

QUALITY CONTROL WITH ANALYTICAL PLOTTERS

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1. Introduction

One of the most important applications of an on-line triangulation is a quality-controlled data acquisition followed by an off-line adjustment. One of the simplest ways of achieving this goal is to perform bridging in individual strips by consecutive relative orientation and scaling of models followed by a cross-tying of strips into a block. After each orientation is carried out by the computer, y-parallaxes and tie discrepancies are computed and displayed for each point in the model. These parallaxes can indicate if a gross observational error or blunder has been committed. However, in many cases it is difficult, or even impossible, to locate correctly the point where the faulty observation took place. This is due to two reasons. The first is the low redundancy available in the model as a unit and the second is the high correlation between the residuals (or parallaxes) of some points. Therefore, relying on the magnitude of parallaxes for locating gross errors could be inefficient and misleading. The ability of the system to detect and locate gross errors improves by upgrading the geometry of intersecting rays and by using additional constraints. In addition, rigorous statistical testing, such as data snooping, or "tau" criterion, may be applied. However, due to the characteristics of this type of data (small sample and high correlation), the statistical test should be modified. The computation of the standard error of unit weight and the choice of critical values for statistical tests are discussed in detail in this paper. The NRC on-line triangulation, developed for the ANAPLOT, employs a mathematical model for relative orientation with additional scale transfer constraint and statistically tests the standardized residuals at measured image points. The full weight cofactor matrix of the residuals (Q_{yy}) is computed for each point configuration used. From this matrix, points with highly correlated residuals can be identified. If these points have large residuals, the operator is able to identify the suspected point and reobserve only this point even though other points may display larger residuals. The choice to remeasure the point may be left either to the operator to decide or to the computer. There are advantages and disadvantages in either case as it is shown in the paper.

The objective of this paper is to describe the aspects of quality control with analytical plotters using the NRC on-line triangulation system as example. Using actual and simulated data, many of the factors affecting the ability of the technique to detect gross errors can be studied. Here, the effects of point configuration, density and location are presented.

2. Basic Mathematical Model

In the ANAPLOT software the scale transfer from model to model in the process of bridging is determined simultaneously with the relative orientation of the model. In this instance the model coordinates of suitably chosen tie points in the preceding model directly constrain the intersections of corresponding rays in the new model.

Relative orientation of the first model is based on the determination of five unknown parameters whereas the simultaneous solution of the scaled orientation in subsequent models contains six unknowns. In order to preserve the uniformity of programming and an adequate real-time speed, the orientation of all models is formulated primarily with the use of coplanarity condition. In addition, all models except the first one, enforce the connection with preceding models by using a modified collinearity condition. Collinearity condition is applied only to tie points. Since the intersection with corresponding rays is already enforced through the coplanarity condition, which is very strong in YZ plane, it is sufficient to check the ties with the previous model by specifying a single collinearity equation related to XZ plane only.

Both conditions are applied in the following form:

$$\begin{aligned} - \text{ coplanarity } F_p &= (bx'x'') = 0, \\ - \text{ collinearity } F_L &= \Delta Xz'' - \Delta Zx'' = 0, \end{aligned} \quad (1)$$

where vector $b = (1 \ \beta_y \ \beta_z)^T$ represents the photogrammetric base normalized through its x -component into values $\beta_i = b_i/b_x$. Vectors $x' = (x' \ y' \ z')^T$ and $x'' = (x'' \ y'' \ z'')^T$ are derived by orthogonal transformation of original camera coordinates using rotation matrices which are functions of the attitude elements x, φ, ω . Values ΔX and ΔZ are model coordinates of tie points reduced with respect to the right projection centre. All unknowns ($\beta_x, \beta_y, \beta_z, x, \varphi, \omega$) are dimensionless and expected to be of the same order of magnitude.

For more detail on the solution, see [Kratky 1980 or Kratky and El-Hakim 1983].

3. Computation of Redundancy Numbers

The effect of a gross error Δl_i on the residual v_i of an observation l_i is:

$$\Delta v_i = -r_i \Delta l_i \quad (2)$$

where r_i , called the redundancy number, is the i -th diagonal element of matrix $Q_{vv}P$. The weight cofactor matrix of the residuals, Q_{vv} is given by:

$$Q_{vv} = Q - BN^{-1}B^T \quad (3)$$

where N is the normal equation matrix. The standardized residual w_i is computed by:

$$w_i = v_i / \sigma_{v_i} = v_i / (\sigma_0 \sqrt{q_i}) \quad (4)$$

where q_i is the i -th diagonal element of matrix Q_{vv} .

Assuming that w_i is a standardized normally distributed variable [$w_i \sim N(0, 1)$], the null hypothesis, H_0 , that no gross error exists in observation l_i is respected if

$$|w_i| < c$$

where c is a critical value computed for a specific confidence level as will be discussed later.

Table 1, based on real measurements, illustrates the distribution of an artificial error of 100 μm introduced in point 3, after relative orientation was completed (the point numbering as shown in Figure 1). The table also gives the y-parallax and standardized residual for each of the six points. From the examination of y-parallaxes, point 2 seems to be the one in error, although this is not true. However, the largest standardized residual appears at point 3 where the error actually took place. It is obvious that the standardized residuals show the real significance of distributed discrepancies and that they should be tested for gross errors rather than the magnitude of parallaxes.

4. Effect of Point Distribution and Density

When analysing a suitable point distribution and density, two aspects of on-line triangulation must be considered - the accuracy of collected data and the efficiency of used procedures. The efficiency is not only judged by the time needed to perform the operations, but also by the intrinsic reliability of data. The on-line calculations are always applied to volume limited data and are not so much important for the final, usually independent, block adjustments, as they are for the crucial function of quality control. One should take great care that statistical testing is not too adversely affected by the limited on-line geometry. The geometry will ultimately be improved in the final, simultaneous processing of data, but then statistical tests are more difficult to run. It is important to consider suitable point configurations which guarantee high reliability of statistical testing rather than the highest possible accuracy of space limited photogrammetric solutions.

4.1 Models with no Scale Transfer

First models in every strip are computed only by relative orientation at a given scale. Since no scale transfer is applied, this computation is not typical for the on-line bridging process. Nevertheless, it is instructive to start our analysis with a model in which the coplanarity formulation is separated from other constraints.

It should be noted that the geometry of pure relative orientation is sensitive to relations governed by y-positions of corresponding points, with little or no respect to changes in coordinates x which primarily affect only the height definition of a measured point and not the orientation proper. Any statistical tests are, therefore, capable of detecting only gross errors in y-coordinates, but even significant blunders in x may go undetected.

Figure 2 shows the redundancy numbers for measured y-coordinates (or rather y-parallaxes) for points ranging from 6 to 15 in six different configurations a) to f). In general, the geometry of intersecting rays which decides on the success of relative orientation is determined by the distribution of image points. A good geometry results in a high reliability of the orientation. The redundancy numbers r then indicate the local reliability of the adjustment as reflected by any particular observation. Low redundancy number reflects limited reliability, while its increased value means an improved reliability. For instance, the residuals at all corners of the six-point pattern a) in Figure 2 will show only 8% of the actual local error.

The mechanism of the error distribution by Equation (2) is clearly illustrated by the full matrix Q_{vv} . Figure 3 lists its values corresponding to the previous example. One should realize that the listed values are two times larger than the corresponding redundancy numbers r . These are derived from the product $Q_{vv}P$ for which the weight matrix for y-observations is, as shown in [Kratky, 1983], $P_{yy} = 0.51$. The six values in any i-th column of Q_{vv} express the proportional distribution of the error in the i-th observation over the six points used in the solution. Whenever an off-diagonal element exceeds or matches the diagonal one, the error distribution distorts the testing of errors by the magnitude of residuals, which becomes misleading and worthless. This is clearly documented in Figure 3 in each column of the matrix. All critical values are marked by boxes. The same effect is graphically represented in Figure 2 by heavy arrows indicating the direction of all critical, misleading transfers in pattern a). The numbers at arrow lines are the critical weight cofactors from matrix $Q_{vv}P$, characterizing the degree of distortion. Also shown for each of the patterns in Figure 2 are the number of observations n and trace tr of the derived weight cofactor matrix Q_{gg} of unknowns, which could be used to assess and compare the expected accuracy of individual solutions.

The remaining sketches in Figure 2 can be analyzed in the same manner. The heavy arrows in patterns b, d, e, again indicate a critical transfer of errors. Dotted arrows in pattern c) show an error transfer which is less critical, but still serious to be misleading. It is apparent that pattern a), b), c) and d) are very poor and, obviously, it is not only the number of points which improves the stability. The lowest redundancy numbers show in corners and it appears to be very efficient to strengthen the stability by doubling points in critical areas, as shown in cases e), f). These twin points will also show a high correlation between their r -values, however, this is not critical any more. Since their location is almost identical, the problem area is uniquely identified. Also apparent from Figure 2 is the fact that the critical error transfers appear between neighboring points of low and high redundancy numbers showing an imbalance in the error distribution. Ideally, one should strive for a balanced, uniform distribution of r -values, however, still with reasonable, not excessively high, number of observed points. Since, by theory, the sum of r_i -values is equal to the number of redundant observations in the system, the redundancy numbers average here at $r_0 = (n - 5)/n = 1 - 5/n$. Pattern f) is definitely the best configuration in Figure 2 as for the reliability of detecting gross errors and also rates well in the accuracy assessment. A very good distribution pattern, not shown in the Figure, is achieved by measuring only 13 points (nine standard positions with four doubled corners). It yields r_i ranging from 0.58 to 0.80 with an average $r_0 = 0.62$ and with the trace of Q_{gg} equal to 14.3.

4.2 Models with Scale Transfer in three Tie Points

The analysis of the scale constrained orientation of models can be conducted in a similar way; however, with taking into account the additional collinearity conditions. We will again consider a few standard point configurations and compare their reliability and accuracy potentials. The scale transfer is considered to be based on the use of three tie points distributed in the narrow overlap of three consecutive photographs.

Figure 4 illustrates a group of six different configuration patterns, each with three tie points and with the number of orientation points ranging from 6 to 13.

The redundancy numbers are displayed separately for x'' -observations at tie points and y'' -observation at orientation points. Also listed for each pattern are the total number of observation points, the average redundancy number $r_0 = (n-3)/(n+3)$ and the trace tr of the cofactor matrix Q_{gg} .

We have demonstrated in the previous section that the standard six- and nine-point patterns are not suitable for on-line triangulation because of their critical error transfer, which prevents an efficient quality control. Nevertheless, they are included in Figure 4 as patterns a) and d) to allow for a comparison with corresponding patterns of Figure 2. The y -redundancy numbers are slightly increased and the critical transfers reduced due to the strengthening effect of scaling. The correlations among redundancy numbers within and between groups of observations x'' and y'' are demonstrated for the nine point pattern d) by matrix Q_{yy} in Figure 5. The first three diagonal elements represent r -values for observations x'' and the remaining diagonal elements are related to observations y'' in the sequence shown in Figure 1. The rest of the matrix shows correlations. For instance, there is no correlation between an x -error in point 2 and y -residuals for any other point.

The reliability of checking errors in y'' for patterns with 10 to 13 points in Figure 4 is generally good. However, it is obvious that three tie points do not support an adequate control of gross errors in the scale transfer. These poor conditions are practically unaffected by the number and configuration of other orientation points. The shift of errors x'' from side points to the centre, as documented numerically in the left upper 3×3 submatrix in Figure 5, remains critical in all patterns of Figure 4.

With reference to Figures 4d and 5, Appendix A illustrates the effect of a gross error in a practical example of NRC ANAPLOT operations. The computer printout shows the way in which the statistical evaluation is displayed to the operator for his decision on which point should be remeasured. Parallaxes and tie discrepancies are followed by a table which lists, for each measurement, the corresponding redundancy number (RED-N), x - or y -parallaxes both computed (PARX) and statistically expected (EXPX), as well as the error significance (SIGF) represented by the standardized residual. Otherwise, the Appendix is self-explanatory.

4.3 Models with Scale Transfer in five Tie Points

It is logical to expect an improvement in the scale transfer reliability by increasing the number of the points, from three to five, but not necessarily raising the total number of observations in the model. The number of 10 to 15 points has already proved to be useful. If the distribution of existing tie points on the left side of the model is mirrored on the right side for future tie points in the next model, one can chose one of the patterns described in Figure 6. Values r are graphically distributed in the same format as in previous Figures and complemented by information on n , r_0 and tr . Pattern a) with a regular distribution of ten points again displays weak upper and lower sides with a critical or serious transfer for three pairs of points. An alternate selection of double points in weak model parts works very well and all other patterns in Figure 6 guarantee a good quality control. Variant c) with 13 points represents an excellent practical choice. It almost matches the accuracy and reliability of the 15 point pattern while surpassing it in efficiency due to the lower number of observations.

5. Statistical Testing and Choice of Critical Value

Statistical tests and critical values chosen for on-line data snooping should be different from those usually applied for off-line block adjustment. This is due to the significant difference in the characteristics of the types of data in the two cases. Data in an on-line solution, as demonstrated earlier, usually have the following characteristics:

1. Small degree of freedom. This means that we are dealing with a small sample. This causes problems with any statistical testing since the small sample does not represent a specific distribution. Also, an estimated $\hat{\sigma}_o$ from this sample may not be as close to population σ_o as we would like.
2. High correlation between residuals. As shown earlier, this is a problem in locating gross errors. It is also a problem with any statistical testing since these tests are based on the assumption that no correlation exists.

3. Remeasuring cost, at observation stage, is negligible. On analytical plotters, going back to any specific point in the model is fast and automatic which makes remeasuring any point rejected by the test, before removing the photographs from the instrument, a very simple task. Therefore we can afford to become less conservative in choosing the significance level of test, α , and increase the probability of type I error (rejecting good observation). Thus, smaller error can be detected, and replaced immediately, than in case of off-line triangulation.

With the above characteristics in mind, the following is a discussion of some alternatives in choosing critical values for statistical testing of on-line data with some examples.

5.1 Test of standardized Residuals (w_j) using $\hat{\sigma}_o$ and $\alpha = 0.001$ - Normal Distribution

This is basically the test used in most block adjustment tests for gross error detection (data snooping). In this case the critical value $C = 3.29$. The test doesn't take into consideration the sample size in computing C .

Figure 7 is a print out of statistical analysis as it comes from the ANAPLOT. The results shown are for case d) in Figure 2 [9 points]. When no errors exist, the standard error of unit weight $\hat{\sigma}_o = 1.0$ micron. An error of 100 microns was then introduced at point 1 (ID No. 11 in the print out) which caused $\hat{\sigma}_o$ to be 17.2 microns. However, the statistical test didn't reject any point since the largest $w_j = 2.00$ (at the faulty observation) while $C = 3.29$. When introducing different sizes of error, either smaller or larger than 100 microns, still no observations were rejected. It is clear now that:

1. The significance level, $\alpha = 0.001$, is too small.
2. The size of the sample (number of observation) should be taken into consideration in computing the critical value C .
3. $\hat{\sigma}_o$ is largely affected by the gross error (by almost 20 % of the error) that it does not represent the population σ_o .

Therefore a statistical test of this type cannot be applied successfully to on-line gross-error detection.

5.2 Test of Standardized Residuals (w_j) using σ_o of Population

The same example is used here except that $\hat{\sigma}_o$ in the computation of w_j is replaced by σ_o , the expected standard error of unit weight of the population, as determined from previous experience (5 microns, for example).

Figure 8 displays the results of this test which shows that too many points are rejected. Large correlation between observation is the main reason for this.

5.3 Test of Standardized Residuals (w_j) using $\hat{\sigma}_o$ and $\alpha = 0.1$ - τ -Criterion

The τ -criterion [Pope, 1976] is a modified student-distribution which takes into account the size of the sample and doesn't require the knowledge of σ_o of population. For 9 points, the critical value C is 2.159 at 90 % confidence level ($\alpha = 0.10$). This is still not sensitive enough to detect the gross error for this number of points (see again figure 7). This may indicate that α is still too small. The question now is how to chose the significance level α so that errors of certain size can be detected. The answer to this question will definitely vary from one point distribution to another and from one point location to another, again a characteristic of on-line gross-error detection, where the geometry significantly vary from one point to another. For example, if the same test is applied to point distribution c) in figure 6 (13 points with double corner points) with an error at a point on one corner (point ID #12), the error is clearly detected by the test even at $\alpha = 0.01$ (99 % confidence level, where $C = 2.793$, τ -distribution) as shown in Figure 9.

Therefore, it is the opinion of the author that no statistical test is to be applied on w_j in an on-line adjustment which involves one model and small degree of freedom, or not a strong geometry. Instead, it is suggested that a test is to be performed on $\hat{\sigma}_o$. If the test shows that it varies significantly from the population (or expected) σ_o then the point with largest w_j is to be remeasured.

6. Conclusion

The method of data snooping, as applied on the NRC ANAPLOT, is capable of locating potential gross errors affecting the scale constrained orientation of consecutive models in bridging. The standardized residual, w_j , is always the largest at the point where the gross error took place, which is not the case with the residuals. However, due to the characteristics of on-line data, relying solely on statistical test with certain critical value will only succeed with strong point geometry and using a distribution that takes into consideration the degree of freedom (sample size), such as τ -distribution. It is also recommended that a test is applied on $\hat{\sigma}_0$ (e.g. X^2 -test) and if it shows a significant variation from the population σ_0 , then the point with largest w_j is to be remeasured. Based on numerous on-line simulations and practical experiments, a standard configuration of 13 orientation points including 5 tie points is considered best for a practical routine use.

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Abstract

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The objective of this paper is to describe the aspects of quality control with analytical plotters using the NRC on-line triangulation system as example. Using actual and simulated data, many of the factors affecting the ability of the technique to detect gross errors can be studied. Here, the effects of point configuration, density and location are presented.

QUALITÄTSKONTROLLE AN ANALYTISCHEN AUSWERTEGERÄTEN

Zusammenfassung

Eine der wichtigsten Anwendungen der rechnergestützten direkten (on-line) Triangulation ist die Datenerhebung mit Qualitätskontrolle für die nachfolgende (off-line) Ausgleichung. Diese Aufgabe kann auf einfache Weise durch aufeinanderfolgenden Folgebildanschlüsse und nachfolgende Verknüpfung der Streifen zu einem Block erfolgen. Nach jedem Folgebildanschluß werden die y-Parallaxen für jeden Modell-Daten können auf grobe Beobachtungs- oder andere Datenfehler hinweisen. Zwei Gründe machen es schwierig oder sogar unmöglich, fehlerhafte Punkte zu lokalisieren: die Redundanz in der Modelleinheit ist gering, und die y-Parallaxen bzw. Restfehler verschiedener Punkte können stark miteinander korreliert sein. Die Suche nach groben Fehlern kann daher sowohl wenig effektiv als auch irreführend sein, wenn sie auf Parallaxen und Restfehlern basiert. Die Aufdeckung grober Fehler kann so-wohl durch eine Verstärkung der Geometrie der sich schneidenden Strahlen und durch die Einführung zusätzlicher Bedingungen als auch durch statistische Tests wie das sogenannte data snooping oder das "tau"-Kriterium verbessert werden. Geringe Redundanz und hohe Korrelationen erfordern jedoch eine Modifizierung dieses statistischen Tests.

Der Bericht beschreibt die Berechnung des Gewichtseinheitsfehlers und die Wahl der kritischen Werte für die statistischen Tests. Das am NRC für den ANAPLOT entwickelte Verfahren für rechnergestützte direkte Triangulation schließt in die relative Orientierung eine zusätzliche Maßstabsbedingung und statistische Tests für normalisierte Restfehler an den gemessenen Bildpunkten ein. Die volle Matrix Qvv der Gewichtskofaktoren der Restfehler wird für jede Punktanordnung benutzt. Sie erlaubt es, Punkte mit stark korrelierten Restfehlern zu identifizieren. Sollten diese Punkte grobe Fehler aufweisen, ist der Auswerter in der Lage, den wahrscheinlich fehlerhaften Punkt zu identifizieren und nochmals zu messen, selbst dann, wenn andere Punkte größere Restfehler haben. Die Entscheidung für eine Nachmessung eines Punktes kann entweder dem Beobachter oder dem Rechner überlassen werden. Im Bericht werden die Vor- und Nachteile beider Fälle aufgezeigt.

Der Bericht beschreibt Qualitätskontrollgesichtspunkte der rechnergestützten direkten Triangulation an analytischen Auswertegeräten und benutzt das am NRC für den ANAPLOT entwickelte Verfahren als Beispiel. Viele Faktoren können sowohl mit tatsächlichen als auch mit simulierten Daten untersucht werden. Der Bericht behandelt Auswirkungen von Punktanordnung, -dichte und -lage.

CONTROLE DE QUALITE AVEC DES SYSTEMES DE RESTITUTION ANALYTIQUES

Résumé

L'une des applications les plus importantes de la triangulation on-line (directe) est l'acquisition de données avec un contrôle de qualité, suivie d'une compensation off-line. L'un des moyens les plus simples de parvenir à ce résultat consiste à réaliser un enchaînement de clichés formant des bandes individuelles avec une orientation relative et une mise à l'échelle des modèles, ces bandes étant ensuite rattachées en un bloc. Après chaque orientation, l'ordinateur calcule et sort les parallaxes y et les erreurs résiduelles de chaque point du modèle. Ces parallaxes peuvent signaler des erreurs d'observation grossières ou d'autres erreurs dans les données. Toutefois, dans plusieurs cas il est difficile, voire impossible, de localiser correctement le point entaché de cette erreur et ce, pour deux raisons: la première est la faible redondance à disposition dans le modèle en tant qu'unité, la seconde est la grande corrélation qui peut exister entre les erreurs résiduelles (ou les parallaxes) de certains points. Par conséquent, se baser sur les parallaxes pour localiser des erreurs grossières peut tout aussi bien être inefficace que prêter à de fausses conclusions. La capacité du système à déceler et à localiser des erreurs grossières peut être améliorée par une géométrie plus rigoureuse des rayons d'intersection et par l'introduction de conditions supplémentaires, cette façon de procéder pouvant encore être complétée par des tests statistiques rigoureux tel celui nommé "data snooping" (fouille des données) ou le critère "tau". Toutefois, une redondance faible et une corrélation élevée exigent de modifier ce test statistique et l'exposé décrit en détail le calcul de l'erreur-type de l'unité de poids et le choix des valeurs critiques pour les tests statistiques. Le procédé de triangulation directe (on-line) développé au National Research Council of Canada pour l'ANAPLOT utilise un modèle mathématique pour l'orientation relative, avec une condition de transfert d'échelle additionnelle et des tests statistiques pour les erreurs résiduelles standardisées aux points mesurés des clichés.

La matrice complète des cofacteurs de poids (Q_{yy}) des erreurs résiduelles est calculée pour chaque configuration de points utilisée. Cette matrice permet d'identifier les points avec une grande corrélation des erreurs résiduelles. Si ces points présentent des grandes erreurs résiduelles, l'opérateur peut identifier le point suspect et le remesurer même si d'autres points présentent eux-aussi des erreurs résiduelles importantes. La décision de remesurer un point peut être prise par l'opérateur ou par l'ordinateur. L'exposé fait part des avantages et désavantages rattachés à chaque cas.

Cet exposé a pour objet de décrire les aspects du contrôle de qualité de la triangulation on-line avec des appareils de restitution analytique, à l'exemple du procédé de triangulation on-line développé au National Research Council of Canada. En se servant de données réelles ou simulées, il est possible d'étudier les nombreux facteurs qui affectent la détection des erreurs grossières. L'exposé traite des effets de la configuration, de la densité et du lieu des points.

CONTROL DE CALIDAD CON RESTITUIDORES ANALITICOS

Resumen

Una de las aplicaciones más importantes de la triangulación on-line apoyada por computadora es la recopilación de datos con control de calidad para la subsiguiente compensación off-line. Este problema puede resolverse fácilmente por empalmes sucesivos de fotografías consecutivas y el enlace posterior de las fajas para formar un bloque. Tras cada empalme de fotografías consecutivas se calculan las paralajes "y" de cada punto modelo así como los errores existentes en los puntos de transferencia y se las visualiza. Estos valores pueden llamar la atención sobre graves errores de observación o bien de otros datos. Por dos motivos resulta difícil, o incluso imposible, localizar puntos erróneos: la redundancia es escasa o bien existen fuertes correlaciones entre las paralajes "y" o bien entre los errores residuales de los distintos puntos. La localización de los errores graves puede ser a la vez muy poco eficiente como inducir a confusiones, si se basa en paralajes y errores residuales. La detección de errores graves se perfeccionará tanto incrementando la geometría de los haces que se intersectan e introduciendo condiciones adicionales, como por pruebas estadísticas, tales como el llamado "data snooping" o el criterio "tau". Sin embargo, la escasa redundancia y las fuertes correlaciones hacen necesario modificar esta prueba estadística. Se describen pues el cálculo del error standard de peso 1 y la selección de los valores críticos para las pruebas estadísticas.

El método de la triangulación on-line aplicado en el National Research Council of Canada (Consejo Nacional de Investigación del Canadá) con destino al ANAPLOT incluye en la orientación relativa una condición adicional de escala y las pruebas estadísticas para errores residuales normalizados en los puntos imagen medidos. La matriz completa Q_{yy} de los co-factores de peso de los errores residuales se utiliza para cada configuración de puntos. Dicha matriz permite identificar puntos de errores residuales fuertemente correlacionados. En caso de que dichos puntos ostentasen errores graves, el operador está en condiciones de identificar el punto posiblemente erróneo y volver a medirlo incluso cuando otros puntos tuvieran errores residuales mayores. La decisión de repetir la medición podrá dejarse sea al criterio del operador, sea al de la computadora. En la presente conferencia se estudian las ventajas y desventajas de ambos casos.

Se describen los aspectos de control de calidad en triangulación on-line apoyada por computadora con restituidores analíticos, tomando como ejemplo el método para el ANAPLOT, desarrollado en el NRC. En la posible detección de errores graves influyen muchos factores que se analizan tanto con datos concretos como simulados. Se tratan los efectos que causan las configuración, densidad y situación de puntos.

| ID | PY | DX | DY | DZ |
|-----|------|-----|-----|-----|
| 151 | 5. | 0. | 2. | 1. |
| 152 | -6. | 0. | -5. | 1. |
| 153 | 12. | -0. | 5. | -2. |
| 161 | -5. | | | |
| 162 | 6. | | | |
| 163 | 2. | | | |
| 154 | -3. | | | |
| 155 | 4. | | | |
| 156 | -16. | ← | | |

Gross error of 50 μm in #153
misleadingly shows largest
discrepancy at #156

| ID | RED-N | PARX | EXPD | SIGF | SNOOPING STATISTICS |
|-----|-------|-------|------|--------|---------------------|
| 151 | 0.25 | -0.9 | 3.4 | 0.27 | |
| 152 | 0.67 | -0.3 | 5.5 | 0.06 | |
| 153 | 0.26 | 1.2 | 3.4 | 0.36 | |
| 151 | 0.27 | 5.4 | 5.0 | 1.09 | Statistically |
| 152 | 0.81 | -6.1 | 8.7 | 0.71 | most likely |
| 153 | 0.27 | 12.1 | 5.0 | 2.42 ← | error location |
| 161 | 0.43 | -4.9 | 6.3 | 0.78 | |
| 162 | 0.50 | 6.0 | 6.8 | 0.88 | |
| 163 | 0.43 | 1.5 | 6.3 | 0.24 | |
| 154 | 0.69 | -2.7 | 7.9 | 0.34 | |
| 155 | 0.75 | 4.4 | 8.3 | 0.54 | |
| 156 | 0.69 | -15.7 | 7.9 | 1.98 | |

STANDARD ERROR OF UNIT WEIGHT = 6.8 MICRONS

MORE POINTS?

ANY REJECTION? ENTER ID! 153 0

REPLACEMENT? Y

153 3 3.

| ID | PY | DX | DY | DZ | |
|-----|-----|-----|-----|-----|---------------------|
| 151 | -1. | -0. | 0. | -1. | Discrepancies after |
| 152 | -1. | 0. | -2. | 0. | correcting #153 |
| 153 | -6. | 0. | -2. | 1. | |
| 161 | 5. | | | | |
| 162 | -5. | | | | |
| 163 | 0. | | | | |
| 154 | -1. | | | | |
| 155 | 2. | | | | |
| 156 | 7. | | | | |

STANDARD ERROR OF UNIT WEIGHT = 3.4 MICRONS

APPENDIX A

| Point # | 1 | 2 | 3 | 4 | 5 | 6 |
|-----------------------------|------|-------|------|------|------|------|
| Redundancy number r | 0.09 | 0.77 | 0.07 | 0.32 | 0.40 | 0.23 |
| y-parallax in μm | 6.6 | -20.7 | 6.6 | -2.6 | 11.8 | -2.0 |
| Standardized residual w | 1.38 | 1.44 | 1.51 | 0.27 | 1.14 | 0.25 |

Table 1: Distribution of a Gross Error in Pattern a) in Fig. 4

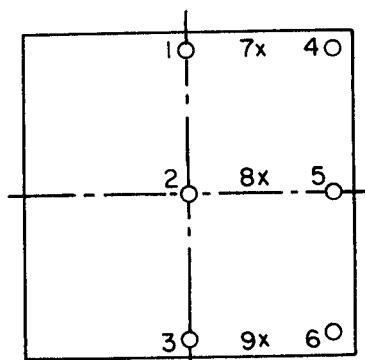


Fig. 1:
 Standard Point Distribution

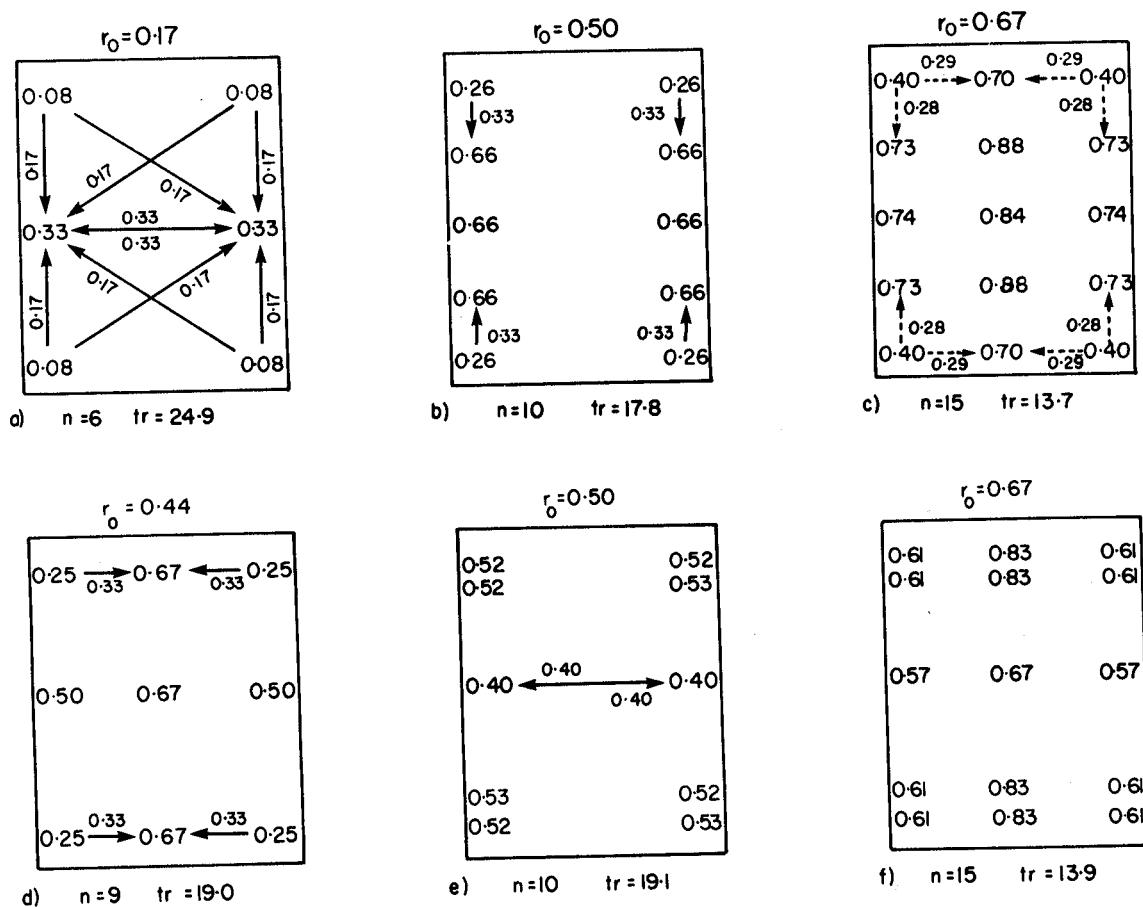


Fig. 2: Redundancy Numbers for Relative Orientation without Scale Transfer

$$\begin{array}{ccccccc}
 & \underline{0.17} & -0.33 & 0.17 & -0.17 & 0.33 & -0.17 \\
 \boxed{-0.33} & & \boxed{0.67} & \boxed{-0.33} & \boxed{0.33} & \boxed{-0.67} & \boxed{0.33} \\
 \\
 0.17 & -0.33 & \underline{0.17} & -0.17 & 0.33 & -0.17 & \\
 \\
 -0.17 & 0.33 & -0.17 & \underline{0.17} & -0.33 & 0.17 & \\
 \\
 \boxed{0.33} & \boxed{-0.67} & \boxed{0.33} & \boxed{-0.33} & \boxed{0.67} & \boxed{-0.33} & \\
 \\
 -0.17 & 0.33 & -0.17 & 0.17 & -0.33 & \underline{0.17} &
 \end{array}$$

Fig. 3 : Matrix Q_{VV} for Point Pattern a) in Fig. 2

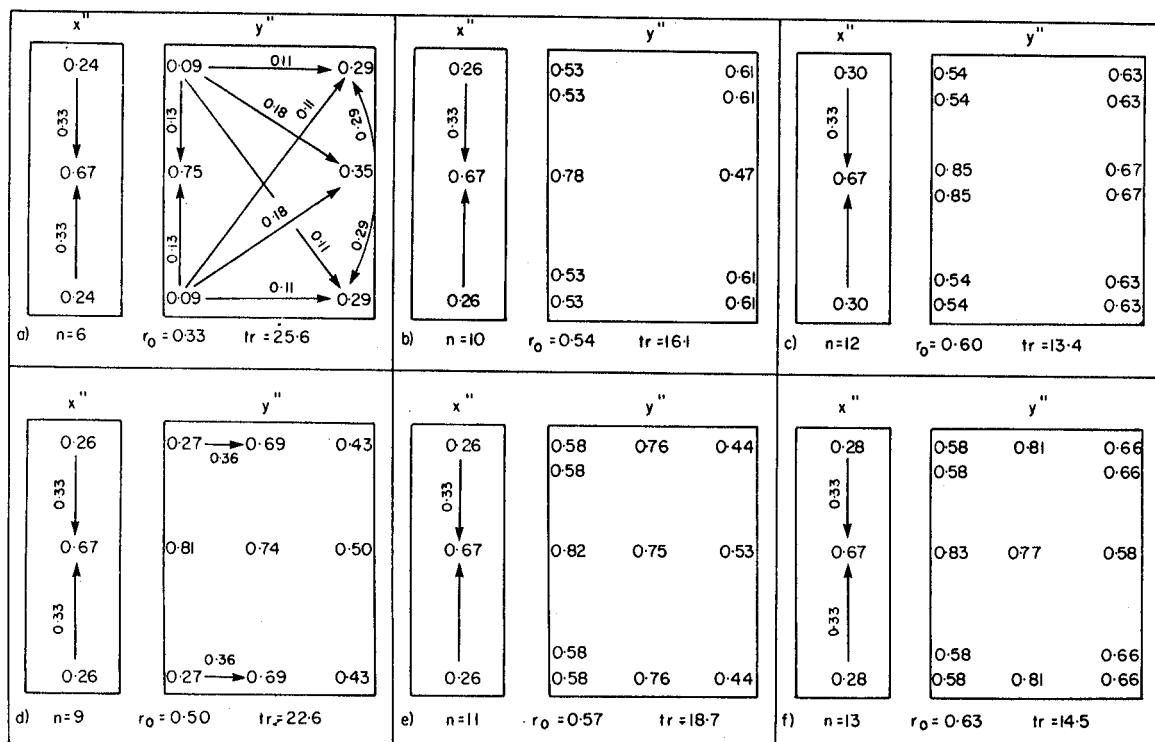


Fig. 4 : Redundancy Numbers for Scale Constrained Relative Orientation Using Three Tie Points

| | | | | | | | | | | | |
|-------------|-------------|--------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| <u>0.26</u> | -0.33 | 0.08 | -0.06 | -0.24 | -0.06 | 0.18 | 0.00 | 0.18 | 0.06 | -0.12 | 0.06 |
| -0.33 | <u>0.67</u> | <u>-0.33</u> | -0.00 | -0.00 | 0.00 | 0.00 | -0.00 | -0.00 | 0.00 | -0.00 | 0.00 |
| 0.08 | -0.33 | <u>0.26</u> | 0.06 | 0.24 | 0.06 | -0.18 | 0.00 | -0.18 | -0.06 | 0.12 | -0.06 |
| -0.06 | -0.00 | 0.06 | <u>0.54</u> | -0.18 | 0.21 | 0.05 | 0.33 | -0.28 | -0.71 | 0.08 | -0.04 |
| -0.24 | -0.00 | 0.24 | -0.18 | <u>1.63</u> | -0.18 | -0.14 | -0.33 | -0.14 | -0.16 | -0.35 | -0.16 |
| -0.06 | 0.00 | 0.06 | 0.21 | -0.18 | <u>0.54</u> | -0.28 | 0.33 | 0.05 | -0.04 | 0.08 | -0.71 |
| 0.18 | 0.00 | -0.18 | 0.05 | -0.14 | -0.28 | <u>0.85</u> | -0.33 | 0.52 | -0.55 | -0.24 | 0.12 |
| 0.00 | -0.00 | 0.00 | 0.33 | -0.33 | 0.33 | -0.33 | <u>1.00</u> | -0.33 | -0.00 | -0.67 | 0.00 |
| 0.18 | -0.00 | -0.18 | -0.28 | -0.14 | 0.05 | 0.52 | -0.33 | <u>0.85</u> | 0.12 | -0.23 | -0.55 |
| 0.06 | 0.00 | -0.06 | -0.71 | -0.16 | -0.04 | -0.55 | -0.00 | 0.12 | <u>1.37</u> | -0.08 | 0.04 |
| -0.12 | -0.00 | 0.12 | 0.08 | -0.35 | 0.08 | -0.24 | -0.67 | -0.23 | -0.08 | <u>1.49</u> | -0.08 |
| 0.06 | 0.00 | -0.06 | -0.04 | -0.16 | -0.71 | 0.12 | 0.00 | -0.55 | 0.04 | -0.08 | <u>1.37</u> |

Fig. 5: Matrix Q_{VV} for Point Pattern d) of Fig. 4

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|------|---|--------------|-------------|--------|--------------|------|--|------|------|------|--|--|------|------|------|------|------|------|------|------|------|------|------|---|------|------|------|--|---|------|---|------|------|------|------|--|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| a) | x'' | y'' | b) | x'' | y'' | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | <table border="1"> <tr><td>0.47</td><td>0.37</td></tr> <tr><td>0.72</td><td></td></tr> <tr><td>0.80</td><td></td></tr> <tr><td>0.72</td><td>0.37</td></tr> <tr><td>0.47</td><td></td></tr> </table> | 0.47 | 0.37 | 0.72 | | 0.80 | | 0.72 | 0.37 | 0.47 | | <table border="1"> <tr><td>0.26</td><td>0.35</td></tr> <tr><td>0.76</td><td>0.30</td></tr> <tr><td>0.86</td><td>0.66</td></tr> <tr><td>0.76</td><td>0.35</td></tr> <tr><td>0.26</td><td>0.67</td></tr> </table> | 0.26 | 0.35 | 0.76 | 0.30 | 0.86 | 0.66 | 0.76 | 0.35 | 0.26 | 0.67 | | <table border="1"> <tr><td>0.58</td><td>0.58</td></tr> <tr><td>0.80</td><td></td></tr> <tr><td>0.58</td><td>0.58</td></tr> </table> | 0.58 | 0.58 | 0.80 | | 0.58 | 0.58 | <table border="1"> <tr><td>0.53</td><td>0.62</td></tr> <tr><td>0.53</td><td>0.62</td></tr> </table> | 0.53 | 0.62 | 0.53 | 0.62 | | | | | | | | | | | | | | | | |
| 0.47 | 0.37 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.72 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.80 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.72 | 0.37 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.47 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.26 | 0.35 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.76 | 0.30 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.86 | 0.66 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.76 | 0.35 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.26 | 0.67 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.58 | 0.58 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.80 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.58 | 0.58 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.53 | 0.62 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.53 | 0.62 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | $n=10$ | $r_0 = 0.60$ | $tr = 18.6$ | $n=10$ | $r_0 = 0.60$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| c) | x'' | y'' | d) | x'' | y'' | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | <table border="1"> <tr><td>0.58</td><td></td></tr> <tr><td>0.58</td><td></td></tr> <tr><td>0.80</td><td></td></tr> <tr><td>0.58</td><td></td></tr> <tr><td>0.58</td><td></td></tr> </table> | 0.58 | | 0.58 | | 0.80 | | 0.58 | | 0.58 | | <table border="1"> <tr><td>0.59</td><td>0.81</td><td>0.67</td></tr> <tr><td>0.59</td><td></td><td>0.67</td></tr> <tr><td>0.86</td><td>0.78</td><td>0.59</td></tr> <tr><td>0.59</td><td>0.81</td><td>0.67</td></tr> <tr><td>0.59</td><td></td><td>0.67</td></tr> </table> | 0.59 | 0.81 | 0.67 | 0.59 | | 0.67 | 0.86 | 0.78 | 0.59 | 0.59 | 0.81 | 0.67 | 0.59 | | 0.67 | | <table border="1"> <tr><td>0.58</td><td></td></tr> <tr><td>0.80</td><td></td></tr> <tr><td>0.58</td><td></td></tr> </table> | 0.58 | | 0.80 | | 0.58 | | <table border="1"> <tr><td>0.62</td><td>0.84</td><td>0.69</td></tr> <tr><td>0.62</td><td>0.84</td><td>0.69</td></tr> <tr><td>0.87</td><td>0.78</td><td>0.59</td></tr> <tr><td>0.62</td><td>0.84</td><td>0.69</td></tr> <tr><td>0.62</td><td>0.84</td><td>0.69</td></tr> </table> | 0.62 | 0.84 | 0.69 | 0.62 | 0.84 | 0.69 | 0.87 | 0.78 | 0.59 | 0.62 | 0.84 | 0.69 | 0.62 | 0.84 | 0.69 |
| 0.58 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.58 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.80 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.58 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.58 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.59 | 0.81 | 0.67 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.59 | | 0.67 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.86 | 0.78 | 0.59 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.59 | 0.81 | 0.67 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.59 | | 0.67 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.58 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.80 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.58 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.62 | 0.84 | 0.69 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.62 | 0.84 | 0.69 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.87 | 0.78 | 0.59 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.62 | 0.84 | 0.69 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.62 | 0.84 | 0.69 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | $n=13$ | $r_0 = 0.67$ | $tr = 14.0$ | $n=15$ | $r_0 = 0.70$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

Fig. 6: Redundancy Numbers for Scale Constrained Relative Orientation Using Five Tie Points

CONFIDENCE LEVEL? (1/2) : 1

CONFIDENCE LEVEL? (1/2) : 1

| ID | RED-N | PARX | EXPD | SIGF | | ID | RED-N | PARX | EXPD | SIGF | SNOOPING STATISTICS |
|----|-------|------|------|------|--|----|-------|-------|------|------|---------------------|
| 11 | 0.25 | -0.8 | 0.7 | 1.15 | | 11 | 0.25 | 24.3 | 12.2 | 2.00 | |
| 12 | 0.50 | 1.0 | 1.0 | 0.98 | | 12 | 0.50 | -15.7 | 17.2 | 0.91 | |
| 13 | 0.25 | -0.8 | 0.7 | 1.16 | | 13 | 0.25 | 7.5 | 12.2 | 0.62 | |
| 21 | 0.25 | -0.5 | 0.7 | 0.69 | | 21 | 0.25 | 7.8 | 12.2 | 0.64 | |
| 22 | 0.50 | 0.3 | 1.0 | 0.33 | | 22 | 0.50 | 17.2 | 17.3 | 1.00 | |
| 23 | 0.25 | -0.5 | 0.7 | 0.70 | | 23 | 0.25 | -9.0 | 12.2 | 0.74 | |
| 14 | 0.67 | 1.3 | 1.2 | 1.13 | | 14 | 0.67 | -32.3 | 19.9 | 1.62 | |
| 15 | 0.67 | -1.3 | 1.2 | 1.13 | | 15 | 0.66 | -1.2 | 19.9 | 0.06 | |
| 16 | 0.67 | 1.3 | 1.2 | 1.13 | | 16 | 0.67 | 1.4 | 19.9 | 0.07 | |

STANDARD ERROR OF UNIT WEIGHT = 1.0 MICRONS STANDARD ERROR OF UNIT WEIGHT = 17.2 MICRONS

a) No gross errors

b) 100 microns error at point 11

Fig. 7 : Statistical test using $\hat{\sigma}_o$ - normal distribution

EXPECTED STANDARD ERROR OF UNIT WEIGHT (IN MICRONS) = 5

CONFIDENCE LEVEL? (1/2) : 1

| ID | RED-N | PARX | EXPD | SIGF | SNOOPING STATISTICS |
|----|-------|-------|------|------|---------------------|
| 11 | 0.25 | 24.0 | 3.5 | 6.81 | @#? |
| 12 | 0.50 | -15.5 | 5.0 | 3.11 | |
| 13 | 0.25 | 7.4 | 3.5 | 2.10 | |
| 21 | 0.25 | 7.8 | 3.5 | 2.20 | |
| 22 | 0.50 | 17.1 | 5.0 | 3.41 | @#? |
| 23 | 0.25 | -8.9 | 3.5 | 2.52 | |
| 14 | 0.67 | -31.9 | 5.8 | 5.53 | @#? |
| 15 | 0.66 | -1.2 | 5.8 | 0.20 | |
| 16 | 0.67 | 1.4 | 5.8 | 0.23 | |

Fig. 8 :

STANDARD ERROR OF UNIT WEIGHT = 17.1 MICRONS

Statistical test using expected σ_o

CONFIDENCE LEVEL? (1/2) : 1

CONFIDENCE LEVEL? (1/2) : 1

| ID | RED-N | PARX | EXPD | SIGF | SNOOPING STATISTICS |
|----|-------|------|------|------|---|
| 11 | 0.58 | -1.3 | 1.1 | 1.16 | |
| 12 | 0.57 | 0.7 | 1.1 | 0.59 | 11 0.58 -43.0 20.2 2.13 |
| 13 | 0.57 | -0.3 | 1.1 | 0.24 | 12 0.57 56.9 20.1 2.82 @#? |
| 14 | 0.58 | -0.5 | 1.1 | 0.42 | 13 0.57 -9.9 20.0 0.50 |
| 15 | 0.57 | 1.5 | 1.1 | 1.34 | 14 0.58 1.9 20.2 0.09 |
| 21 | 0.57 | -1.9 | 1.1 | 1.64 | 15 0.57 4.0 20.2 0.20 |
| 22 | 0.58 | 1.1 | 1.1 | 0.99 | 21 0.57 0.5 20.1 0.03 |
| 23 | 0.57 | -0.1 | 1.1 | 0.07 | 22 0.58 3.5 20.2 0.18 |
| 24 | 0.57 | -0.5 | 1.1 | 0.46 | 23 0.57 9.8 20.1 0.49 |
| 25 | 0.58 | 1.5 | 1.1 | 1.29 | 24 0.57 -3.0 20.2 0.15 |
| 16 | 0.80 | 1.4 | 1.3 | 1.04 | 25 0.58 -1.0 20.2 0.05 |
| 17 | 0.66 | 0.3 | 1.2 | 0.27 | 16 0.80 -18.2 23.8 0.77 |
| 18 | 0.80 | -2.0 | 1.3 | 1.48 | 17 0.66 0.5 21.7 0.02 |
| | | | | | 18 0.80 -2.0 23.8 0.08 |

STANDARD ERROR OF UNIT WEIGHT = 1.1 MICRONS

STANDARD ERROR OF UNIT WEIGHT = 18.8 MICRONS

a) No gross errors

b) 100 micron error at point 12

Fig. 9 : Statistical test using $\hat{\sigma}_o$ - τ -distribution