Earth Rotation as a Geodesic Flow, a challenge beyond 2000 ?

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Abstract

The title can be interpreted in different ways; the question may be treated alternatively. In other words, what part of energy is dissipated in such a way that energy is lost, as in case of coastal tides along shelf areas, so that the path of the pole is no longer a geodesic? And what part is preserved, as in tidal dissipation within the (closed) earth-moon-system where part of the energy is transferred within the closed system to the accelerated path of the moon from the decelerated earth? For instance, the role of the anelastic mantle was in detail discussed by Molodensky and Groten (1998).

In spite of increasing knowledge on global earth parameters it is still impossible to model completely earth rotation so that prediction would be possible for various applications in space science. The transition from celestial to terrestrial reference frames is a perpetual problem in a satellite geodesy where centimeter solutions are of practical interest. In spite of a lot of progress in recent time, we still rely on empirical approaches based on world-wide networks. Also in the interpretation of earth rotation observations a variety of unexplained phenomena prevails. F. Stacey's mentioning of a „never ending" story in his geophysical textbook still holds beyond the year 2000. We know that our present official nutation formulas and the precession constant are no longer up-to date now but precise celestial space systems are now independently defined and implemented and purely conventional terrestrial systems of ITRF-type represent more an artificial model than the actual earth. Insofar the geophysical interpretation of earth rotation data is mainly affected by those modelling effects. Then care is necessary, also beyond the year 2000, when IAU in 2001 updates present official formulas.

If IERS and WGAS of IAU could agree on the removal of existing inconsistencies, the way would be open for a new fundament system in a pure relativistic frame in 2001 and also for a new consistent GRS 2001 of SC-3 in IAG. This would indeed be progress along a geodesic.

1. Introduction

Everybody knows that I was always fascinated by Erik's work on exterior calculus, related to geodesia intrinseca. So when he showed me an excellent book on mathematics by Cushman and Bates (1997), I realized in a nice way the distinct formulation of geophysical problems in terms of mathematics, of physics using a limited number of parameters and the actual world. For the rotating earth the model of a Euler top etc. illustrates in an obvious way the problems we usually face with exact formulations of questions in earth rotation and geodynamics.

In talking about integrability and integrable systems we go "back to the roots" of E. Grafarend's work. He pointed out to me quite an interesting new discussion (Cushman and Bates 1997) of that topic. Erik treated this topic in relation to heights, deflections of the vertical and other classical "integrable" or "non-integrable" quantities. The idea to relate it to earth rotation is straight forward and leads to surprising results, as far as the motion of the pole is concerned. Unfortunately, its beauty gets lost when we face dissipative systems. Nevertheless, the mathematical beauty stands for itself and is as fascinating as in case of classical integrable systems as those in exterior or intrinsic geodesy or calculus.

Earth rotation studies, in particular polar motion research where we consider the rotation parameters of a deformable earth model from a terrestrial view point – i.e. in an earth-fixed frame of reference -, is often called a „never ending story“ because with each answer to questions we are confronted with several new open questions.

The rotation of an elastic or even anelastic earth in a relativistic frame is one of the most complex and intricate problems of relativistic hydrodynamics and still basically unresolved if contemporary accuracies of better than $\pm 10^{-10}$, as now possible and realistic, are asked for.

Even in a non-relativistic frame, the problem of a heterogeneous fluid outer core surrounding a stratified inner core and surrounded by an anelastic inhomogeneous mantle with a rather irregular ocean on top has never been solved as far as the frequencies from subdiurnal up to 18.6 years are concerned.

If we focus on particular aspects, such as core-mantle coupling at the core-mantle boundary (CMB), the variety of aspects becomes evident; starting from electromagnetic coupling and related „bumps“ which were long-time ago first contemplated by my good old friend S.K. Runcorn up to topographic coupling in connection with non-hydrostatic flattening at CMB, the unresolved physics behind all this is almost endless.

2. Various aspects of earth rotation research

Basically, the formulation of earth rotation in an earth fixed frame by classical means is done in terms of Liouville’s equation; for instance, for the Chandlerian motion we thus get (Sekiguchi, 1994)

$$\begin{aligned}\dot{x}_1/\sigma + x_1 &= \psi_1\\ \dot{x}_2/\sigma - x_2 &= \psi_2\\ LOD/86400 &= \psi_3\end{aligned}$$

with $\sigma = 2\pi/435$ days (Chandlerian wobble frequency), Ψ_i (i=1,2,3) = excitation functions components, LOD = length of day, $\dot{x}_i = dx_i/dt$ and x_i (i=1,2) polar motion components, as usually defined. Using Euler-angles we find geodesic solutions in terms of Legendre series.

The above equation is derived for uniform (mean) motion from its general form

$$\frac{d}{dt}\left[\mathbf{I}(t)\omega + \mathbf{h}(t)\right] + \omega \times \mathbf{I}(t)\omega + \mathbf{h}(t) = \mathbf{L} \qquad \text{where} \qquad \mathbf{H} = \mathbf{I}(t)\omega + \mathbf{h}(t)$$

with

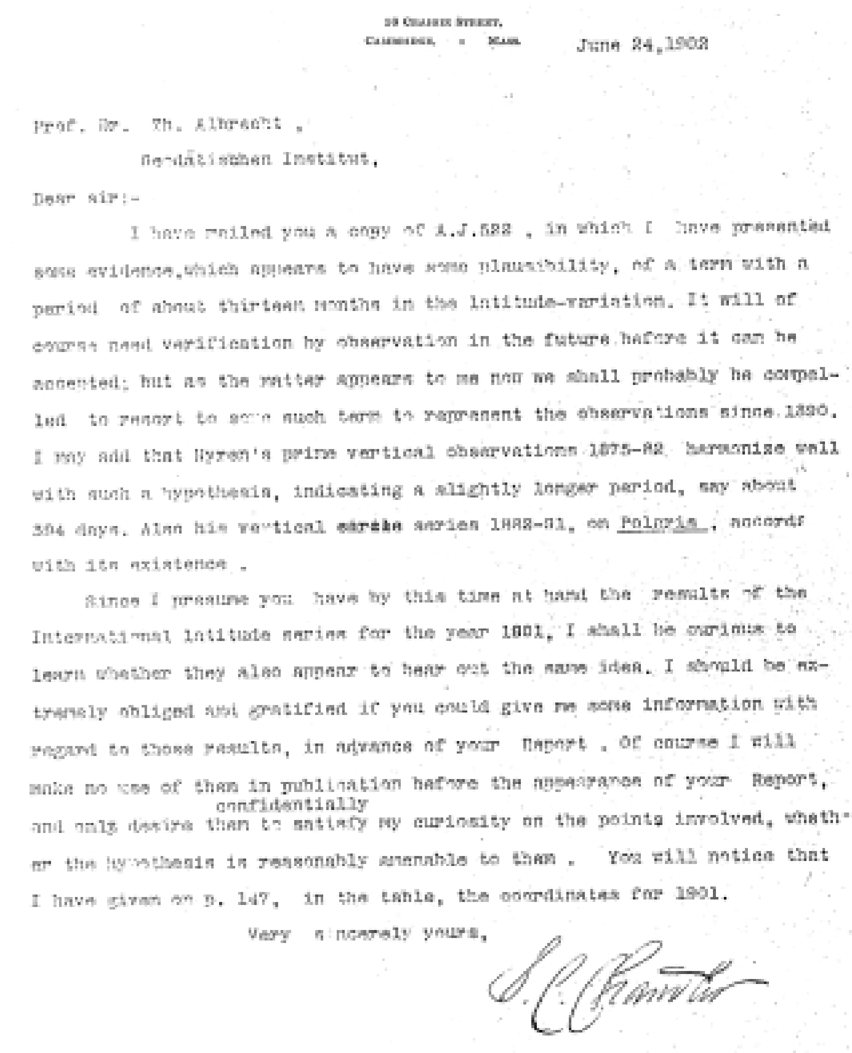
$\mathbf{I}$ = inertia tensor, $\mathbf{H}$ = angular momentum (AM); $\mathbf{h}$ =AM due to motion wrt x_i;
$\mathbf{L} = \dot{\mathbf{H}}$ = torque (dot indicates time derivatives), t = time
ω = angular velocity, x_i = body fixed axes, wrt = with respect to.

Solutions of Liouville equations for earth rotation in form of geodesics were the reason for the title of this paper suggested by Erik Grafarend. Nevertheless, it can also be interpreted in quite different ways, even in relativistic frames.

However, with the introduction of VLBI on the one side and superconducting gravimeters on the other side attenuation became a major topic which was already known from tidal (secular) friction down to seismic frequencies in terms of wave attenuation and associated quality factors Q_i which were assumed to be more or less frequency-dependent. There is still a lack of reconciliation and agreement of quality factors derived from various types of observations, e.g. at FCN frequency, but even at tidal frequencies the impact of attenuation became obvious which affects the unified "geodesic" concepts and attenuation itself became a significant tool of geophysical investigation. The parallelism of (1) a rotating body (like the earth), (2) motion of bodies around each other (as in case of the earth-moon or satellite systems) and (3) associated tides is always fascinating but has its limits. Nevertheless, by interrelating Love and load Love numbers we may even extend that principle to load tides and I have learnt a lot on that from Peter Varga. Together with my collaborators we used superconducting gravimetry and VLBI data mainly to constrain theoretical models.

From a mathematical viewpoint, the deficient knowledge of structure parameters for the earth (density, temperature, pressure, quality factor Q for different frequencies, anelasticity parameters etc.) is one reason for the failure of exactly modelling the earth's rotation, so that the question of non-linear „ill-posed" problems becomes serious, as even small deviations in the input lead to large errors in the results of inverse (or direct) problem formulations. S.M. Molodensky adopted the name „pathological" vibrations for such ill-posed solutions in Hadamard's sense.

Consequently, the observational approach dominated earth rotation and the beginning of related investigations is indeed fully characterised by such approaches where, e.g., S. Chandler's letter to the Geodetic Institute in Potsdam is an excellent demonstration; see Fig. 1.


16 Craigie Street,
Cambridge, Mass.

June 24,1902

Prof. Dr. Th. Albrecht ,

Geodätischen Institut,

Dear sir:-

I have mailed you a copy of A.J.522 , in which I have presented some evidence, which appears to have some plausibility, of a term with a period of about thirteen months in the latitude-variation. It will of course need verification by observation in the future before it can be accepted; but as the matter appears to me now we shall probably be compelled to resort to some such term to represent the observations since 1890. I may add that Nyren's prime vertical observations 1875-82 harmonize well with such a hypothesis, indicating a slightly longer period, say about 394 days. Also his vertical circle series 1882-91, on Polaris , accords with its existence .

Since I presume you have by this time at hand the results of the International latitude series for the year 1901, I shall be curious to learn whether they also appear to bear out the same idea. I should be extremely obliged and gratified if you could give me some information with regard to those results, in advance of your Report . Of course I will make no use of them in publication before the appearance of your Report, and only desire them confidentially to satisfy my curiosity on the points involved, whether the hypothesis is reasonably amenable to them . You will notice that I have given on p. 147, in the table, the coordinates for 1901.

Very sincerely yours,

S. C. Chandler

Fig. 1: Letter of S. Chandler to Th. Albrecht (Potsdam) after he had recovered the Chandler wobble (courtesy Prof. Jochmann, Berlin).

We might continue to point out other rotational components in polar motion such as the Nearly Diurnal Free Wobble (NDFW) related to free core nutation which appeared as a resonance phenomenon in the daily frequency band of earth tide observations where M.S. Molodensky started in 1957 a discussion of fluid outer core model alternatives parallel to Fedorov's polar motion consideration of related astronomical observations.

The empirical aspects of present definitions in earth rotation is obvious in the controversial definition of the Celestial Ephemeris Pole (CEP) which is presently reconsidered as only forced daily motion is subtracted from the earth rotation axis motion and free daily motion (because it cannot yet be modelled) is absent. Consequently, CEP still contains (contrary to its popular definition) daily perturbations such as NDFW.

For investigations, as we presently do at IPGD, in view of sub-diurnal interpretations we are hampered by the unsatisfying definition of CEP. Basically, this and other similar deficiencies of definitions lead to imprecise separation between nutation-precession and polar motion in the classical sense.

Until now we have still ignored the atmosphere, but if we look deeper into the subdiurnal or, generally speaking, high-frequency part of polar motion the interaction of ocean and atmosphere becomes one of the leading parts in generating functions.

Recently, even in long-periodic components the effects of „El Nino“ and „La Nina“ implied significant earth-rotation perturbations associated with „high“ and „low“ sea level variations in equatorial latitudes of the Pacific Ocean, also associated with low and high temperature and consequent density effects.

It is by no means clear, to what extent such climatic variations associated with strong wind and thunderstorms regionally affect long-term earth motion besides Milankovitch-cycles which originate from the motion of the earth in the ecliptic. B.F. Chao has recently discussed the climatic effects of variations of the obliquity, defining the angle between CEP and the ecliptic, as well as the influence of water reservoir water level variations and (global) earth rotation.

Even though monitoring the earth's rotation by VLBI and similar techniques now leads to relative accuracy of about $\pm 10^{-9}$ we still suffer in the interpretation of such data from the incomplete separation of plate tectonic motion at the VLBI-observatories from polar motion as deduced from such data.

Even completely independent techniques, such as inertial systems using INS-laser ring technology where, based on the Sagnac-effect, absolute motion is derived for such an INS-station, are effected by this deficient separation between plate tectonics and polar motion.

Consequently, purist people like H. Eichhorn in Gainsville/Florida always questioned the possibility to define a global rotation of a deformable earth but rather insisted in an individual rotation vector for every earth surface element at any observatory site.

With new possibilities to derive temporal variations of harmonic coefficients of the earth's gravitational potential, such as C_2^1, S_2^1 , the motion of the principal axis of inertial became an observable quantity but this axis is not identical with and difficult to relate to the conventional terrestrial pole of the IERS system.

In speaking of a „geodesic flow“ instead of a „geodetic flow“ of earth rotation we introduce basically a relativistic thinking. To model the motion of the earth in absolute space or in terms of a relativistic frame is still more controversial than ever before. One of the reasons for the intricacy is the lack of clear definitions and the deficiencies of implementing related corrections and reductions.

Take a simple example: Until 1998 it was clear what we mean by luni-solar and planetary precession. With $\pm 10^{-9}$ astrometry the planetary effects in luni-solar precession became significant as planetary attraction can no longer be ignored besides luni-solar attraction. Take another example:

Free motion due to ocean tides in the definition of Universal Time is significant and it therefore illogical to ignore free motion in CEP. "„Shifting" the CEP definition by including free motion (as far as it can be modelled) is simply a matter of definition but free motion is not constant in time but varying. Third case: Ignoring secular corrections in the transition of one dynamic time (such as TDT) to another, such as TDB, did not seriously affect results in the past; we may define theoretical „times" in a way similar to aberration where, we also leave out certain terms, by definition. However, if the „physical" meaning is not fully compatible with the abstract definition, difficulties arise wherever a step higher in accuracy is demanded. Similar problems arise with „geodesic precession" in case of defining a „non-rotating frame of reference" according to B. Guinot and others.

3. Outlook

As a result, IAU is now in a process of clarification in order to end up in the year 2001 with a relativistic frame of reference to which motion can be related exactly. The limits, however, are obvious as the definition of an inertial system is even not completely satisfying: an unaccelerated system has a clear dynamic meaning but its kinematical behavior is not defined: and we are close to the „Big Bang"-problem where the expanding world model does not answer all questions, as far as the kinematics are concerned.

So let us finally return to classical physics in an attempt to solve geophysical questions related to „generating functions". There are so many open questions in classical physics that, besides the $\pm 10^{-9}$ domain, still a lot of progress has to be made in classical physics applications. The basic formulae applied by us are shown in Table 1.

Table 1: The work from 1963 to 1998 in dealing with earth rotation at IPGD

1963: Hough equation:

$$\left(\mu\nabla^2 - \frac{\partial^2}{\partial t^2}\right)\nabla^2 p + 4\Omega^2 \frac{\partial^2}{\partial t^2} \quad \frac{\partial^2 p}{\partial z} = 0 \tag{1}$$

$\partial/\partial t$ = Eulerian derivative; t = time; p = pressure

1973: Euler's equation

$$p[\partial q/\partial t + (q\cdot\nabla)q] = -\nabla P - p\nabla\phi \tag{2}$$

q = velocity in an inertial system, ϕ = Potential of external forces
$-\nabla P$ = force per unit volume; Ω = spin, μ = shear modulus.

1983: Navier's equation (without rotation; Stokesian form):

$$\frac{\partial q}{\partial t} + (q\cdot\nabla)q = -\nabla p + \nu\nabla^2 q \tag{3}$$

$\nu = \eta/p$; p = density; η = viscosity (dynamical shear)

1993: Poincare's equation:

$$\nabla^2 p - \frac{4\Omega^2}{\omega_i^2} \frac{\partial^2 p}{\partial t^2} = 0 \tag{4}$$

1996: Boussinesq's equation:

$$\rho \frac{Dq}{Dt} = -\nabla P + \rho g \tag{5}$$

with g = gravity

1998:

$$\frac{\partial \omega_i}{\partial t} = \nabla x(q \times \omega_i) + \upsilon \nabla^2 \omega_i \tag{6}$$

with $\omega_i = \nabla$xq.

Liouville equation, as given above.

To a certain extent, the situation in earth rotation studies appears similar to the study of gravimetric problems, e.g., the determination of big „G", where observational intricacies lead to various assumptions and complicated theoretical explanations where, at the end, observational difficulties finally may primarily explain the controversial points. Take, e.g., the varying results obtained recently for big „G", i.e. Newton's Gravitational Constant (Schwarz et al., 1998, Kestenbaum, 1998).

Similarly, we still discuss frequency modulation of polar motion without finding appropriate models which could explain temporarily varying periods. As far as the Chandler period is concerned it may indeed be assumed that the Chandlerian period is not at all a free period but rather a conglomeration of forced vibrations around a period of 1.2 years. Who knows? In case of our results with sub-diurnal varying periods around the tidal bands oceanic effects of time-varying periods could be caused by non-linear effects in the ocean-atmosphere interaction at very high frequencies. But as in case of big „G" some people may agree with the Birmingham physicist C. Speake (see also Speake (1988) and Groten (1988)) who was quoted in Science, vol. 282, 18 Dec 1998 on page 2181 „Nobody gives a damn about Big G" also the earth rotation study will be a never ending story, so it will continue well beyond the year 2000.

The final answer to Erik's question in the title has certainly to be postponed but it is sure that we do not fully understand energy dissipation so that the deviations of pole path from a geodesic cannot be described in detail. The non-conservative forces for ocean-sea interaction may be partly understood for well surveyed systems such as the zonal winds in Antarctic regions, even for El-Nino-events but, e.g., for the short-period (sub-diurnal) phenomena investigated by us we are far from understanding what part is actually dissipative in the sense of non-conservative, then leading to deviations from an geodesic as a pole path.
If we follow Dziewonski, Kanamori etc. in assuming that part of the origin of Chandler wobble is due to (large) earthquakes there is a similar dissipation problem as the exact mechanism is unknown in which way energy due to seismic moments is radiated to polar motion, to heat and other forms of dissipation.

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