D. Fritsch, M. Englich & M. Sester, eds, 'IAPRS', Vol. 32/4, ISPRS Commission IV Symposium on GIS - Between Visions and Applications, Stuttgart, Germany.

DATA UNCERTAINTY IN A HYBRID GIS

Michael Glemser and Dieter Fritsch
Institute for Photogrammetry
University of Stuttgart
Geschwister-Scholl. Str. 24
D-70174 Stuttgart, Germany
Phone: +49-711-121-3384
Fax: +49-711-121-3297
Michael.Glemser@ifp.uni-stuttgart.de
Dieter.Fritsch@ifp.uni-stuttgart.de

KEY WORDS: Data Uncertainty, Data Quality, Hybrid GIS, Raster-Vector-Conversion, Polygon Overlay

ABSTRACT

The paper presents a new approach for integrating data uncertainty into a hybrid GIS environment. Such a hybrid system can process raster data as well as vector data in an integrated manner taking uncertainty into account. For this purpose a hybrid data model is defined and extended to manage data uncertainty. The uncertainty description is based on probability measures. The consequences for the analysis methods are demonstrated by two selected operations. First, we concentrate on raster-vector-conversion which is often applied in a hybrid system and secondly treat polygon overlay. Some examples help to clarify the modifications made for these functions.

1 INTRODUCTION

The increasing availability of data for geographic information systems (GIS) is indicated by the current discussions about data warehouses, internet access to databases and open GIS. Even today a variety of data sets are offered like topographic information, cadastral data, statistical data or digital orthophotos and satellite images. Providers of data can be found in public administration as well as in private companies. For the user community the data availability has a large impact on the realization of projects. It is not a vision that data collection will widely change from digitizing own data sources to transferring or accessing data from existing databases. This will result in an enormous reduction of costs what will promote the further growing of GIS and its applications. For the GIS users this means, that two sub groups have to be distinguished: a smaller group of data producers and a larger group of data consumers where producers and consumers normally are different persons or institutions. So we can think of applications that are able to run without creating own data. To exploit the real potential of the offered data (i.e. to use whatever data source is available) and to concentrate completely on the thematic aspects, a hybrid system is required. A hybrid system can handle both fundamental data types - raster and vector data - in an integrated manner. For example, topographic data in vector format derived from analogous maps can be easily combined with landuse information in raster format gained by satellite images. The management and processing is done by the hybrid system without any special user interaction.

The importance of data quality comes along with the increase in data availability. By accessing various sources the combination of data with significantly different quality features is enabled (e.g. small scale geological data and highly accurate cadastral data). Any combinations of data are imaginable, also such combinations that might be without any practical use. Two major questions arise in this context (Glemser, Klein, 1998):

- Which data fit the requirements for a given application ?
- How accurate is the result of an analysis ?

The first question contributes to the avoidance of senseless combinations. It can be answered by an examination of the meta description of the data set. Among others the meta description should consists of a list of quality features. Possible features of the list are lineage, logical consistency, geometric, thematic and temporal accuracy (CEN, 1996). Corresponding to these features the user has to define his requirements and has to check them against the given meta description of each set. Only such data should pass the examination that fulfill all requirements. The test especially becomes important if there is more than one data source available with nearly the same thematic contents. Then the best selection of data for the application can be determined. Even though the first question is an important one the present paper concentrates on the second question, which is more difficult to solve. It is not sufficient to select and use only fitting data because this would not guarantee the quality and reliability of analysis results. In addition, also the uncertainty of the outcome has to be evaluated. For this purpose some extensions in data structure and GIS functionality are necessary. Presumed that the uncertainty of input data is known the hybrid data model has to be extended in order to manage the uncertainty information. Furthermore the complete set of analysis methods has to be adapted to propagate the uncertainty onto the output results.

In the following sections a hybrid data structure is presented which is enhanced with respect to the management of data uncertainty. A probabilistic approach is used as uncertainty model. In this approach the

D. Fritsch, M. Englich & M. Sester, eds, 'IAPRS', Vol. 32/4, ISPRS Commission IV Symposium on GIS - Between Visions and Applications, Stuttgart, Germany.

geometric accuracy of an object is related to the probability that any point belongs to the object. On the other hand, the thematic accuracy is related to the probability, that a specific attribute has the assigned value. The hybrid data structure is altered with respect to both aspects. Another focus is set on the GIS functionality and its accompanying modifications when uncertainty is integrated. We limit ourselves to the discussion of two basic methods. The first one is the conversion from raster to vector representation and vice versa which plays a main role in hybrid data processing. The problem is to avoid a loss in accuracy during the conversion step. The geometry of a transformed object should be identical (within the limits of discretization) compared to the original. The second method is the well-known polygon overlay of two data sets which comes out with new objects containing the attributes of both input sets. Therefore the propagation of the input probabilities onto the output is defined. A set of examples demonstrates how the propagation operates and which results are obtained for different cases.

2 HYBRID DATA

Spatial phenomena can be represented geometrically either in vector or in raster format (Bill, Fritsch, 1991). The raster format results from a regular x,y-sampling in connected raster cells. To each cell one attribute value is assigned. The vector format uses geometrical primitives like points and lines to build up a variable spatial structure which can possess a number of attributes. The kind of format used is determined by the digitizing method applied. For example, if a digitizer tablet is used the result will be vector data whereas a scanner produces raster data. On the other hand the choice of equipment primarily depends on the properties of the considered spatial phenomena. In general two types of phenomena can be distinguished: fields and objects (Burrough, 1996). Fields are unbounded characteristics of space which often possess a continuous variation in their values. Examples are temperature, various kinds of density distributions or height. In contrast, objects are phenomena that can be spatially and thematically separated from each other and can be grouped together in classes. Each object consists of a set of attributes representing the thematic aspect. In contrast to fields the attributes are seen as spatially discrete (which does not mean that their values have to be discrete).The spatial component is defined by the position and extension of the phenomena, i.e. the spatial validity of the attribute set. Examples for classes are roads, houses or landuse areas.

Looking at the two possible representation (raster and vector) we can state that raster is the adequate structure to represent fields because the resolution of the raster can easily be adapted to the continuous spatial variation inherent in fields. Isolines are possible vector representation but they do not recover the continuous aspect of fields. An additional interpolation algorithm is needed. For objects, the critical aspect to model is the representation of the spatial position and extension of the phenomena. This can be done in either structures, i.e. in raster and in vector, respectively. Vector format is more often used in practice because of its compact structure which saves computer resources. Another advantage of vector structure is found if there is a demand for high precision in position (e.g. in the domain of cadastre). Vector data is more suitable than raster for this purpose because of the continuous spatial reference of coordinates accessed for vector data. In the scope of the paper we limit ourselves to the representation of objects in a hybrid manner, i.e. by using raster and vector format, without including fields.

Hybrid data are created if raster and vector data are used together in one system environment. For the system architecture different stages of integration can be distinguished (Ehlers et al., 1989; Yang, 1992):

1. **Hybrid Visualization:** Vector and raster data are visualized together. One format is the main processing structure of the system, the other one serves as background information supplier.

2. **Hybrid Processing:** Different analyses can be performed in both structures independently from each other. Explicit conversion allows the change in format.

3. **Hybrid Data Structure:** The data structures for raster and vector are integrated based on a hybrid data model. Analysis methods are able to process the hybrid data which means that they work independently from the present format.

Only the third stage fulfills the requirements for a fully hybrid processing of data. It enables the user to process data without knowing if it is actually structured in vector or raster format. The user has access to the complete functionality of the system, so that he can fully concentrate on his application.

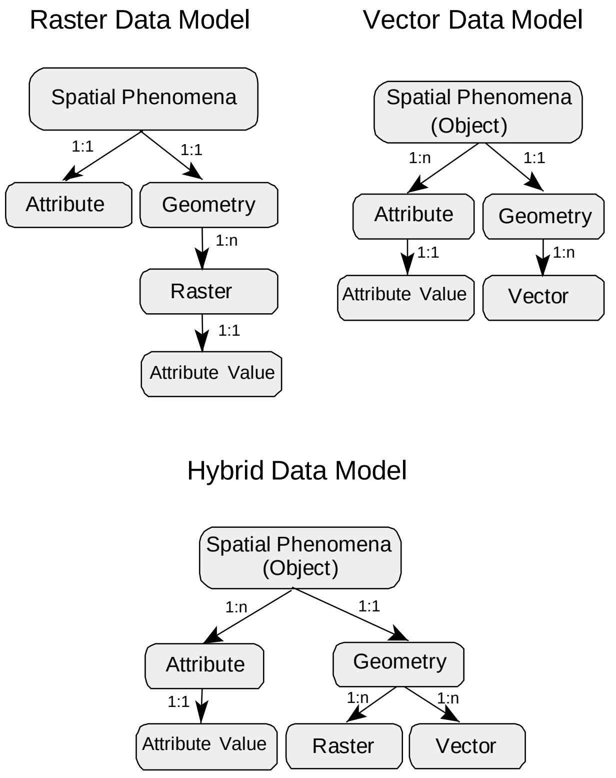


Figure 1: Data models for raster data, vector data and hybrid data

D. Fritsch, M. Englich & M. Sester, eds, 'IAPRS', Vol. 32/4, ISPRS Commission IV Symposium on GIS - Between Visions and Applications, Stuttgart, Germany.

The third stage is based on a hybrid model. It can be developed by combining the models for raster and vector data. Figure 1 shows typical data models for both. Beside the treatment of geometry the models have obvious differences in representing the thematic component. In the raster model multiple attribute values for each thematic aspect are allowed. This is absolutely necessary if fields have to be modeled. The vector model assumes that the attribute value is constant over the whole extension of an individual phenomenon. The approach applied here for building a hybrid model is derived from Molenaar and Fritsch (1991). It requires the existence of objects as spatial phenomena. In the case of fields a transformation has to be defined. The hybrid model used in this work is shown on the lower half of Figure 1. The thematic component is the same as in the vector model whereas the geometric component allows a raster or a vector representation as alternative possibilities. A more detailed description of the model in object oriented representation (Rumbaugh et al., 1991) is shown in Figure 2. The model defines that an object possesses either a vector or a raster representation and has multiple attributes attached to it.

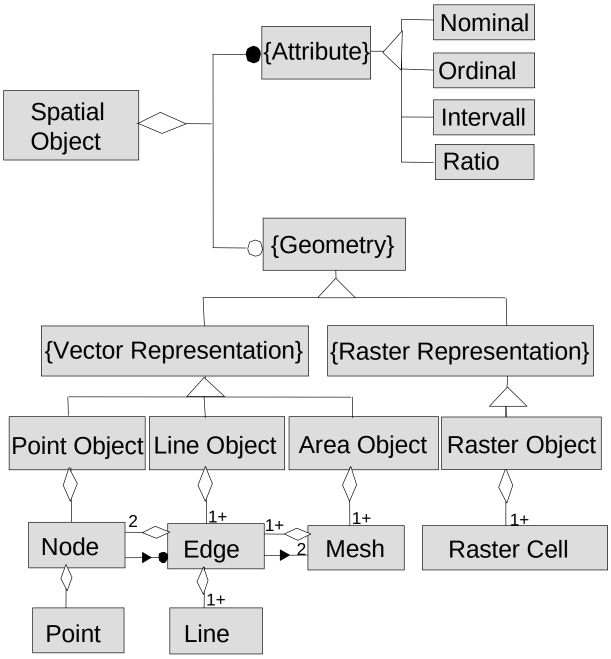


Figure 2: Object oriented hybrid data model (OMT-syntax: Rumbaugh et al., 1991)

3 DATA UNCERTAINTY

Spatial data can be divided in a geometric and a thematic component. The geometric component defines the position and the extension of the spatial phenomenon, and the thematic component includes all descriptive information. Various influences are responsible that the data cannot be collected completely error-free (Burrough, 1986; Caspary, 1992). For example, measurements are always of limited accuracy due to resolution of the equipment and human operator interactions. These influences cause some amount of uncertainty. Because we are not able to avoid uncertainty, we have to deal with it (Arronoff, 1989). For this purpose a model has to be defined. In the following appropriate models are discussed for the different components of spatial data.

3.1 Geometric Uncertainty

Uncertainty in geometry refers to the variation in position of geometric primitives (points, lines, areas) which represent a spatial object. An example for such a variation is shown in Figure 3 where one object is collected ten times by different operators.

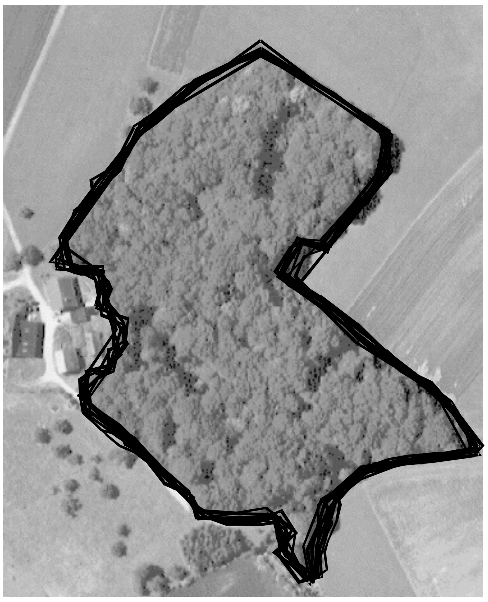


Figure 3: Ten different geometric versions of an object collected by different operators.

An uncertainty model should describe the variation in a quantitative way by a set of parameters. Different approaches are possible (Glemser, 1994). One is based on the well-known epsilon-band (Blakemore, 1984) where an error-band is laid around each primitive. It is assumed that somewhere within the epsilon-band the unknown true position can be found. The shape of the band was subject in various investigations (Caspary, Scheuring, 1992; Shi, 1994). Another approach looks at the object as a fuzzy set of points. Here the uncertainty is handled with the theory of fuzzy sets developed by Zadeh (1965).

A series of approaches use stochastic methods to describe the variation. For example, the variable extension of the object can be defined by evaluating the frequency of slivers belonging to an object, if the object is multiple collected like shown in Figure 3 (Molenaar, 1996). Another possibility is to treat primitives as random variables and characterize them by a distribution function (Glemser, 1996). In general the standard Gaussian distribution is applied for this purpose. One important parameter of the function is defined by the standard deviation of the random variable which has to be determined for each object. The value can either be estimated individually through comparison with a reference set or can be determined by prior knowledge. In the stochastic approach probabilities define an alternative uncertainty description (Kraus, Haussteiner, 1993). They can be calculated for every position in space indicating the belonging to an object. The probability is dependant on the distribution function and the distance measured between the certain position and the object boundary.

This research focuses on probability measures to describe the geometric uncertainty of an object. For this purpose the spatially continuous probability function is approximated for each object by a discrete raster, which forms a probability matrix. This corresponds to a rasterization of the object and an assignment of a probability value for each pixel of the object. The size of the raster cells depends on the standard deviation of the primitives. The size should be small enough to enable a reconstruction of the function. This can be achieved by setting the size d = σ / 4. The formulas for the calculation of the probabilities vary for the different types of primitives. For area objects the probability is calculated as

$$p(x,y) = p(d) = \int_{-\infty}^{d} f(t)\ dt = F(d)$$

with f(t) as the density function of the Gaussian distribution and F(d) as its distribution function. The value d is defined as the smallest distance measured between the position P(x,y) of the raster cell and the mean boundary of the object. It is positive if P lies inside the object and negative if outside. An additional parameter is applied for points and lines which use the width w of the object for calculation. Then a probability measure for lines is given as

$$p(d,w) = \int_{-\infty}^{\frac{w}{2}+d} f(t)\ dt = F(\frac{w}{2}+d)$$

and for points

$$p(x,y) = p(d,w)^2 \cdot p(-d,w)^2 = F(\frac{w}{2}+d)^2 \cdot F(\frac{w}{2}-d)^2$$

The probability matrix is calculated for each object and is stored with the original data set as additional information.

3.2 Thematic Uncertainty

Attribute values can be distinguished according to the scale they belong to. In general the following four scales exists: nominal, ordinal, interval and ratio scale. Nominal and ordinal scale create discrete values whereas the values belonging to the interval or ratio scale are continuous. Uncertainty of continuous values can be described with the same models that are used for the geometric component, as positional information is also continuous (Drummond, 1995). But we want to concentrate on discrete values in the discussion. Sources for such data can be found for example in the field of remote sensing where classification algorithms produce landuse values from satellite images. Uncertainty in such values is reflected by the degree of truth that the specific attribute possesses the assigned value. The degree of truth can also be expressed using probability measures (e.g. the probability that the landuse type of an area object is meadow). Statistical methods (e.g. maximum likelihood classification) are able to estimate the probabilities (Shi, 1994; Stehman, 1997). The possibility to use probabilities confirms the choice of the stochastic model for geometric uncertainty. Now both components are based on the same model which facilitates the integration in joint analyses. This is essential for hybrid data processing and defines an advantage compared to other approaches. Probabilities can also be used to perform a segmentation on the classified images in an object building process (Klein et al., 1998).

3.3 Uncertainty of Raster Data and Vector Data

Raster and vector data consist both of a geometric and a thematic component. This means that in general both types are influenced by uncertainties of each component. The situation changes if we look at the formation of the data types. Using the definition by Chrisman (1991) a discernable data type is built through fixing one aspect and measuring the other. The two aspects correspond to the two components of an object. Raster data fixes the geometry by the definition of a strict raster matrix and allows variable attribute values. In contrast to that vector data fixes the thematic component in the way that the attribute values are determined in advance and the valid spatial extension of these attributes is measured. Because the fixed component is always constant, it can be taken as error-free. The remaining uncertainty is included only in the measured component. We can follow easily, that in case of raster data the attribute values are uncertain but not the position of the raster cell, whereas in case of vector data geometry is uncertain but not the attribute values.

4 COMMON MODEL FOR UNCERTAIN HYBRID DATA

Following the previous discussions a hybrid data model should include both aspects of uncertainty.

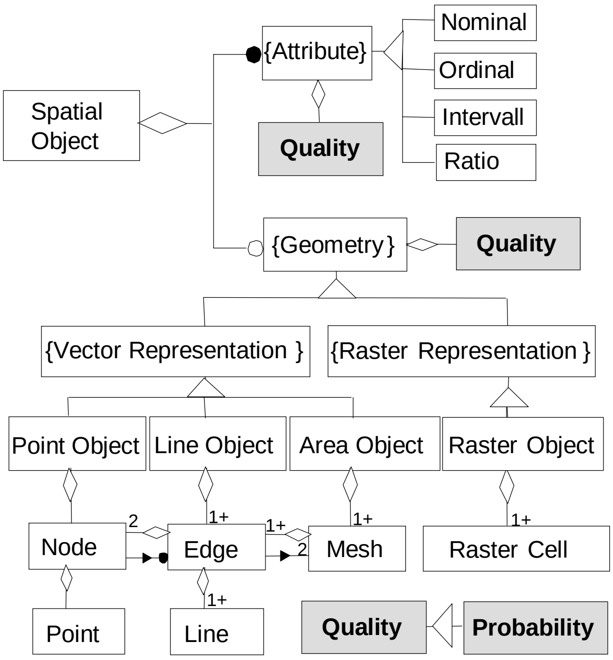


Figure 4: Hybrid data model with integration of uncertainty (OMT-syntax: Rumbaugh et al., 1991).

D. Fritsch, M. Englich & M. Sester, eds, 'IAPRS', Vol. 32/4, ISPRS Commission IV Symposium on GIS - Between Visions and Applications, Stuttgart, Germany.

Figure 4 shows the extensions of the presented hybrid model. Uncertainty descriptions (quality measures) are attached to the attributes and to the geometry. As discussed before only vector representations possess geometric uncertainty. But to emphasize that the geometry is the uncertain factor, the description is attached to the geometry component. In case of a raster representation this part remains unused even it would be possible to assign uncertainty values. The same problem is found in the attribute component where uncertainty is attached to all attributes independent of the type of representation.

The model is based on probabilities as uncertainty measures. The storage of the probabilities is managed by a raster matrix. Such a matrix has to be built for each object if the geometry is uncertain. A single matrix for each attribute is sufficient to hold probabilities for all objects.

5 HANDLING UNCERTAINTY IN HYBRID METHODS

The integration of uncertainty requires additional steps during the data input. More steps means more data. This increases the efforts and therefore costs for the production of spatial information. Such an effort is only justified if the outcome is of higher value for the user. For this purpose also analysis methods have to be extended taking uncertainty into account. In the following the impact on two methods is discussed. First, raster-vector conversion is considered followed by a polygon overlay. The overall objective reaching a higher level in data interpretation is demonstrated with some examples.

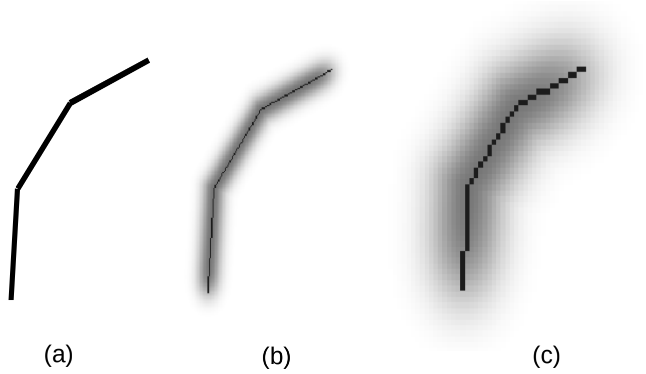


Figure 5: Example of a line object (a) converted into raster object, with low uncertainty (b) or with high uncertainty (c). Geometry is visualized in black. Probabilities are shown in gray (white = low probability, dark gray = high probability).

5.1 Raster-Vector/Vector-Raster-Conversion

Raster-vector conversion is part of the basic functionality in a hybrid GIS. It allows the changing between the geometric representations of an object. This is necessary if a certain analysis method expects data of a specific type. One reason for the expectation is that the result can be processed much easier in that specific format than in the other. For example, the overlay problem discussed in section 5.2 is easier and faster to solve with raster data than with vector data. Thus the conversion very often serves as a background operation which runs before or after the main function. It would be comfortable, if it ran automatically without any user interaction, even without being noticed by the user. The problem is that normally some parameters have to be set which control the conversion. With the integration of uncertainty these values can be estimated. In the following the definition of the parameters according to uncertainty is discussed for the two directions of conversion.

5.1.1 Vector-to-Raster Conversion

Vector-to-raster conversion is a simple task which can be carried out in a single step. Many algorithms are known performing that task (Jäger, 1990). In general the user has to define the size of the raster cells generated. In our approach the cell size is taken from the existing cell size used for generating the uncertainty matrix of the object geometry. Using this cell size has the advantage that the resolution of the raster data is adapted to the uncertainty of the data (e.g. low uncertainty means small cells). Then the uncertainty of the object can directly be seen in the visualization (see Figure 5).

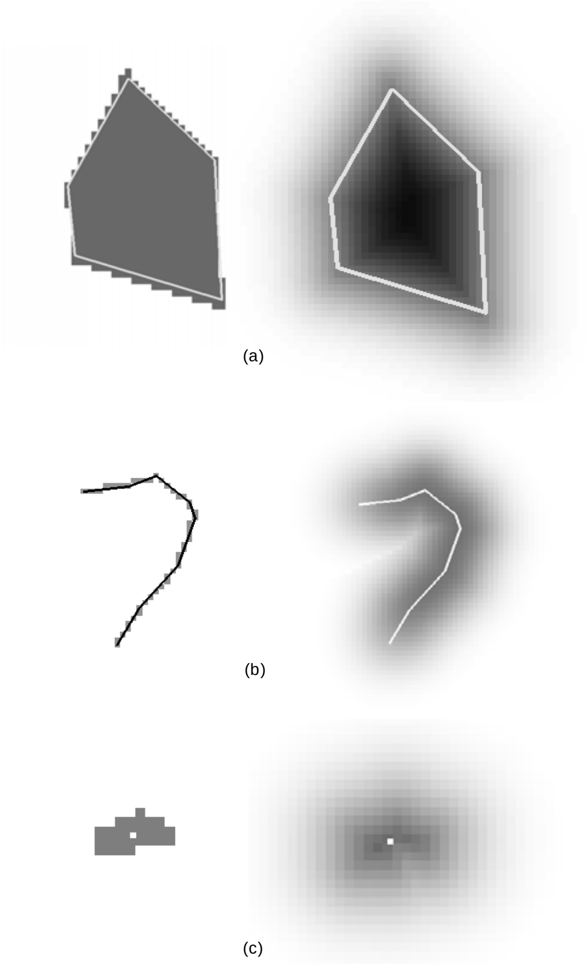
5.1.2 Raster-to-Vector Conversion

Raster-to-vector conversion is a more complex operation. One main problem is that each primitive type needs an own algorithm for generating that type. Thus at least three algorithms have to be implemented according to points, lines and areas (see Figure 6). Generating a point is simply calculating the point of gravity for the raster cells. The conversion into a line consists off several steps. The approach used here is described by Cramer (1993). The following steps are performed: distance transformation, skeleton building, line extraction and line thinning. The line thinning in the end requires a value for the degree of thinning (Douglas, Peuker, 1973). This can be set depending on the original cell size of the raster object. The conversion into an area is similar to the line generation except that we have to apply the discussed algorithm for the boundary of the raster object. We receive again the boundary of the object converted into vectors. The vectors have to fulfill the constraint that they build a closed polygon.

5.1.3 Conversion of Uncertainty

For raster and vector data the uncertainty is described in the same way using probability measures. For both formats an uncertainty matrix stores the values. Thus the conversion of the uncertainty during raster/vector conversion is a trivial step because no new information is generated and thus the values remain unchanged. Only the interpretation changes (see Figure 7). For vector data the value describes the probability that the specific position is part of the object whereas for raster data the value means the probability that the assigned attribute value is correct.

D. Fritsch, M. Englich & M. Sester, eds, 'IAPRS', Vol. 32/4, ISPRS Commission IV Symposium on GIS - Between Visions and Applications, Stuttgart, Germany.


(a)

(b)

(c)

Figure 6: Examples for the conversion of a raster object into an area object (a), a line object (b) and a point object (c). The left side shows the original raster objects overlayed with the conversion results (white). On the right the uncertainty (gray) is displayed as background information.

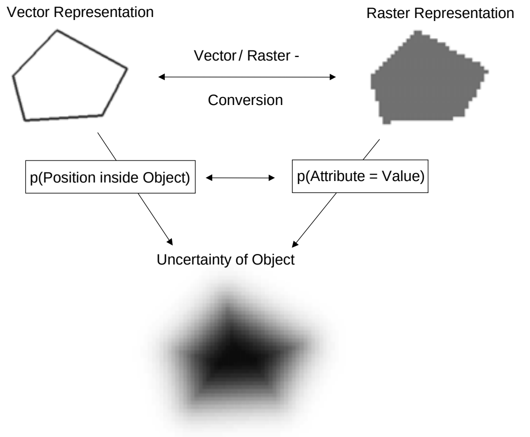


Figure 7: Interpretation for probabilities of raster and vector objects.

5.2 Hybrid Overlay Analysis

Polygon overlay is a basic functionality which is offered by most GIS products. It takes two sets of input objects and intersects them geometrically. The result is a set of new objects consisting only of the intersection parts. Mostly the implementations are limited to data of equal type (e.g. areas can only intersected with areas). The use of hybrid data requires an extension of the normal implementations. For this purpose a new hybrid overlay function is developed. It is called hybrid, because of two aspects. First, the function is independent of the type of primitive (point, line or area) and of the type of data representation (vector or raster). Second, the method takes advantage of the fact that with raster data the overlay operation is easier than with vector data. In the raster domain it is a simple Boolean-And-operator on the raster cells. Thus polygon overlay is realized here using raster overlay technique. If the data is given in vector format it is automatically converted into raster using the described method. Then the overlay is performed completely with raster data. The propagation of the probabilities is defined by

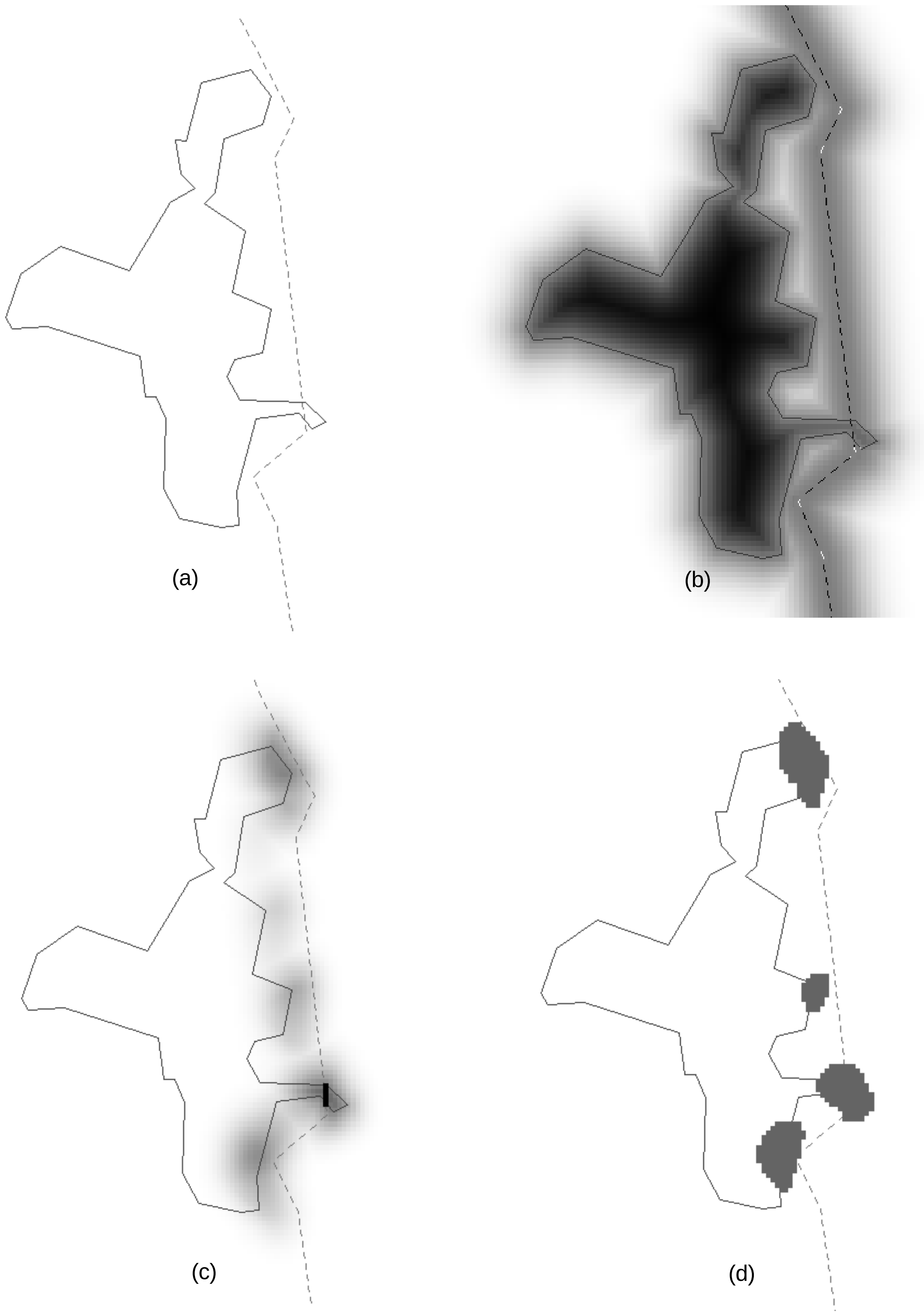
$$p_{xy}(A_1 \cap A_1) = p_{xy}(A_1) \cdot p_{xy}(A_2)$$

with $p_{xy}(A_1)$ and $p_{xy}(A_2)$ as the probabilities of the two objects possessing their assigned attributes values (A_1,A_2) at a certain position P(x,y) (Shi, 1994; Henneberg, 1997). Independence of the attributes is assumed. The result is stored in a new uncertainty matrix. After all the new objects are in the same structure as all others and can be treated in other operations in the same way. Some examples of polygon overlay with uncertainty are shown and explained in Figure 8.

6 CONCLUSION AND OUTLOOK

The paper showed the integration of uncertainty in a hybrid system environment. For this purpose first a hybrid data structure was defined and then extended to hold uncertainty values for raster and vector data. The impact of the uncertainty on analysis tools was discussed focusing on raster/vector conversion and polygon overlay. There are some limitations of the current approach which are subject to further investigations and research. The presented approach is based on objects as spatial phenomena. If fields have to be included they must be transformed into objects. For such phenomena, where an adequate representation can not be found (i.e. phenomena which are highly variable, e.g. temperature) the model has to be extended allowing fields as well. Up to now only discrete values for attributes are considered. The problem defining an extension to continuous attributes is that probabilities are not suitable to describe the uncertainty of such attributes. A possible solution is to open the uncertainty modeling and allow multiple uncertainty measures in parallel. The conversion between these measures is a main problem to be solved. For the working with a GIS, it is necessary that all methods take uncertainty into account. Some methods can be adapted already, others should be investigated in more detail in the future.

D. Fritsch, M. Englich & M. Sester, eds, 'IAPRS', Vol. 32/4, ISPRS Commission IV Symposium on GIS - Between Visions and Applications, Stuttgart, Germany.



(a) (b) (c) (d)

Figure 8: Two intersecting objects (a) – a town (area object) and a power supply line (line object). Uncertainty of the two objects is shown in (b). The darker the gray value the higher the probability is. The result of the polygon overlay is a raster object consisting of a line of raster cells (c) with uncertainty of the object as background information. Obviously there are several intersection regions with varying probabilities (indicated by different gray values). Based on the uncertainty the geometry of the object can be transformed (d). Here all cells are building the geometry which have a probability of more than p = 0.25.

D. Fritsch, M. Englich & M. Sester, eds, 'IAPRS', Vol. 32/4, ISPRS Commission IV Symposium on GIS - Between Visions and Applications, Stuttgart, Germany.

7 REFERENCES

Arronoff, S. (1989): Geographic Information Systems: a Management Perspective. WDL Publications, 1989, 294 p.

Bill, R., Fritsch, D. (1991): Grundlagen der Geo-Informationssysteme. Band 1: Hardware, Software und Daten. Wichmann, Karlsruhe.

Blakemore, M. (1984): Generalization and Error in Spatial Data Bases. Cartographica, Vol. 21, 131-139.

Burrough, P. A. (1986): Principles of Geographical Information Systems for Land Resources Assessment. Oxford Science Publications, Clarendon Press, Oxford.

Burrough, P. A. (1996): Natural Objects with Indeterminate Boundaries. ESF-GISDATA, Vol. 2., Taylor & Francis.

Caspary, W. (1992): Genauigkeit als Qualitätsmerkmal digitaler Datenbestände. In: Grünreich, D., Buziek, G. (Hrsg.): Gewinnung von Basisdaten für Geo-Informationssysteme. DVW-Schriftenreihe, Heft 4, 157-166.

Caspary, W., Scheuring, R. (1992): Error-bands as Measures of Geometrical Accuracy. EGIS ‘92, Vol. 1, 226-233.

CEN (1996): Geographic Information – Data Descritption – Quality. Technical Committee CEN/TC 287, draft document , prEN 12656.

Chrisman, N. R. (1991): The Error Component in Spatial Data. In: Maguire, D., Goodchild, M., Rhind, D. (Eds.): Geographical Information Systems – Principles and Applications. Longman Scientific & Technical, 165-174.

Cramer, M. (1993): Implementation von Raster-Vektor-Konvertierungsbausteinen als Basis für eine GIS-Teachware. Diploma-Thesis at the Institute for Photogrammetry, University of Stuttgart (unpublished).

Douglas, D. H., Peuker, T. K. (1973): Algorithms for the Reduction of the Number of Points Required to Represent a Digitized Line of Caricature. The Canadian Cartographer, Vol.10.

Drummond, J. (1995): Positional Accuracy. In: Guptill, S., Morrison, J. (Eds.): Elements of Spatial Data Quality. Pergamon Press.

Ehlers, M., Edwards, G., Bedard, Y. (1989): Integration of Remote Sensing with Geographic Information Systems: A Necessary Evolution. PE&RS, Vol. 11, No. 11, 1619-1627.

Glemser, M. (1994): Behandlung der Genauigkeit räumlicher Daten in Geo-Informations-systemen. In: Die benutzte Erde, Alfred-Wegener-Stiftung (Hrsg.). Ernst&Sohn, Berlin.

Glemser, M. (1996): Integration geometrischer Datenqualität in GIS-Funktionen. In: Proceedings Workshop Datenqualität und Metainformation in Geo-Informationssystemen, Universität Rostock.

Glemser, M., Klein, U. (1998): Datenqualität in hybriden Geoinformationssystemen. Nachrichten aus dem Karten- und Vermessungswesen. Reihe I, Verlag des Instituts für Angewandte Geodäsie, Frankfurt am Main (in press).

Henneberg, C. (1997): Fortpflanzung der geometrischen Genauigkeit von Objekten bei der Flächenverschneidung. Diploma-Thesis at Institute for Photogrammetry, Universität of Stuttgart (unpublished).

Jäger, E. (1990): Untersuchungen zur kartographischen Symbolisierung und Verdrängung im Rasterdatenformat. Wissenschaftliche Arbeiten der Fachrichtung Vermessungswesen, Uni-versität Hannover, Nr. 167.

Klein, U., Sester, M., Strunz, G. (1998): Segmentation of Remotely Sensed Images Based on the Uncertainty of Multispectral Classification. In: ISPRES Commission IV Symposium, Stuttgart, Germany.

Kraus, K., Haussteiner, K. (1993): Visualisierung der Genauigkeit geometrischer Daten. GIS, Vol. 6, Heft 3, 7-12.

Molenaar, M. (1996): The Extensional Uncertainty of Spatial Object. 7th Spatial Data Handling, Delft, Vol. 2, 9b.1-9b.13.

Molenaar, M., Fritsch, D. (1991): Combined Data Structures for Vector and Raster Representations in Geographic Information Systems. GIS, Vol. 4, Heft 3, 26-32.

Rumbaugh, J., Blaha, M., Premerlani, W., Eddy, F., Lorensen, W. (1991): Object-Oriented Modelling and Design. Prentice Hall.

Shi, W. (1994): Modelling Positional and Thematic Uncertainties in Geographic Information Systems. ITC Publication, No. 22, Enschede.

Stehman, S. V. (1997): Selecting and Interpreting Measures of Thematic Classification Accuracy. Remote Sensing of the Environment, Vol. 62, 77-89.

Yang, H. (1992): Zur Integration von Vektor- und Rasterdaten in Geo-Informationssystemen. DGK, Reihe C, Nr. 389, München.

Zadeh, L. A. (1965): Fuzzy Sets. Information and Control, Vol. 8, 338-353.