

# Robust Geodetic Parameter Estimation under Least Squares through Weighting on the Basis of the Mean Square Error

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## Abstract

A technique for the robust estimation of geodetic parameters under the least squares method when weights are specified through the use of the mean square error is presented. The mean square error is considered in the specification of observational weights instead of the conventional approach based on the observational variance. The practical application of the proposed approach is demonstrated through computational examples based on a geodetic network. The results indicate that the least squares estimation with observational weights based on the mean square error is relatively robust against outliers in the observational set, provided the network (or the system) under consideration has a good level of reliability, as to make the network (or system) stable under estimation.

## Introduction

The classical approach in the estimation of geodetic parameters is through the least squares method within the framework of the Gauss-Markov model given as:

$$\tilde{y} = Ax + \varepsilon; \varepsilon \sim (0, \sigma_0^2 W^{-1}) \quad (1)$$

where  $y$  is an  $n \times 1$  vector of observations,  $A$  is an  $n \times m$  design matrix,  $x$  is an  $m \times 1$  vector of unknown parameters,  $\varepsilon$  is an  $n \times 1$  vector of observational errors,  $\sigma_0^2$  is the variance of unit weight, and  $W$  is an  $n \times n$  positive-definite weight matrix.

This estimation model is based on the assumption that the observational errors collected in the vector  $\varepsilon$  occur randomly and are distributed according to the normal distribution. with this assumption and under the least squares condition that  $\varepsilon^T W \varepsilon$  be minimum, the following estimates may be obtained:

$$\begin{aligned} \hat{x} &= (A^T W A)^{-1} A^T W \tilde{y} \\ D(\hat{x}) &= \Sigma_{\hat{x}\hat{x}} = \hat{\sigma}_0^2 (A^T W A)^{-1} \\ \hat{y} &= Ax = A(A^T W A)^{-1} A^T W \tilde{y} \\ D(\hat{y}) &= \Sigma_{\hat{y}\hat{y}} = A \Sigma_{\hat{x}\hat{x}} A^T = \hat{\sigma}_0^2 A(A^T W A)^{-1} A^T \\ \hat{\varepsilon} &= y - \hat{y} = y - A\hat{x} = y - A(A^T W A)^{-1} A^T W \tilde{y} \\ D(\hat{\varepsilon}) &= \Sigma_{\hat{\varepsilon}\hat{\varepsilon}} = \Sigma_{\tilde{y}\tilde{y}} - A \Sigma_{\hat{x}\hat{x}} \\ \hat{\sigma}_0^2 &= \hat{\varepsilon}^T W \hat{\varepsilon} / (n - m) \end{aligned} \quad (2)$$

In the event however that the observational vector  $y$  may be contaminated with a bias parameter  $b$  (whereby the bias may be as a result of gross errors, or systematic errors, or a combination of both),

then the assumption  $\varepsilon \sim (0, \sigma_0^2 W^{-1})$  gets invalidated, in that the errors on  $y$ , which now also comprise  $b$  can no longer be considered to be distributed according to the normal distribution. The consequence of this is that if the estimation of the unknown parameters still be performed according to the least squares condition, under the Gauss-Markov model as in (1), then the so obtained estimates will be biased as a result of  $b$ . To deal with this problem, two options come into consideration: (i) one performs the estimation under the least squares under the model (1) but seeks to identify and remove outliers (biased observations) from the observational dataset in what we may refer to as *outlier detection*, or (ii) one adopts estimation techniques that are robust with respect to the biases under *robust estimation*.

The propagation of outlier detection in geodesy and surveying was motivated by the works of W Baarda [2, 3, 4]. Today outlier isolation forms an integral component of any major geodetic data processing and analysis. However the detection and isolation of outliers within the framework of the Gauss-Markov model as specified in (1), still suffers from the tendency of the ordinary least squares method to spread out the effect of outliers among observations, thereby rendering the isolation of the outliers difficult, and sometimes altogether impossible. To cope with this problem, robust estimation techniques offer real alternatives.

The objective in robust estimation is to perform an estimation of the parameters from the observations in such a way that the estimates of the parameters so obtained are virtually unaffected by any biases or outliers that may be present in the observations. An extensive study of the application of robust estimation in geodesy is reported in [5]. Robust estimation techniques in the estimation of parameters in general were however brought to the fore through the works of P J Huber [8, 9, 10], while a further extensive treatment of the subject has been presented by [7]. The core of Huber's technique is the M-estimator, which is based on the maximum-likelihood method.

A general characteristic of the robust estimation techniques is that they restrict a range of observational error within which the observations may be accepted, and observations associated with observational error outside the specified range are 'cut off' from the estimation process within the process of 'winsorisation'. The problem in this approach however is that the decision on where the 'cut-off' point itself should be is rather subjective. As an alternative approach in robust estimation, a procedure for robust estimation based on iterative weighting of observations was suggested in [1]. This was an attempt at a procedure that would avoid excluding any observations from the estimation procedure, but include all the observations within the estimation procedure except with appropriate weighting.

In this presentation, we extend the concept of iterative weighting by considering it from the point of view of the observational weights based on the *mean square error* (MSE), and evaluate the effectiveness of the method through the computation of a practical network.

### The Mean Square Error

Let us consider a parameter vector  $\xi$ , whose realisation (obtained through estimation or otherwise) is  $\hat{\xi}$ , then the mean square error of  $\hat{\xi}$  is given as

$$M(\hat{\xi}) = E[(\hat{\xi} - \xi)(\hat{\xi} - \xi)^T] \quad (3)$$

In general we have that  $E(\hat{\xi}) = \xi + \beta$  where  $\beta$  is a bias vector. Thus we may rewrite (3) as

$$\begin{aligned} M(\hat{\xi}) &= E[(\hat{\xi} - (E(\hat{\xi}) - \beta))(\hat{\xi} - (E(\hat{\xi}) - \beta))^T] \\ &= E[(\hat{\xi} - E(\hat{\xi}))(\hat{\xi}^T - E(\hat{\xi})^T)] + \beta\beta^T \end{aligned} \quad (4)$$

But we have that the dispersion  $D(\hat{\xi})$  of  $\hat{\xi}$  is given as

$$D(\hat{\xi}) = E[(\hat{\xi} - E(\hat{\xi}))(\hat{\xi}^T - E(\hat{\xi})^T)] \quad (5)$$

Thus

$$M(\hat{\xi}) = D(\hat{\xi}) + \beta\beta^T \quad (6)$$

(see e.g. [11] and [6]).

In the special case that  $\beta = 0$ , we have then that

$$M(\hat{\xi}) = D(\hat{\xi}) \quad (7)$$

From the fact that the mean square error incorporates the biases in the realisation of a parameter, the mean square error is a much more effective and efficient estimate of the quality of the parameter in the sense of *accuracy*. The dispersion on the other hand, respectively the variance, as is ordinarily known, gives the *precision* of the estimate or realisation, which however only becomes also a measure of accuracy in the special case when  $\beta = 0$ , in which case (7) obtains.

We have from (6) that in the special case that it is a single independent parameter being considered, the mean square error is given as: *mean-square-error* = *variance* + *bias*<sup>2</sup>.

### The Estimation Model

In the event that the observation  $\tilde{y}$  in (1) is contaminated with a bias  $b$ , then we have that

$$E(\tilde{y}) = y + b \quad (8)$$

where  $y$  is the 'true' value of the parameter.

But we have that

$$\tilde{y} = E(\tilde{y}) + \varepsilon; \varepsilon \sim (0, \Sigma_{\tilde{y}\tilde{y}}) \quad (9)$$

Then with (8) and (9) we have

$$\tilde{y} = y + b + \varepsilon \quad (10)$$

which with  $v := b + \varepsilon$ , becomes

$$\tilde{y} = y + v \quad (11)$$

For

$$y = Ax \quad (12)$$

we then have that  $\tilde{y} = Ax + b + \varepsilon$  or

$$\tilde{y} = Ax + v, E(v) = E(b + \varepsilon) = b, M(\tilde{y}) = \Sigma_{\tilde{y}\tilde{y}} + bb^T \quad (13)$$

We adopt this as the model for the estimation of the parameters within the framework of least squares.

We note therefore from (13) that if we can estimate  $v$  such that

$$E(v) = E(b + \varepsilon) = b + E(\varepsilon) = b, \quad (14)$$

then we would have been able to obtain an unbiased estimate of  $x$  that is relatively free from the influence of the bias  $b$ .

In the conventional least squares approach, whereby the model is defined according to (1), if the model had a bias parameter as to be described according to (13), but with the stochastic part described

through  $\varepsilon \sim (0, \Sigma_{\tilde{y}\tilde{y}} = \sigma_0^2 W^{-1})$ , then the model would have not been appropriately specified, so that the parameters estimated with the model will be biased. We seek to overcome the bias effect in that we define the estimation model through (14) and weight the observations according to the mean square error (MSE), which already incorporates the bias effect. We propose then to define the weight  $W$  of the observations as

$$W = \sigma_0^2 M_{\tilde{y}\tilde{y}}^{-1} \quad (15)$$

in which we have taken  $M_{\tilde{y}\tilde{y}} = M(\tilde{y})$ .

If we assume independence of observations  $\tilde{y}_i (i = 1, \dots, n)$ , then we have that for an observation  $\tilde{y}$ , the mean square error may be given as

$$m_i = \sigma_i^2 + b_i^2 \quad (16)$$

for  $\sigma_i^2$  and  $b_i$  being respectively the variance and bias of  $\tilde{y}_i$ . Then the weight of  $\tilde{y}_i$  can now be defined as

$$w_i = \frac{\sigma_0^2}{m} \quad (17)$$

With the weights so defined, we notice that  $M_{\tilde{y}\tilde{y}}^{-1}$  will exist due to the fact that  $M_{\tilde{y}\tilde{y}}$  has been taken to be a diagonal matrix, and hence  $W$  according to (15) can be evaluated.

The question however is how does one evaluate the mean square error in the first place, when the bias  $b$  itself is in the first instance unknown, and must in any case be evaluated. We seek to deal with this problem in that we evaluate  $b$  iteratively and hence also  $W$ .

### The Estimation Process

We begin the estimation process by assuming nominally that  $b = 0$ . With this, we notice that we will simply be having the Gauss-Markov model as described in (1). From this, the first estimates of  $b$  as ‘residuals’ will have been obtained. With the residuals  $v_i$ , a new value for  $m_i$  is obtained according to (16), however with  $\sigma_i$  being as originally set, since these are the original variances of the observations, which are assumed known a priori. With the new mean square error values, the estimation process is repeated. The process is repeated until convergence for the estimated parameters is achieved at the specified level of tolerance. In particular, since the main parameters being estimated are the unknown parameter vector  $x$ , the convergence of the  $x$  parameters would be more appropriately adopted as control for the iteration.

Through the iterative process, the mean square error of an observation is estimated for simultaneously as well and consequently the mean-square-error weight of the observations. The robustness of the procedure is thus contained in the mean-square-error weight, which is a much more comprehensive and realistic representation of the observational weights.

## The Test Example

### *The test network*

A two-dimensional network as shown in Fig. 1 was adopted for the test example. The network comprises 9 points, which are linked by distance observations. A single distance observation was considered to have been measured with a standard error of 3mm+0.5ppm; with this the eventual standard error for the mean distance adopted was then deduced from the number of individual measurements from which the particular mean distance is obtained. The network has a total of 30 distance measurements.

### *Experimental design*

Four versions of the network were computed; these were designated as Net-0, Net-1, Net-2, and Net-4. The networks were specified according to the numbers of gross errors they contained as follows: Net-0 - no gross errors; Net-1 - one gross error; Net-2 - two gross errors; and Net-4 - four gross errors. The gross errors were simulated into the networks as given in Table 1.

Table 1: The simulated gross errors

Line	Error [metres]	Network
4-7	+0,780	1,2,4
2-3	-5,067	2,4
7-11	+0,355	4
3-4	-0,055	4

Each version of the network was then computed on the basis of both the ordinary least squares and the least squares method with mean-square-error weights as proposed here. The network was computed throughout in free-network mode.

### *Results*

In the results presented below,  $X$  and  $Y$  are estimated point coordinates in metres;  $\sigma_X$  and  $\sigma_Y$  are estimated positional standard errors in metres;  $a$  and  $b$  are the major and minor axes of the positional error ellipse in metres, while  $\phi$  is the orientation of the major axis of the ellipse in degrees taken with respect to the  $X$  axis.

### *Net-0*

Table 2: Conventional least squares

Point	X	Y	$\sigma_X$	$\sigma_Y$	a	b	$\phi$
1	5428972.186	3462429.367	0.0044	0.0059	0.0063	0.0039	155.4
2	5439065.854	3468259.524	0.0044	0.0053	0.0056	0.0040	150.5
3	5457025.454	3476522.257	0.0054	0.0059	0.0059	0.0054	170.3
4	5465079.259	3433374.141	0.0045	0.0064	0.0065	0.0042	163.0
5	5448374.040	3427727.520	0.0039	0.0043	0.0047	0.0034	143.1
6	5439527.166	3423319.106	0.0053	0.0066	0.0076	0.0036	145.4
7	5447601.042	3443324.505	0.0060	0.0038	0.0060	0.0037	9.8
8	5411104.687	3454335.360	0.0064	0.0092	0.0097	0.0056	158.0
11	5464986.157	3457965.548	0.0086	0.0053	0.0086	0.0053	179.8

Table 3: Robustified least squares (3 iterations)

Point	X	Y	$\sigma_X$	$\sigma_Y$	a	b	$\phi$
1	5428972.186	3462429.370	0.0030	0.0042	0.0045	0.0026	155.4
2	5439065.857	3468259.522	0.0030	0.0043	0.0045	0.0026	157.1
3	5457025.455	3476522.260	0.0036	0.0040	0.0040	0.0036	3.4
4	5465079.258	3433374.142	0.0030	0.0041	0.0043	0.0028	161.1
5	5448374.040	3427727.518	0.0027	0.0032	0.0034	0.0024	146.8
6	5439527.168	3423319.103	0.0036	0.0049	0.0055	0.0025	147.8
7	5447601.039	3443324.504	0.0041	0.0028	0.0042	0.0027	12.9
8	5411104.688	3454335.360	0.0042	0.0060	0.0064	0.0036	156.0
11	5464986.155	3457965.548	0.0057	0.0035	0.0057	0.0035	2.1

Observations treated as containing gross errors in the adjustment: Nil

### *Net-1*

Table 4: Conventional least squares

Point	X	Y	$\sigma_X$	$\sigma_Y$	a	b	$\phi$
1	5428972.119	3462429.461	0.0616	0.0834	0.0882	0.0545	155.4
2	5439065.822	3468259.569	0.0625	0.0745	0.0796	0.0559	150.5
3	5457025.475	3476522.203	0.0767	0.0828	0.0830	0.0765	170.3
4	5465079.471	3433373.962	0.0631	0.0899	0.0922	0.0597	163.0
5	5448374.111	3427727.603	0.0556	0.0606	0.0666	0.0484	143.1
6	5439527.284	3423319.082	0.0741	0.0932	0.1078	0.0505	145.4
7	5447600.816	3443324.577	0.0841	0.0537	0.0848	0.0525	9.8
8	5411104.631	3454335.397	0.0898	0.1302	0.1367	0.0795	158.0
11	5464986.118	3457965.474	0.1209	0.0752	0.1209	0.0752	179.8

Table 5: Robustified least squares (6 iterations)

Point	X	Y	$\sigma_X$	$\sigma_Y$	a	b	$\phi$
1	5428972.189	3462429.366	0.0033	0.0047	0.0051	0.0027	153.8
2	5439065.857	3468259.524	0.0031	0.0042	0.0044	0.0028	156.4
3	5457025.454	3476522.262	0.0038	0.0043	0.0043	0.0038	178.6
4	5465079.254	3433374.146	0.0042	0.0049	0.0056	0.0032	143.1
5	5448374.039	3427727.516	0.0031	0.0035	0.0036	0.0029	153.1
6	5439527.165	3423319.103	0.0043	0.0053	0.0061	0.0029	144.4
7	5447601.044	3443324.502	0.0055	0.0030	0.0055	0.0030	177.8
8	5411104.690	3454335.360	0.0045	0.0065	0.0069	0.0038	155.0
11	5464986.154	3457965.550	0.0061	0.0038	0.0061	0.0038	4.8

Observations treated as containing gross errors in the adjustment

Line	Gross error as isolated (metres)	Redundancy
4 – 7	-0.7952	100%

## Net-2

Table 6: Conventional least squares

Point	X	Y	$\sigma_X$	$\sigma_Y$	a	b	$\phi$
1	5428972.886	3462429.822	0.4272	0.5778	0.6114	0.3775	155.4
2	5439067.003	3468259.524	0.4332	0.5164	0.5515	0.3876	150.5
3	5457022.867	3476521.657	0.5314	0.5738	0.5750	0.5301	170.3
4	5465079.528	3433373.759	0.4374	0.6229	0.6391	0.4135	163.0
5	5448374.140	3427727.373	0.3855	0.4203	0.4615	0.3351	143.1
6	5439527.264	3423318.920	0.5135	0.6457	0.7470	0.3502	145.4
7	5447600.915	3443324.313	0.5827	0.3720	0.5880	0.3637	9.8
8	5411105.229	3454336.092	0.6223	0.9021	0.9475	0.5507	158.0
11	5464986.014	3457965.650	0.8379	0.5210	0.8379	0.5210	179.8

Table 7: Robustified least squares (8 iterations)

Point	X	Y	$\sigma_X$	$\sigma_Y$	a	b	$\phi$
1	5428972.187	3462429.365	0.0037	0.0049	0.0052	0.0032	153.5
2	5439065.854	3468259.524	0.0038	0.0043	0.0045	0.0035	147.7
3	5457025.460	3476522.263	0.0061	0.0045	0.0062	0.0043	14.5
4	5465079.254	3433374.146	0.0043	0.0051	0.0058	0.0033	144.4
5	5448374.039	3427727.516	0.0030	0.0035	0.0036	0.0029	157.4
6	5439527.165	3423319.103	0.0043	0.0052	0.0061	0.0028	143.5
7	5447601.044	3443324.503	0.0054	0.0030	0.0054	0.0030	176.4
8	5411104.689	3454335.358	0.0046	0.0065	0.0068	0.0042	158.9
11	5464986.154	3457965.550	0.0061	0.0039	0.0061	0.0039	0.5

Observations treated as containing gross errors in the adjustment

Line	Gross error as isolated (metres)	Redundancy
4 – 7	-0.7951	100%
2 – 3	+5.0783	100%

## Net-4

Table 8: Conventional least squares

Point	X	Y	$\sigma_X$	$\sigma_Y$	a	b	$\phi$
1	5428972.864	3462429.832	0.4259	0.5760	0.6096	0.3764	155.4
2	5439066.998	3468259.726	0.4319	0.5149	0.5498	0.3864	150.5
3	5457022.859	3476521.637	0.5298	0.5721	0.5733	0.5284	170.3
4	5465079.501	3433373.799	0.4361	0.6210	0.6371	0.4122	163.0
5	5448374.131	3427727.363	0.3843	0.4190	0.4601	0.3341	143.1
6	5439527.268	3423318.877	0.5119	0.6437	0.7447	0.3491	145.4
7	5447600.832	3443324.287	0.5810	0.3709	0.5862	0.3625	9.8
8	5411105.206	3454336.094	0.6204	0.8993	0.9446	0.5491	158.0
11	5464986.187	3457965.713	0.8354	0.5194	0.8354	0.5194	179.8

Table 9: Robustified least squares (8 iterations)

Point	X	Y	$\sigma_X$	$\sigma_Y$	a	b	$\phi$
1	5428972.136	3462429.354	0.0061	0.0066	0.0073	0.0052	142.2
2	5439065.800	3468259.517	0.0071	0.0058	0.0077	0.0049	148.5
3	5457025.431	3476522.214	0.0085	0.0089	0.0091	0.0083	29.9
4	5465079.216	3433374.153	0.0060	0.0082	0.0088	0.0051	152.9
5	5448374.005	3427727.512	0.0042	0.0060	0.0060	0.0042	2.0
6	5439527.134	3423319.092	0.0061	0.0074	0.0085	0.0046	145.3
7	5447600.999	3443324.497	0.0077	0.0055	0.0077	0.0055	176.9
8	5411104.642	3454335.337	0.0065	0.0085	0.0086	0.0063	163.0
11	5464986.482	3457965.652	0.0123	0.0083	0.0124	0.0081	167.4

Observations treated as containing gross errors in the adjustment

Line	Gross error as isolated (metres)	Redundancy
4 – 7	-0.7951	100%
2 – 3	+5.0783	100%
2 – 11	+0.3184	99%
5 – 11	+0.2669	100%

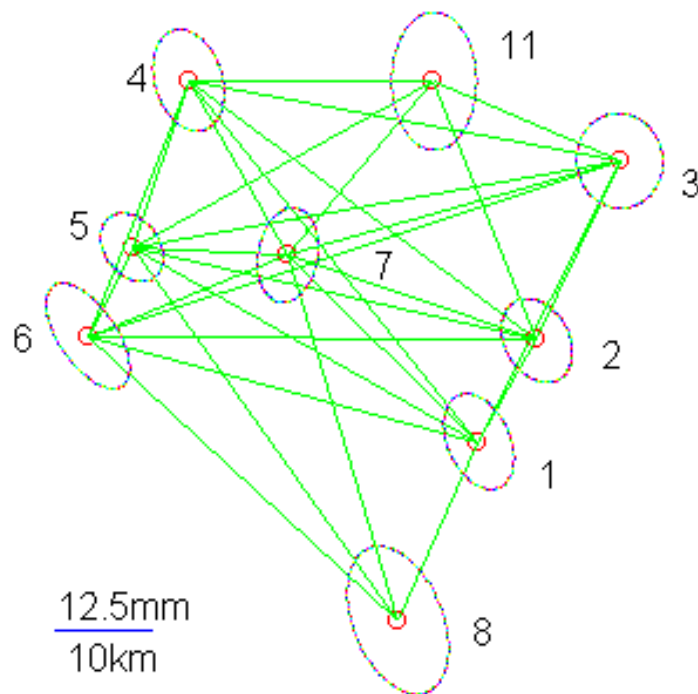


Fig. 1: Net 0 - Conventional least squares



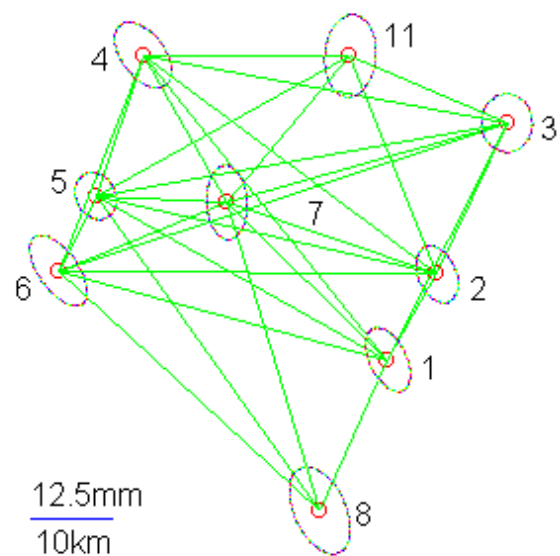


Fig. 3: Net 1 - Robustified least squares

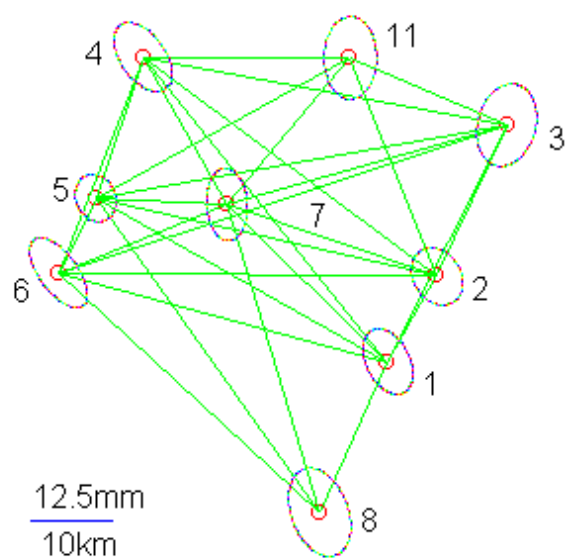


Fig. 4: Net 2 - Robustified least squares

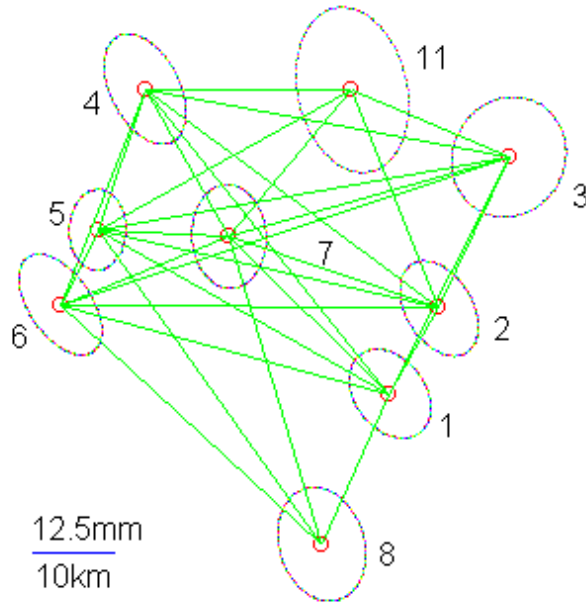


Fig. 5: Net 4 - Robustified least squares

## Discussion

The technique described here, like most techniques for robust estimation and management of outliers in observations, depends considerably for its effectiveness on the reliability of the network. The technique is only able to isolate outliers and damp their effects on the estimation process through the fact that the bias-free observations are in a position to estimate effectively the unknown parameters and at the same time resist the influences from the outlying observations. This way, the effects of the outlying observations on the estimated parameters are rendered minimal.

If however the bias-free observations should be overwhelmed by the outlying observations, either through sheer numbers or through geometric distribution within the observational set, then an adequate solution of the estimates may be rendered difficult, or altogether impossible. For instance, in the present study, in the case with four gross errors in the network a converging solution was only obtained after eight iterations. However, although the results indicate that the estimated parameters have been obtained with relatively acceptable precision, the space of convergence of the parameters is biased, as can be ascertained through comparing the results in Table 9 with those in Table 3. This bias has been caused by the fact that the network was not sufficiently robust in configuration (i.e. in geometry, as well as observational type, number and quality) as to be able to isolate the observations containing gross errors, which in the first place were rather ‘unsuitably’ distributed. The gross errors were here distributed such that out of the five network points, 2,3,4,7,11, connected with gross-error-contaminated observations three of the points, namely 3,4,7, were each connected with gross-error-contaminated observations. The result of this was that the gross errors in lines 3-4 and 7-11 could not adequately be isolated, and instead lines 2-11 and 5-11 were interpreted as the ones containing the gross errors.

In the cases with one and three gross errors, whose results are presented in Tables 5 and 7, the biases were effectively isolated, even though in this case point 7 was still connected by two gross-error-contaminated observations. The results for these two cases were found to be even more precise than

those from the ordinary gross-error-free least squares case presented in Table 2. In the initial case with no gross errors we notice from Table 3 that the results for the robustified least squares technique are considerably more precise than the case with ordinary least squares. Thus we have that even with observations that are effectively gross-error free one obtains more efficient estimates than with the ordinary least squares approach.

## Conclusion

The results of this study demonstrate that the definition of the observational weights through the mean square error results in robustified least squares estimates. The technique tested was able to cope effectively with outliers in the observational set. The effectiveness of the technique however, as can be expected, is dependent on the reliability of the network, and especially on the particular observations contaminated with outliers. When the network reliability is sufficiently high, the technique of weighting observations on the basis of the mean square error instead of the variance can be relied on to yield fairly reliable estimates even with gross errors in the observational set. The computational process is rendered rather slower than in the case of weights based on variances, due to the fact that the mean square error has essentially to be determined iteratively.

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