
DIGITAL IMAGE CORRELATION WITH THE ANALYTICAL PLOTTER PLANICOMP C100

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Abstract: The paper presents a method of digital image correlation matching the matrices of grey values of 2 homologous image windows by a linear geometric and radiometric transformation. The system is implemented at the Zeiss Planicomp C 100 for on-line operation.

First the mathematical formulation of the correlation method is given. The unknown transformation parameters are determined by a least squares solution by minimizing the differences of grey values.

Then the extension of the hardware-configuration of the system Planicomp C 100 by 2 video cameras is described.

Finally the results of experimental investigations with the correlator are discussed with respect to accuracy and time consumption.

1. Introduction

Parallax measurements are used in a wide range of applications in stereo observation techniques. Examples in photogrammetry are relative orientation, digital elevation model, point transfer etc. The quality of such operations depends directly on the quality of parallax measurements.

The idea to execute parallax measurement by matching small image areas by digital correlation was taken up at Stuttgart Institute of Photogrammetry about five years ago. A correlation method was adopted and further developed which achieves optimum matching of two digitized image areas by a least squares solution for adequate geometric and radiometric transformations.

As the initial investigations have been most successful and promising (ACKERMANN/PERTL, 1983) it was decided to integrate this method into a photogrammetric instrument for direct application to standard photogrammetric operations. The direct aim of this project is automatic high precision measurement of parallaxes by digital image correlation in connection with existing photogrammetric instruments.

The instrument in question is an analytical plotter Zeiss Planicomp C 100 which has been equipped with two Hamamatsu CCD video cameras which can digitize image areas of up to 58 mm². The analog signals, after passing an interface and being decoded, are processed in the standard HP 1000 computer of the instrument by our own software program for digital image correlation.

The system is operational and has at present the following capabilities:

- Approximate overlap of homologous image windows is obtained by the human operator
- Scanning of image windows of 32×32 picture elements of 20 μm linear pixel size
- Selection of a central (16×16) area within the (32×32) area
- Phase correlation with (16×16) grey value matrices (for safe and quick convergence), and
- Correlation by least squares technique comparing a (16×16) mask matrix and a (22×22) search matrix
- Transfer of a representative point from mask matrix to search matrix
The complete process of digitization and digital image correlation is a real time process and takes altogether, at present, about 5 sec of time. The process is interfaced with the ordinary operation mode of the analytical plotter without any disturbing effects.

In this paper first the correlation method is explained. Thereafter the hardware configuration is described and empirical results of automatic relative orientation are presented. The paper will be concluded with a discussion of the results and an outlook to further investigations and developments.

2. The two-dimensional correlation by a least squares solution

(1) The basic idea of the correlation method consists in optimal matching of 2 homologous image areas by minimizing the square sum of the remaining grey-value differences between the two grey value matrices. This is achieved by a least squares adjustment introducing the grey value differences as observations.

For the mathematical formulation it is assumed that two homologous digitized image windows

\[ g_m = g_m(x^m, y^m) \]

and

\[ g_s = g_s(x^s, y^s) \]  \hspace{1cm} (2.1)

of a pair of overlapping photographs are representing the same object. The digital images are assumed to be contaminated by noise

\[ n_m = n_m(x^m, y^m) \]

and

\[ n_s = n_s(x^s, y^s) \]  \hspace{1cm} (2.2)

We then obtain for the observed grey-value functions:

\[ \bar{g}_m = g_m + n_m \]

and

\[ \bar{g}_s = g_s + n_s \]  \hspace{1cm} (2.3)

The grey value functions and coordinate axis of both scanning systems are indexed with \( m \) and \( s \), standing for mask matrix and search matrix respectively.

In addition it is assumed that the digital images differ both in geometry and radiometry as they belong to different perspective images and different photographic exposures. Therefore, when correlating image windows, which are scanned independently of each other a geometric transformation and a radiometric transformation has to be taken into account. At present, the geometric transformation applied in the program is an affine (6 parameter) linear transformation:

\[ T_x = T_x(x^m, y^m, p_1) = x^s = p_0 + p_1 x^m + p_2 y^m \]  \hspace{1cm} (2.4)

\[ T_y = T_y(x^m, y^m, p_1) = y^s = p_3 + p_4 x^m + p_5 y^m \] .
The radiometric transformation considers, at present, 2 linear parameters:

$$T_R = T_R(r_j, \bar{g}_s) = \bar{g}_m = r_o + r_j \bar{g}_s \ . \quad (2.5)$$

The functions $T_x$, $T_y$ thus compensate for geometric distortions, as they occur locally in perspective images. The function $T_R$ compensates for difference in brightness level and contrast scale.

Connecting eqs. (2.4) and (2.5) leads to the mathematical formulation of the relation between the digitized images:

$$g_m(x^m, y^m) + n_m(x^m, y^m) = T_R(r_j, g_s(T_x, T_y) + n_s(T_x, T_y)) \ . \quad (2.6)$$

If the symbol $\nu(x^m, y^m)$ is substituted for the differences of the noise components

$$\nu(x^m, y^m) = n_m(x^m, y^m) - T_s(r_j, n_s(T_x, T_y)) \quad (2.7)$$

equation (2.6) can be written as

$$g_m(x^m, y^m) + \nu(x^m, y^m) = T_R(r_j, g_s(T_x, T_y)) = T_R(r, g) \ . \quad (2.8)$$

Assuming that the shifts of the unknown transformation parameters are differential small values, a first order approximation of eq. (2.8) is applied with regard to $p_i$ ($i = 0, \ldots, 5$) and $r_j$ ($j = 0, 1$)

$$g_m(x^m, y^m) + \nu(x^m, y^m) = T_R^0(r_j, g_s(T_x^0, T_y^0))$$

$$\quad + \sum_{i=0}^5 \frac{\partial T_R(r, g)}{\partial r_j} \cdot \frac{\partial T_R(r, g)}{\partial p_i} \cdot dp_i \ . \quad (2.9)$$

With

$$g_x = \frac{\partial T_R(r, g)}{\partial T_x} = \frac{\partial T_R(r_j, g_s(T_x, T_y))}{\partial T_x}$$

$$g_y = \frac{\partial T_R(r, g)}{\partial T_y} = \frac{\partial T_R(r_j, g_s(T_x, T_y))}{\partial T_y} \quad (2.10)$$

and

$$\Delta g(x^m, y^m) = g_m(x^m, y^m) - T_R^0(r_j, g_s(T_x^0, T_y^0))$$

we obtain from eq. (2.9)

$$\Delta g + \nu = \sum_{j=0}^5 \frac{\partial T_R(r, g)}{\partial r_j} \cdot dr_j + \sum_{i=0}^5 \left(g_x \frac{\partial T_x}{\partial p_i} + g_y \frac{\partial T_y}{\partial p_i} \right) \cdot dp_i \ . \quad (2.11)$$
where \( \Delta g \) is the difference of the observed grey values, \( g_x \) and \( g_y \) are the gradients of \( g_m \) in x- and y-direction.

Equation (2.11) has the form of linearized observational equations with the observations \( \Delta g \) and the unknowns \( p_\perp \) and \( r_j \). For each pair of corresponding pixels one equation is obtained leading to a highly redundant equation system which is solved for the unknowns using the least squares technique. The observations are assumed to have equal weight, thus the disturbing noise is supposed to be white. This least squares solution, minimizing \( v \), corresponds to an image correlation by minimizing the residual grey values.

Normally the approximate values of the unknown transformation parameters are initially not close enough, therefore convergence of this nonlinear problem is obtained by an iterative approach. For one of the two discrete grey value functions (here \( g_a \)) this requires a new set of interpolated grey values after each iteration step known as resampling. The bi-linear interpolation method is used there determining the resampled values from the four surrounding picture elements.

For reasons of symmetry the grey value gradients \( g_x \) and \( g_y \) are computed from both scanned image areas and averaged.

\[ \begin{align*}
    x_c^m &= \frac{\sum_k (x^m \cdot g_x^k)}{\sum g_x^k} \\
    y_c^m &= \frac{\sum_k (y^m \cdot g_y^k)}{\sum g_y^k}
\end{align*} \]

(2.12)

The corresponding point in the transformed image window is obtained by applying the transformations \( T_x \) and \( T_y \) to \( x_c^m \) and \( y_c^m \). The coordinates \( x_c^a \) and \( y_c^a \) of the transferred point in general do not coincide with the centre of gravity in the search matrix.

As can be shown (Förstner 1982) using this special point as transfer point the effect of un-modelled geometric distortions will be reduced.

(3) For quality assignment the variance factor

\[ \sigma_0^2 = \sum \Delta g^2 / (n-u) \]

(2.13)

is estimated from the remaining grey value differences \( \Delta g \) between the two correlated areas. \( n \) is the number of pixels used and \( u \) is the number of unknown transformation parameters (8 in this case). With \( \sigma_0 \) and the elements of the weight coefficient matrix the standard deviations \( \alpha_{x_c} \) and \( \alpha_{y_c} \) of the transferred point are derived using error propagation. \( \alpha_{x_c} \) and \( \alpha_{y_c} \) are a measure for the precision of the point transfer (based on the mathematical model of the least squares approach).
3. Hardware configuration

The analytical plotter Zeiss Planicomp C 100 at the Institute for Photogrammetry at Stuttgart University has been supplemented by 2 Hamamatsu solid state video cameras in June 1983 for on-line image correlation. The system is intended for on-line digitization of local image areas and digital processing on the analytical plotter in real time, without disturbing or hindering the operator. The video cameras were attached to the Planicomp (underneath the plate carriers) without major modifications. On either side the light, coming from the illumination system of the Planicomp and after passing through the diapositive, goes through a beam splitter.

The sensor array is scanned, giving an analog output signal which goes through an A/D converter to be delivered digitally into the internal memory of the HP 1000 computer of the Planicomp system. The selected image window can be presented on a monitor.

![Diagram](image)

Fig. 3.1: Modified Zeiss Planicomp C 100 for on-line digital image correlation

In the following some more features of the system are briefly described:

(1) Scanning of the two image windows is done by the two matrix cameras (C1000-35 M). They are fixed at the bottom side of the basic instrument of the planicomp C100 system. Each video camera has a sensor array of 244(vertical) * 320(horizontal) metal oxide semiconductor elements. Only the central part of 256 semiconductor elements in each horizontal line is sensitive to light. The cameras have a high geometrical stability and low error rate during data recording. The unit cell size of each semiconductor element is 27µm * 27µm, so that the sensor arrays cover an area of size of 6.6mm * 8.8mm.
The energy of light for the passive sensors is obtained from the ordinary planicomp illumination system by splitting the beam after passing the diapositive and before reaching the measuring mark (Fig. 3.1). For this purpose the two prisms between the photo carriages and the measuring marks are replaced by two beam splitters which divert 15% of the energy of light onto the sensor arrays. Between the beam splitters and the video cameras two auxiliary optical systems are inserted with an enlargement factor of 1.35 producing an effective pixel size of 20μm * 20μm at the plane of the photograph. Thus the effective size of the sensor arrays is 4.9mm * 6.4mm in the image.

To complete the system additional hardware components are needed: 2 A/D-converters for quantizing the analog output signal of the video cameras into 8-bit grey levels and 4 interfaces to transfer the signals digitally into the internal memory of the HP 1000 computer of the C 100 system.

(2) The present hardware configuration doesn't permit a pixelwise data recording. The image fields are scanned sequentially vertical line by vertical line. Therefore it is possible to scan single lines, selected parts of the matrix or all 256 vertical lines of the sensor array being sensitive to light. It is possible to mark the selected column for scanning with a cursor on a supplemented monitor. Sampling line and marker positioning is selectable in 1024 steps (10-bit) in each horizontal line. By this it is possible to skip a constant number (1, 2, ..., 16) of steps. Because of the 256 semiconductor elements per line a data compression (selection) of 4 is efficient, so that each sensor element is responding just once.

The addressing range for each vertical line is 256. The data of the first row and of the last 12 rows are uncorrected (there are no video signals). So there are only 243 quantized signals available at each vertical line for further processing.

The process of scanning takes 1/60 second for each group of 256 vertical lines. Quantizing the analog output signal of the cameras in one of 256 (8-bit) grey levels takes 1.5μsec for each signal. So conversion of the whole scanned image takes about 0.10 seconds in total.

For fully utilizing the maximum density of data and transferrate 2 successive 8-bit-grey-values are packed in a 16-bit word and transferred into the computer memory in blocks by 128 words per transfer. The blocksize is not limited to 128 words. It depends on the processing rate which is required by the computer.

After the process of data transfer the 16-bit words must be depacked into 2 (8-bit) bytes corresponding to the previously grey values. This is done by bit-masking and bit-shifting. Data transfer for each block containing 128 words takes 16.7 msec. Thus the transfer of all quantized signals of one whole sensor array takes about 4.3 seconds.

If scanning, A/D-conversion and data transfer is limited to the 2 subsections are needed for correlation (32*32) as described in part 1 the total procedure of data recording in this case takes about 2.2 seconds.

4. Results of first empirical investigations

The analytical system extended to digital image correlation, is expected to automatically measure parallaxes which can be used in different areas of application, for example relative orientation, measurement of digital elevation models or deformation measurements.
(1) Our first investigations, during the past few months, concentrated on the calibration of the system. Here, some results will be shown concerning the first photogrammetric application: parallax measurement for relative orientation.

For this application the absolute calibration of the coordinate systems of the two sensor arrays has only to be known approximately. The calibration of the relative position of the sensors is required with an accuracy only, which guarantees convergence of the correlation algorithms. An accuracy of one pixel is quite sufficient for the approximate relative positioning of the two windows. Any remaining constant error will cancel out in the process of relative orientation.

A rough absolute calibration of the sensors with regard to the image coordinate systems of the analytical plotter can be obtained with the aid of grid plates, for both sensors separately: Either floating mark of the planicon is set on a reseau cross. The position of the measuring mark does not necessarily coincide with the central point of the CCD array. Approximate centering is possible through the signal processing by appropriately addressing the respective elements of the sensor array.

The relative positioning of the two windows is achieved by the digital image correlation. The results of 19 such correlations are shown in fig. 4.1 and 4.2 for x- and y-direction.

![Fig. 4.1: Relative positioning of the sensor arrays in x-direction](image1)

![Fig. 4.2: Relative positioning of the sensor arrays in y-direction](image2)

This method achieves a calibration of the relative position of the scanned image windows (i.e. of the parallax) to each other to less than one pixel. Thus, without further correction approximate values for shifts (part 2) between mask and search matrix are calibrated to within one pixel. This is sufficient for y-parallax measurements as needed for relative orientation. Any remaining constant error is taken out in the orientation procedure, i.e. is of no consequence, and the initial approximation obtained is close enough for convergence of the correlation procedure.

(2) As a representative example the results of relative orientation with automatic y-parallax measurement are presented. The example refers to a pair of photographs from the test field Appenweier (photo scale 1:8000, 60% foreword overlap, RMK 15/23, photo number 285 and 288). Here it was possible to distinguish between natural and signalized points for relative orientation.

The analytical relative orientation was computed separately for 27 signalized orientation points and for 12 natural terrain points. The internal precision of the automatic parallax measurements is indicated by the theoretical standard deviations $\sigma_{x_c}$, $\sigma_{y_c}$ of the digital image correlation at the points used for relative orientation. The standard errors are quite small, amounting to
< 0.05 pixel size (< 1 µm) in case of signalized points and < 0.07 pixel size (< 1.4 µm) in case of natural terrain points (Fig. 4.3 and 4.4). Thus the high local accuracy of digital image correlation, known from previous experiments, is again confirmed.

![Graphs showing precision](image)

**Fig. 4.3:** Internal precision ($\sigma_{x_c}, \sigma_{y_c}$) for signalized points  
**Fig. 4.4:** Internal precision ($\sigma_{x_c}, \sigma_{y_c}$) for natural terrain points

For comparison purposes the same points were also measured in the conventional way by the operator. With those measurements the relative orientation was again computed.

From the residual $y$-parallaxes after relative orientation a standard deviation of observed $y$-parallaxes can be estimated from

$$
\sigma_{\text{op}_y}^2 = \sum p_y^2 / (n-5)
$$

(4.1)

In table 4.1 the resulting $y$-parallax precision is shown for the conventional type of measurements and for the digital image correlation, distinguishing in either case between signalized and natural points.

<table>
<thead>
<tr>
<th>measurement</th>
<th>kind of points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>signalized</td>
</tr>
<tr>
<td>stereo-observation</td>
<td>3.1 µm</td>
</tr>
<tr>
<td>(operator)</td>
<td></td>
</tr>
<tr>
<td>digital correlation</td>
<td>3.7 µm</td>
</tr>
</tbody>
</table>

**Table 4.1:** Standard deviation of $y$-parallax observations, for signalized and natural points

The results of table 4.1 indicate generally, that the precision of relative orientation is about the same in either case. Comparing the figures with the directly obtained precision values $\sigma_{x_c}, \sigma_{y_c}$ of the digital image correlation (of about 1.0 to 1.4 µm) it is evident, that additional sources of errors are acting. Obviously $\sigma_{\text{op}_y}$ is mainly affected by the total image deformations and instrumental errors of the analytical plotter which affect the image coordinate measurements. These additional errors obscure the higher precision of the actual $y$-parallax measurements.
5. Conclusions

This paper has shown that on-line high precision parallax measurement and point transfer by digital image correlation is operational in connection with a standard analytical plotter Planicomp C 100 equipped with CCD video cameras. The results demonstrate not only the high precision which is obtained by digital image correlation. They also show that the system is integrated into the conventional photogrammetric measuring process and can very well be operated economically.

With the present state of hardware and software a single point transfer takes about 2.2 sec for data recording and about 0.5 sec for each iteration of the correlation procedure. With good initial approximate values for shift (<20 μm) 4-5 iterations are sufficient for the correlation procedure. For a number of applications the time is brief enough for practical operations and for further testing of the method.

The operation time can be further reduced by 1) the reduction of the patch size of the grey value matrices (Förstner, 1984) and 2) the full utilization of the possibilities of data transfer and data decoding. Furtheron it might be envisaged in future to use special microprocessors for the computation of the image correlation.

Further investigations in photogrammetric application will be done in deformation measurement and point transfer for the aerial triangulation. For these purposes the correlation algorithm will be extended to multi image correlation.

REFERENCES


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