ON STATISTICAL CORRELATION BETWEEN IMAGE COORDINATE ERRORS

K. Torlegård, Stockholm

1. INTRODUCTION

The development of cameras, photographic material, measuring instruments, and computational methods in analytical photogrammetry has now come to a stage where we can achieve in production environment a relative accuracy in point determination of 10^{-5} of the object size. Under very specialized conditions even better accuracy has been reported. The methods include photography with multiple coverage, i e each object point is imaged in more than two images, and a rigourous least squares adjustment of observations based on the perspective transformation with selfcalibration and/or additional parameters. The computation often includes an automated elimination of blunders and outliers in the observations, so called data snooping.

Measures of accuracy of the determined point coordinates can be determined in various ways. The most direct way is of course to calculate the variance-covariance matrix of the unkonwn point coordinates in the least squares adjustment. Sometimes controlled experiments are organized in which the photogrammetric result is compared with so called given values. Other methods to estimate the accuracy may be based on calibration and estimation of precision of instruments and procedures and propagation of such errors to the final result.

Estimation of accuracy is based on theory of errors of observations. We have well established methods to calculate the propagation of errors and variances-covariances. The theory also should be albe to explain the size, distribution and interrelation of residuals of adjustments, and explain discrepancies when results are checked against other values. The theory has to include assumptions about the properties of the errors of the basic observations in photogrammetry. The majority of practical adjustment calculations are based on the assumption of that observations are indepndent and of equal weight. Resulting discrepancies of controlled experiments, residuals of calibration of cameras and instruments, and of adjustments of highly redundant image blocks show however that errors of image coordinates are not independent and not of equal variance.

2. WHAT IS AN OBSERVATION ERROR IN PHOTOGRAMMETRY?

What is an observation in analytical photogrammetry? In least squares adjustment of bundle blocks it is the image coordinate. What is then the error of this observation? Is it the error made by the observer, may he be human, electronic or algorithmic. No, the observation error is something else. So what is it?

The observation error is the error of the mathematical model used in the calculation of the unknowns. The error is not made by the observer. It is the difference between reality and model, between observation and the value obtaind from the functional model of reality. It is the approximation introduced when we use a model to describe the physical reality. We write

$$\varepsilon_i = (x', y') - F_i$$

where index i indicates the type of function F we have chosen, which gives the approximation or error ε related to this function.

Let us look at three common functional models in photogrammetry. The first is single point resection in space with given interior orientation. Six parameters of the exterior orientation are unknowns, $X_O, Y_O, X_O, \omega, \phi, \kappa$ and three of the interior are given, x'_O, y'_O, c. The perspective transformation from given object coordinates X,Y,Z to image coordinates x′y′ is the functional model. The second model is the projective relation between two planes. Eight parameters are unknown, no interior orientation is needed, and given X,Y object coordinates are transformed to the image x′, y′. The third model is the perspective transformation with unknown parameters also for interior orientation including radial and decentering distortion and affine shrinkage.

Assume that we have used a distortion free high precision camera to take photographs of a completely flat terrain, i e all object points in the same plane. Then all three functional models will give residuals of the same magnitude. But as soon as the object points are located at some varying distances from the average plane, the residuals of the projective model will increase in magnitude compared to those obtained from the other models. Common cameras are seldom or never free from distortion and the film always shows some shrinkage, which means that the perspective transformation with additional parameters for distortion and shrinkage always yields residuals of smaller magnitude than the other models. These examples demonstrate that observation errors are depending on the approximations introduced by the functional model.

3. ELEMENTARY ERRORS

The most common mathematical model is based on the perspective transformation with corrections for affine film shrinkage, radial distortion, atmospheric refraction, and earth curvature in one way or another. This is a functional description of the imaging geometry. What kind of approximations are now made?

After having corrected for the systematic errors mentioned, we regard the geometry to be a perfect central perspective. This means that we assume the imaging ray to be a straight line from object point through perspective centre to image point. This is of course not perfectly true. There are remaining radial and tangential components of lens distortion. They may be modelled by higher order parameters for radial distortion and by parameters of decentering distortion, but there will always be a limit on the number of parameters. The approximation in rays close to each other will be very similar. The approximation is likely to change continously over the field of view without any discontinuities. Errors in adjacent points thus have a positive covariance.

The perspective transformation also assumes the image to be a plane. The flatness of the emulsion is limited and this is another approximation. Image points are displaced in radial direction because of this. Again the displacement error is a continuous function which causes correlation between adjacent points. The effect on image coordinates increases with the radius from the principal point.

The major parts of the film shrinkage are two scale factors along and across the film, and a lacking perpendicularity. They are compensated for by the affine transformation. But there are irregular shrinkage patterns that remain, and again these cause errors which are not independent for adjacent points.

The correction for the atmospheric refraction is based on physical parameters of a standard atmosphere. At exposure the physical conditions may be different from those of the standard, e g other temperature and pressure gradients. If this is the only error, it will cause a radial error component. The variation of the optical density within the field of view can however be irregular, which gives both radial and tangential errors. Again we get correlated image errors. Their variance can be assumed to increase with the angle from the nadir line, as those rays travel longer through the atmosphere.

Image coordinates are measured by an operator with a comparator. The operator setting will be in error, and this is one of the few elementary errors that are likely to be independent. The optical and mechanical parts of the comparator however cause errors that show positive correlation for points close to each other. The coordinate recording device reads with a certain resolution and this causes a rounding-off error which has a rectangular distribution over the least reading interval. This error is independent.

The photogrammetric process includes the use of so called given values of fiducial marks and of control points for absolute orientation. Given values are also used in controlled experiments and in test field calibration. These given values have errors. Some points, e g control, have been signalized before photography. The excentricity of the panels or targets cause errors. They can as a rule be regarded to be independent.

The following table shows an attempt to illustrate which physical errors influence some photogrammetric procedures or adjustments.

Physical error	Influence on accuracy in Fiducial transf	Rel orient	Abs orient	Calibration self-	testfield
Lens	-	x	xx	xx	xx
Flatness	x	xx	xx	xx	xx
Shrinkage	xx	xx	xx	xx	xx
Atm refr	-	x	xx	xx	xx
Instr	xx	xx	xx	xx	xx
Operator	xx	xx	xx	xx	xx
Given fid.	xx	-	x	(x)	(x)
Given XYZ	-	-	xx	-	xx
Targeting	-	-	xx	-	xx

Legend:
- `-` No influence
- `x` Some influence
- `xx` Large influence
- `(x)` Depending on method

4. STOCHASTICAL MODEL

The elementary errors are to be regarded as stochastic variables. We do not know their individual magnitude and direction. But we can make assumptions on the variances and covariances of each elementary error. We can also assume that one type of elementary error is independent of another one.

The errors depending on lens distortion and atmospheric refraction can be divided in two components, radail and tangential. The two components may have different variances. These errors have to be propagated to the image coordinate system.

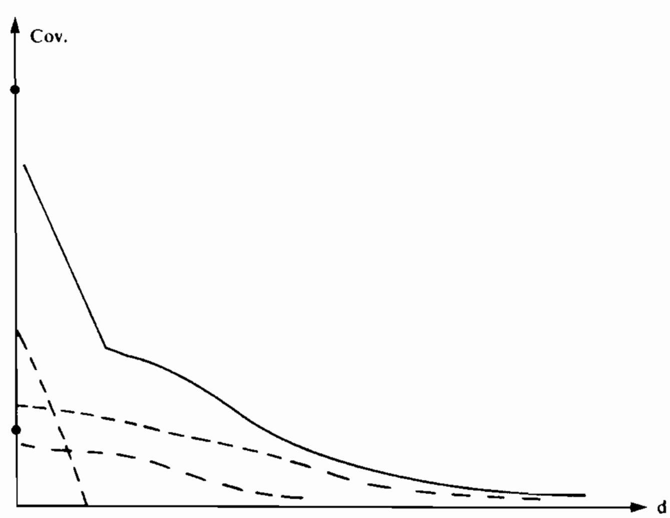


Fig 1.
Covariance functions of
elementary errors - - -
the total error ——
and variance of
independent errors .

Doing so we find that we arrive at a correlation between the errors of x´ and y´ of each particular point. The flatness error causes the same effect.

We have been able to make a statement on the distribution function for one elementary error only, namely the rounding-off error. The distribution of the other elementary errors may follow the normal distribution, but we can not say for sure. But we can assume that the variances of the elementary errors are limited. We may even assume that their magnitudes are similar.

The photogrammetric observation error is the sum of the elementary errors. With the above assumptions we may justify the following conclusions based on statistical theory:
-that the variance-covariance of the sum is the sum of the variance-covariances of the elementary errors,
-that the observation error has a distribution that is approximatively normal,
-that the variance varies over the image,
-that the errors in x´ and y´ of a point are correlated, and
-that errors of pairs of points have a covariance that decreases with distance between the points. See fig 1.

The theory in this and previous paragraph is described in more detail by TORLEGÅRD 1989.

This type of covariance functions are also used in linear least squares interpolation, see e g KRAUSS 1972, who used it in digital terrain modelling and for correction of film deformation.

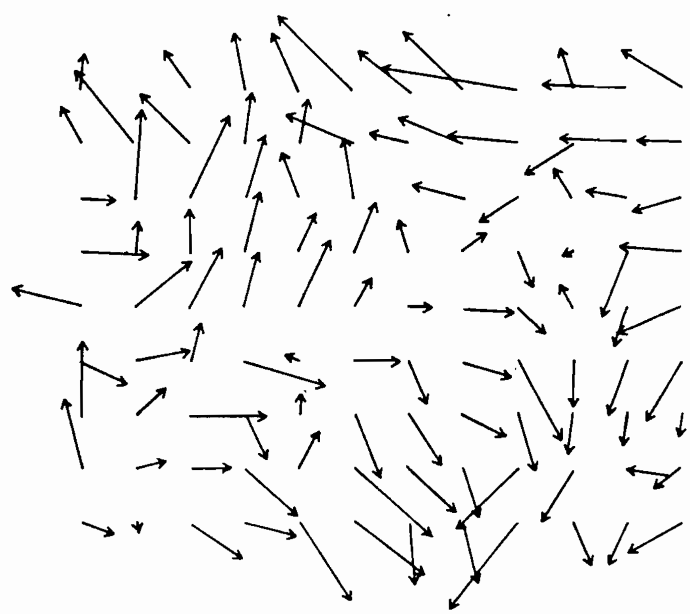
The covariance function is a decreasing function taking positive values, and for distances larger than a certain value the covariance can be neglected. This limiting distance depends on the type of mathematical function chosen to describe the imaging geometry. For a 15*23*23 aerial camera we may assume the following limiting distances:
40 - 60 mm without distortion or additional parameters,
20 - 30 mm with 3 radial and 2 decentering parameters,
20 - 30 mm with 20 additional and no distortion parameters.
The idea is that the covariance function shall describe the systematic effects that remain after the additional parameters have removed the major part of the systematic error. Thinking in terms of spatial frequency, the additional parameters eliminate the lowest frequencies of the systematic error variation, while the covariance function has to describe the effect of the remaining higher ones.

5. EFFECTS EXPLAINED BY THE STOCHASTICAL MODEL

Camera calibration on test fields with a large number of well distributed points as a rule show residual vectors that are of the same size and direction in local areas of the image, and that change slowly to other areas. See e g TORLEGÅRD 1967 and fig 2. Similar patterns of residuals is recognized in controlled experiments. Film shrinkage determined from reseau plates also show such quasi-systematic semi-irregular residual patterns. As a matter of fact it is such results that has been the reason for the development of a theory of correlated errors. The numerical results ask for an explanation and the theory is an attempt to give an answer.

A deep study of the variance-covariance conditions of a stereomodel was presented by STARK 1973. He had a large empirical data material from a test field. The autocorrelation of errors in model coordinates was 80% or more. This correlation can be explained by the propagation of variance-covariance from the image coordinates to the model. The study also demonstrates the effect of correlation on the standard error of distances computed from the model coordinates. The prediction of variance of those distances is much better when the correlation is taken into account.

Fig 2
Typical residual vector pattern showing strong correlation between neighbouring points.



The stochastical model is also in accordance with the results and conclusions of studies by SCHILCHER 1980 and ACKERMANN & SCHILCHER 1978 on auto- and cross correlation of image errors. The new thing in these studies is the experimental estimation of the correlation of observation errors between points with the same image coordinates but in adjacent images on the film. Such a positive covariance can be explained by the above theory. The physical errors related to the camera do not change very much from one exposure to the other in strip photography. In case the photos are oriented in the same way in the measuring instrument, the instrument errors will be very similar, too. Further, parts of the atmospheric error also may contribute to the covariance.

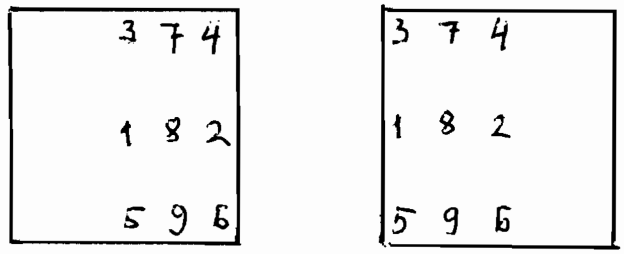
It is wellknown that repeated readings on the same image has a very limited effect on the accuracy of the final result. Taking the average of several readings only reduces the variance component of the operator and the rounding-off (provided the least reading interval is small compared to the operator´s precision). The elementary errors caused by the lens, atmosphere, flatness, film, measuring instrument, targeting, and given coordinates are the same. Their variances are not reduced by the repeated readings. In order to reduce these later variances, the photography has to be arranged in such a way that the elementary errors vary. This is achieved by taking several exposures at each station with different κ-values, and by using more than two photo stations for the point intersection. Multi-station, multi-frame photo blocks are nowadays more or less standard in high precision photogrammetry, both from the air and in industrial environments.

6. EXAMPLES OF APPLICATIONS OF THE THEORY

6.1 Testfield calibaration

Let us begin with the least squares adjustment of a single point resection in space with simultaneous determination of interior orientation parameters based on test field photography. We assume a three dimensional test field with a rather dense net of points (maybe some hundred or so) in a rear plane and some 20 points outside this plane so as to provide a strong geometry for the solution. In most cases the observations are given the same weight and they are regarded to be independent. According to the above theory we should calculate an à priori weight matrix, which is the inverse of the variance-covariance matrix of the observation error. This matrix has to be calculated from known, given or assumed values of the variance-covariances of the elementary errors, which have to be propagated to the sum, the observation error.

Fig 3
Nine points for relative orientation that give an à priori weight matrix as shown in fig 4.



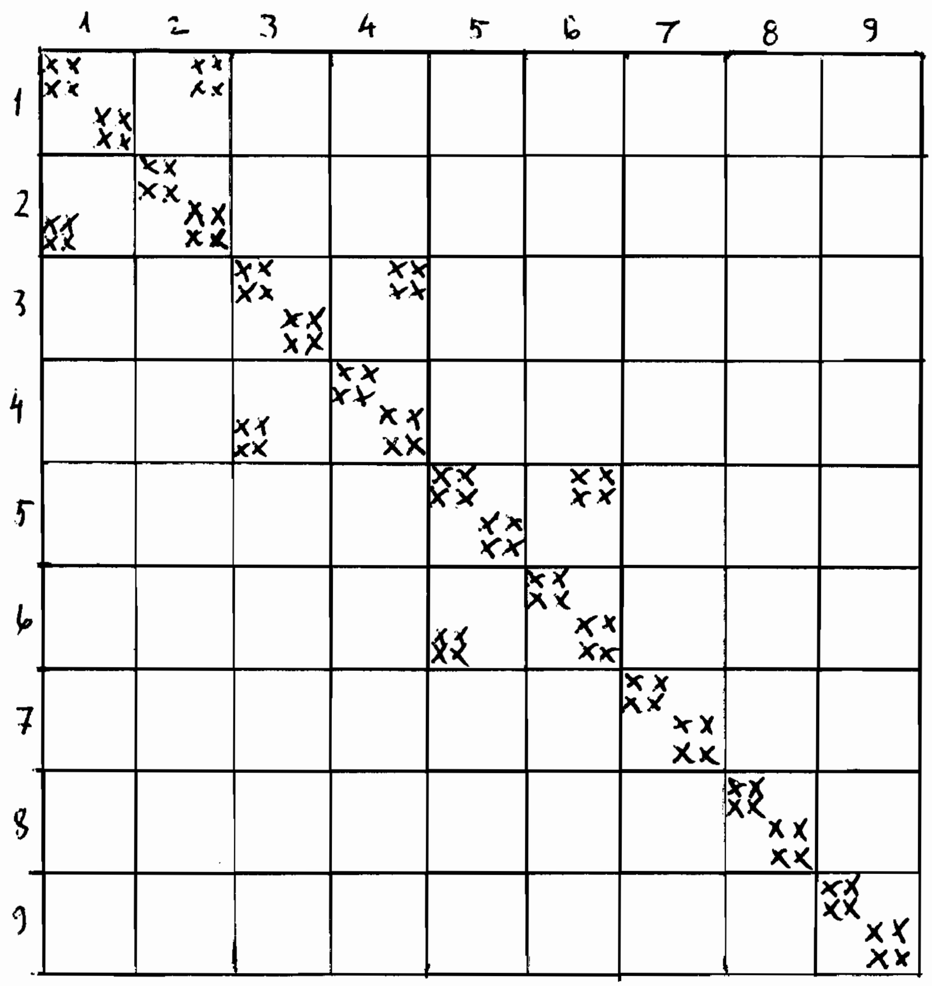


Fig 4. Structure of the à priori weight matrix for nine point relative orientation with point configuration as shown in fig 3.

What would be the difference in the result using à priori correlation and not? The estimates of the unknowns will not change very much, as we have assumed the points to be well distributed over the image with more or less constant distances to neighbouring points. But if some points are clustered in one area with just a few points left in the other part of the image, then one would expect changes in the unknowns.

The estimates of the variance-covariances of the unknowns would change considerably, as well as estimates of variance in functions of the unknowns. This is likely to occur both for homogeneous and heterogeneous point distribution.

6.2 Relative orientation

Let us consider the case of relative orientation based on the coplanarity condition. Observations are made in 9 regularly located points, see fig 3. The shortest distance in the image between two points is 45 mm, which is longer than the max distance for which image errors are correlated. Thus the covariance between points in the same image is zero. But we have a correlation between x´ and y´ of each point. Furthermore, point 1 in the left image is correlated with point 2 in the right, as they have the same location in the image coordinate system. The same holds for the pairs 3-4 and 5-6. The non-zero elements of the à priori variance-covariance matrix is shown in fig 4.

Relative orientation can be subjected to so called data snooping after blunders when there are more than six points observed. Assume that we want to observe 10 points to determine the 5 unknowns. The relative redundancy is 0.5 which is just enough to provide a reliable system for data snooping. How shall we locate the points? First of course we use the six von Gruber positions, but what about the remaining four? If the observations really were independent, then the optimal locations would be in the corners of the model. But our stochastical model tells us that observations are correlated for short distances in the image. Having double observations in the corners just checks blunders made by the operator and the coordinate recording device. If we want to check for blunders and outliers originating also from camera, film and atmosphere, then the points should be distributed in such a way that the distance between them is larger than the effect of correlation. See fig 5.

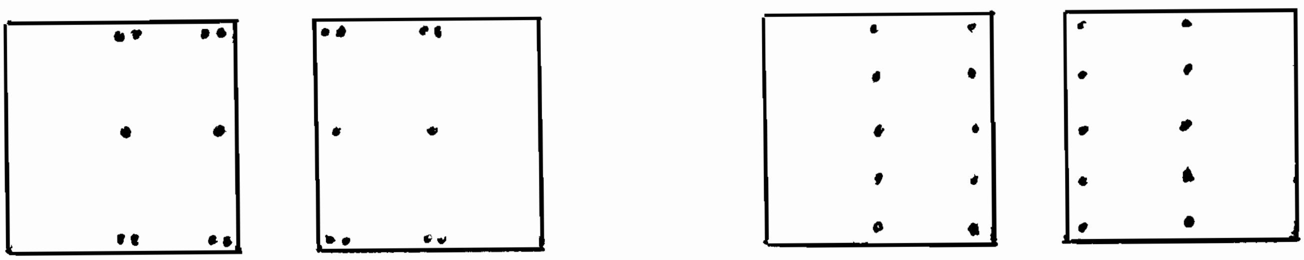


Fig 5. Optimal point positions for blunder detection in relative orientation. To the left assuming zero correlation, to the right with strong correlation

6.3 Absolute orientation of a stereo model

ACKERMANN 1976 has demonstrated the improvement of accuracy when the correlation is taken into account in absolute orientation. He used the same data as STARK 1973. Nine cases of control point configurations were used in a rigourous adjustment with full à priori weight matrices computed from STARK´s empirical variance-covariance matrices. One of the conclusions was that the improvement in height accuracy increased with the number of control points when correlation was taken into account. The relative improvement was larger for few control points. The improvement was a factor 1.6 between 3 and 23 points. A better improvement was expected, but a fairly large random component of the height error covered the effect of the computed propagation of covariances. In the above presented theory given coordinates and targeting contribute to the total error. But Ackermann assumed the control points to be error free. This may explain the effect of the random component in the height error on the improvement factor. With a lower weight on the control point observations, the improvement factor would be larger.

BARD 1989 has developed a computer programme to demonstrate the effect of correlated observations compared to independent ones. The à priori weight matrix for a rigourous adjustment of a stereo model is calculated according to the method described by TORLEGÅRD 1989. Variances and covariance functions can be chosen arbitrarily for the elementary errors described above. Variances and covariances are then propagated to the image coordinates of the stereo pair. The inverse is used in the adjustment. Variances and covariances of new points are calculated. The programme has been used for studies where camera type, flying height, measuring instrument, and control point configuration have been varied.

7. CONCLUDING REMARKS

Covariance between image coordinate errors has been known for a long time. In spite of this, very few studies on the stochastic properties have been reported. To an even lesser extent covariance has been used in practice as basis for à priori weight matrices in adjustment of photogrammetric observations in block adjustment, testfield calibration and prediction of accuracy of photogrammetric processes. One reason for this may be the fact that point coordinates depend very little on the neglection of the à priori covariances in block adjustments with ordinary block geometry.

Another reason for not using à priori covariances may be the difficulty to determine à priori values of the variance-covariance of observations.

A third reason is of course that there has not been an urgent need to take the correlation into account in practical analytical photogrammetry. The interest has instead been devoted to additional parameters, self calibration, and localization and elimination of blunders from observations. This - in combination with multi coverage image blocks and versatile block adjustment programmes - has developed photogrammetry to an efficient method for high precision point determination.

A sound theory of image coordinate errors is however necessary to explain and to understand the accuracy properties of photogrammetric results. Such a theory is also necessary for accurate and unbiased prediction of accuracy of photogrammetric results and functions thereof.

REFERENCES

ACKERMANN, F.: Genauigkeitssteigerung durch Berücksichtigung der Korrelation bei der absoluten Orientierung des Bildpaares. DGK B216, S. 5 - 18. München 1976.

ACKERMANN, F. & SCHILCHER, M.: Auto- and Cross-Correlation of Image Coordinates. Nachrichten aus dem Karten- und Vermessungswesen, Reihe II, Nr 36, 1978. S. 5 - 18.

BARD, P.: Att beräkna viktmatris för utjämning av fotogrammetrisk enkelmodell. (In Swedish) Dipl thesis, KTH, Stockholm 1988. 53 p.

KRAUSS, K.: Interpolation nach kleinsten Quadraten in der Photogrammetrie. BuL 40 (1972), Heft 1, S 7-12.

KRAUSS, K.: Correction of Film Deformation by Least Squares Interpolation. Photogrammetric Engineering, 38:5, 487-493, 1972.

SCHILCHER, M.: Empirisch-statistische Untersuchungen zur Genauigkeitsstruktur des photogrammetrischen Luftbildes. DGK C262, München 1980.

STARK, E.: Die Genauigkeitsstruktur im photogrammetrischen Einzelmodell. DGK C193, München 1973.

TORLEGÅRD, K.: On the Determination of Interior Orientation of close-up Cameras under Operational Conditions using Three-dimensional Test Objects. Thesis, KTH, Stockholm 1967, 100 p.

TORLEGÅRD, K.: Theory of Image Coordinate Errors. Chpt 7 in Non-Topographic Photogrammetry, edited by H M Karara, ASPRS, Falls Church Va, 1989, pp 81-93.