validation of C/No

A comparison was made of predicted C/No against measured C/No[9]. A test site was chosen and data was recorded at 1 second epochs. The terrain, over which the satellite signal had propagated, was a single small building. The results are shown in Figure 3.

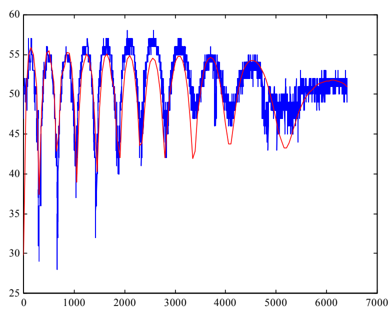

Figure 3 - Comparison of C/No Predicted vs. Observed

Figure 3 shows quite good agreement with measured results. These measurements were made in a relatively simple environment where the satellite signal was reflected from the roof-top of a large metal shed. The fading period and depth of fades agree well. Although this satellite was chosen because it's azimuth variation was small over the observation period there was a finite change which will affect the validity of the terrain profile used in the model.

Forward Specular Reflection Analysis

As a reference problem, a simple forward specular reflection problem is examined. The geometry of this problem is depicted in Figure 4. In this multipath situation we have the direct LOS signal (L) and a single multipath signal (R) arriving at the antenna.

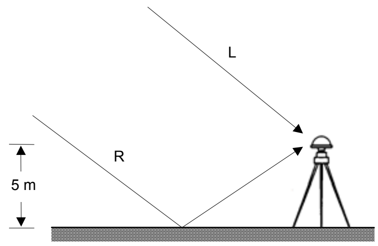

Figure 4 – Geometry of Forward Specular Reflection

A GPS satellite, rising in elevation from 5 degrees to 10 degrees, is modelled. Figure 5 and Figure 6 show, respectively, the calculated PE field, and the PETA result for a propagation angle of 8 degrees.

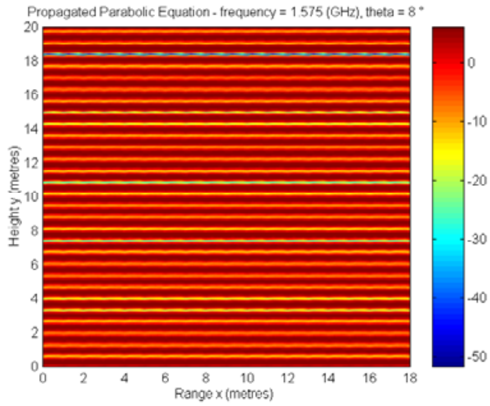

Figure 5 – Computed Field for Forward Scatter problem

This plot of the field strength shows the classical interference region pattern, with constructive and destructive interference clearly evident as a function of height. The computed field shown in this figure is for the full-space field and does not take into account the antenna gain pattern.

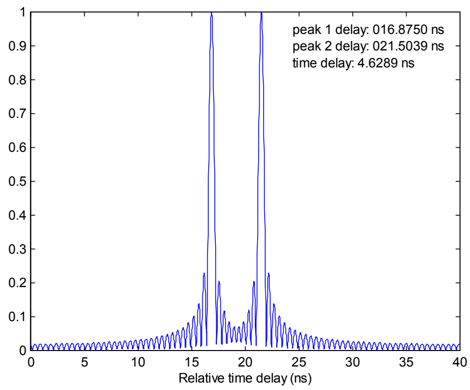

Figure 6 – Time analysis of Forward Scatter Problem

The time-domain analysis clearly shows the LOS and the multipath signals. Each of the multipath parameters is extracted from the PETA results, and using equation 9, we can reconstruct the total field. Figure 7 shows a comparison of the PETA estimated field compared to the full field solution as given by the PE propagation model.

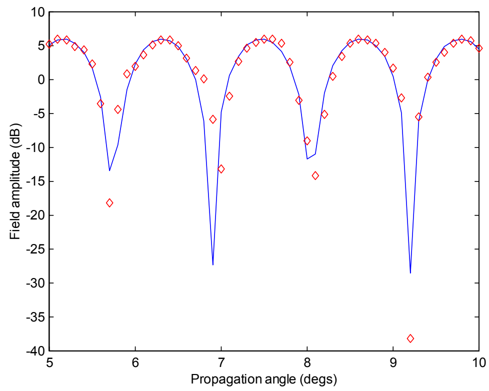

Figure 7 – Comparison PETA field vs. PE field

This figure again shows the classical fading pattern for a single multipath reflection. The results from the time-domain reconstruction are in good agreement with the full field result.

Forward Diffraction Analysis

Another propagation mode to consider is forward diffraction. A GPS satellite is modelled rising over a terrain obstuction, from an initial elevation angle of 5° to a final angle of 15°. In this case we can expect diffraction effects to dominate. The geometry of the problem is depicted in Figure 8, where the LOS signal exists, but *n* diffracted signals may also exist.

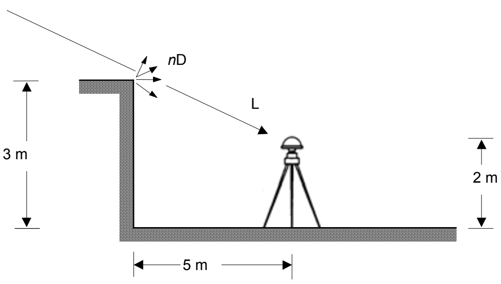

Figure 8 – Geometry of Forward Diffraction Problem

An instantaneous plot of the PE field, for a propagation angle of 10°, is given in Figure 9. The incident shadow boundary (ISB) can be seen (the interface between incident and diffraction illumination as described in [1]) as the -6dB boundary. Typical diffractive effects can be seen below the ISB (25-30dB attenuation of LOS signal) and forward scattering is seen above the ISB (c.f. Figure 5).

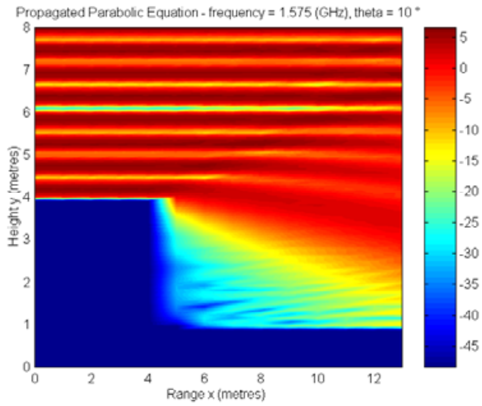

Figure 9 – Computed Field for Forward Diffraction Problem

Diffractive effects are evident in the region below the ISB. The diffractive effect on C/No is presented in Figure 10.

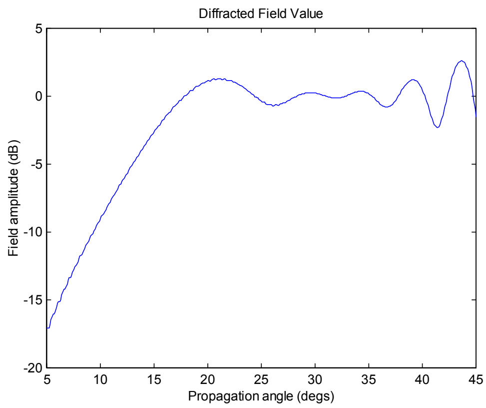

Figure 10 – Diffraction Effect on C/No

The computed field, for a satellite elevation change of 5° to 15°, shows a classical diffraction response. The field strength slowly rises as the antenna comes out of the shadow region, overshooting the incident field strength and oscillating about the 0dB LOS level. At approximately 35° the diffractive effect is almost zero and ground reflection interference starts to dominate. The ideal response for this scenario is for no field until 11° satellite elevation (when the satellite and antenna are LOS) and then a flat 0dB signal. The deviations from this ideal, seen in Figure 10, represent multipath from diffraction and ground reflections (forward scattering).

Figure 11 presents a comparison of the delay of the diffracted signal (upper plot) to that of the unobstructed line-of-sight (lower plot). This clearly indicates the additional path length caused by the diffraction of the signal around the terrain edge. If a receiver has a dynamic range of better than 20 dB then it is able to acquire and maintain track of the diffracted signal. At 5° the diffracted path delay is 0.2 ns representing approximately a 6 cm range error. The convergence of the two lines indicates that the diffractive effect (a function of elevation angle) eventually reduces to zero and the result is the same as that for the LOS case.

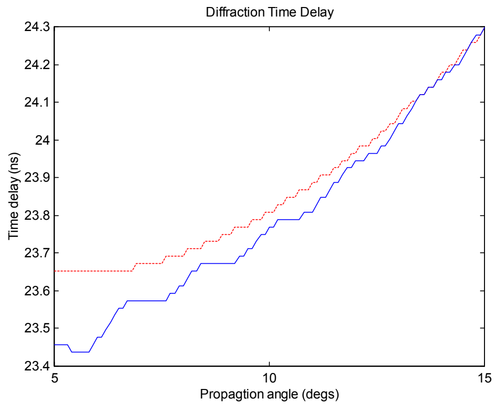

Figure 11 – Time delay profiles for Diffraction Problem

Stepped Back-Scatter Problem

Finally a much more complicated problem is examined. This is an example of an environment that can easily be found in urban environments. The geometry of the problem is seen in Figure 12.

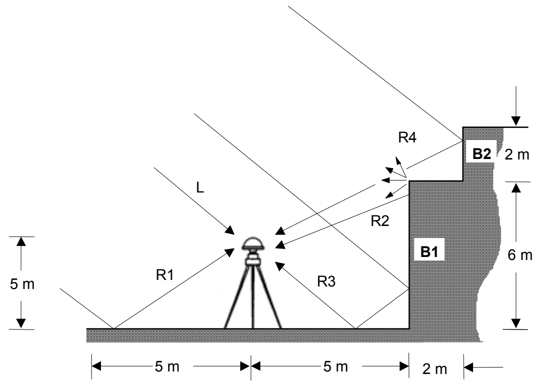

Figure 12 – Geometry of Stepped Back-Scatter Problem

This problem is referred to as a stepped backscatter example, where in addition to the forward scatter, signals are also scattered in the reverse propagation direction, from two distinct interfaces. Figure 13 shows the time-domain results for a 5° propagation angle.

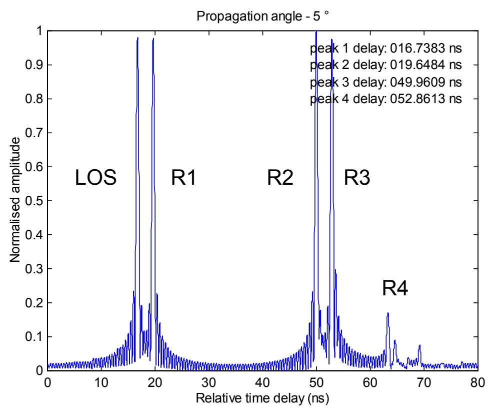

Figure 13 – Time Domain of Stepped Back-Scatter Problem (5°)

Here the LOS signal, forward scatter (R1), and two additional back-scattered multipath signals (R2 and R3) reflected from interface *B1* (see Figure 12) are seen. The first of these back-scattered multi-path signals is identified as a backscatter from above (R2), that is, the LOS is reflected from the *B1* interface and arrives at the antenna location from a positive elevation. The next signal is backscatter from below (R3), and is a reflection of the LOS from a combination of ground and interface (R3). At 5° elevation the *B2* interface is obstructed by the *B1* step. Close examination of Figure 13, shows some low level signal from the *B2* interface arrives at the antenna, but diffractive effects have reduced it's influence (diffracted R4).

The propagation mechanisms in this situation become evident at higher propagation angles. In Figure 14 the results for PETA at 12.5° satellite elevation are shown.

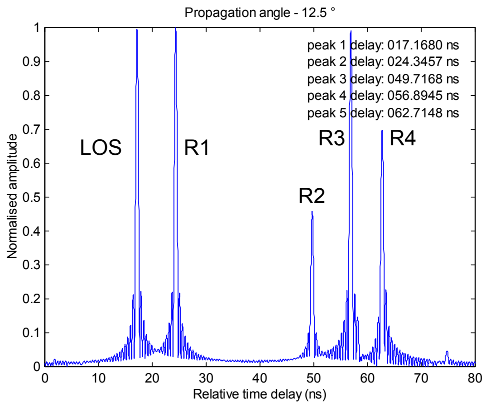

Figure 14 – Time Domain of Stepped Back-Scatter Problem (12.5°)

Figure 14 highlights the change in field due to a slightly higher satellite elevation. The multipath signal (R2) from *B1* (above the antenna) is no longer a direct reflection and is now a diffracted signal, hence its reduced signal level. The multipath signal from *B2* (R4) is becoming a direct reflection and thus it is becoming a stronger signal. At 15° (see Figure 15) the effect is more pronounced and the reflection from *B2* is essentially *in-the-clear*. It is noted that the reflection from *B1/ground* (R3) is unaffected, and that diffractive effects now dominate the reflection (R2) from the *B1* interface.

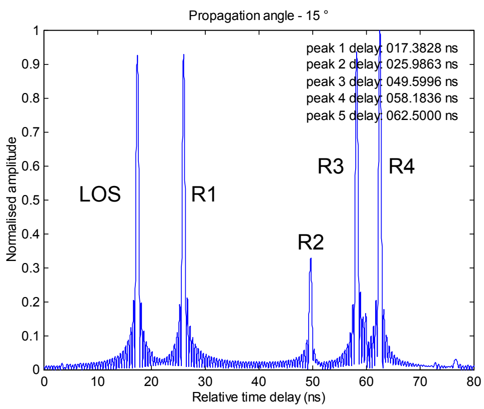

Figure 15 – Time Domain of Stepped Back-Scatter Problem (15°)

The total multipath propagation environment, as a function of elevation angle, is shown in Figure 16. Here we see the full influence of the diffractive effects for this situation.

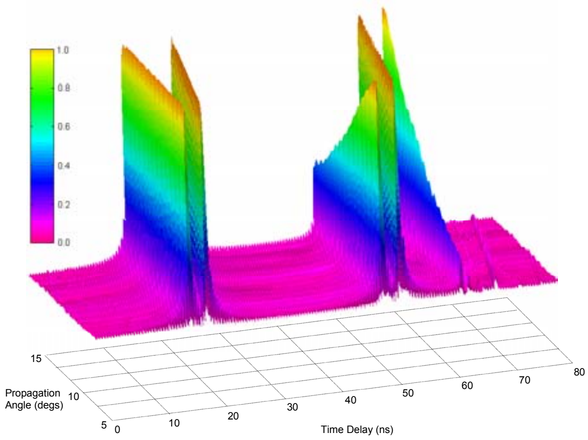

Figure 16 – Time delay profile as a function of satellite elevation

TERRESTRIAL PROPAGATION RESULTS

A common problem on many open cut mines is that of poor communications systems management. The open cut mine environment is an extreme environment for communications systems design due to the 'harshness' of the terrain. This 'harsh' terrain results in large shadow losses, requiring higher power transmissions and strategic placement of communications repeaters. The other factor affecting the communications problem, is that the shape of the mine is constantly changing. A mine communications system that was designed to provide 90% mine pit coverage in June 1997 for a strip mine, may not provide the required pit coverage in June 1998, since the next strip is now being processed. The increased distance from the communications transmitter now results in larger shadow losses, thus reducing the communications coverage in the pit.

Figure 17 shows the propagation over 4 pits on a typical open cut mine site, the 2D terrain profile was obtained from digital photogrammetry of a mine in Queensland. The simulation is for a 157Mhz Gaussian beam, mounted on a 20 m high tower, with the antenna main beam on the horizontal. If we wish to determine the change in performance by tilting the antenna down at 5º, we simply run the PE model again with a changed incident field condition. The results of this simulation appear in Figure 18. Comparison between these two figures shows that an improved coverage of the first pit can be made by tilting the antenna down at 5°. This does not affect the coverage in the second pit as can be seen in the simulations.

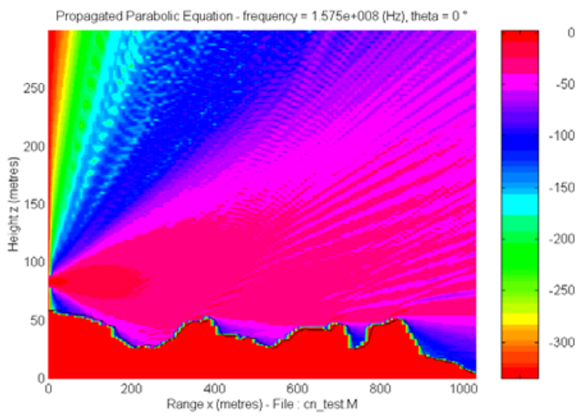

Figure 17 — Mine Repeater Propagation, 0° Antenna Tilt

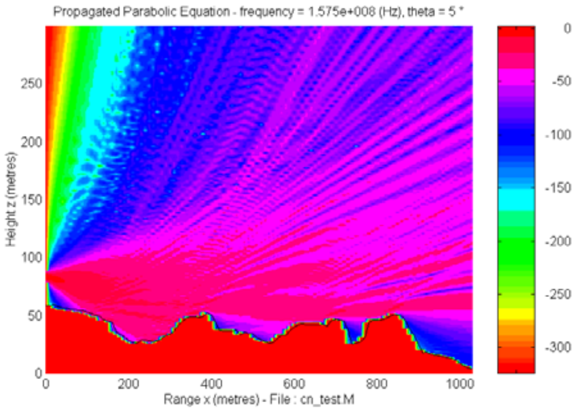

Figure 18 — Mine Repeater Propagation, 5° Antenna Tilt

CONCLUSIONS

It was seen in the stepped back-scatter example, how a relatively simple terrain environment can give rise to multiple delayed replicas of the GPS signal. It is hoped that these examples provide insight into the problems encountered when trying to overcome GPS multipath.

The advantage of numerical techniques, as discussed in this paper, is that the exact multipath nature of a complicated environment can be understood and decomposed. It is hoped that by combining a number of complex receiver models with the PE propagation models presented here, that a complete software-based satellite to user modelling system can be developed.

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