Presented Paper to Commission III, 15th ISPRS Congress, Rio de Janeiro, 1984

REALIZATION OF AUTOMATIC ERROR DETECTION IN THE BLOCK ADJUSTMENT PROGRAM PAT-M43 USING ROBUST ESTIMATORS

Hermann Klein and Wolfgang Förstner

Abstract: The detection of outliers can be automated using robust estimators. The principle is to interpret the residuals v_i of the observations after each iteration as errors in order to calculate new weights based on a weight function $p(v_i)$. The new weights $p_i = p(v_i)$ are then used in the following iteration step.

The paper reports on the realization of this error detection strategy in PAT-M43. Main topic is the extension of the method, especially the choice of a proper weight function, the iteration sequence and the stopping rule. The significant facilitation in handling the program is explained.

1. The original program:

The computer program PAT-M43 performs a blockadjustment by independent photogrammetric models. This approach implies a spatial similarity transformation for each model. The adjustment is based on a least squares solution. The nonlinear observational equations are linearized with respect to the orientation parameters. Because of computational economy the program iterates sequential horizontal and vertical adjustments, applying 4-parameter and 3-parameter transformations, respectively. For each iteration the partially reduced normal equations that contain only the unknown orientation parameters are formed directly from the model and control coordinates and are solved by a modified Cholesky method (Ackermann et. al. , 1970). An extension allows the combined adjustment of photogrammetric models with APR and/or statoscope data, including photogrammetric height measurements of shorelines of lakes (Ackermann et. al. 1972).

2. Manual data cleaning:

One of the main problems of practical blockadjustment is the detection and location of outliers. Dependent on the number and distribution of the observations, errors show up only partly in the residuals of the corresponding observations, the other parts falsify the absolute orientation of the photogrammetric models (Förstner, 1978). The mutual interference of outliers, especially of different size, is a further handicap. For that reason several adjustments for a step by step location and elimination of outliers in accordance with the size of the errors, and some further adjustments in order to avoid wrong decisions are necessary. Nevertheless the quality of manual data cleaning is sufficiently good and comparable with most of the more sophisticated procedures (Förstner, 1982), but in general it requires a great deal of time by fully qualified persons. Thus the main argument of the development of an automatic procedure has been: to shorten the processing time needed by persons in charge of blockadjustment.

3. From least squares to robust adjustment:

The above mentioned problems which arise in the adjustment of data with gross errors are not a specific attribute of the manual data cleaning procedure, but a bad point of the method of least squares. Applying a constant weight p = const for each observation the influence function (first derivative of the minimum function by the residual) shows, that the influence of a defective observation onto the result of the adjustment is directly proportional to the size of the error. Thus as a matter of fact the method of least squares is applicable for errorfree data only and unsuited for automatic error detection procedures.

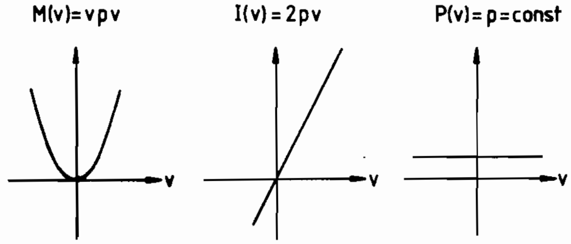

Fig. 1. Least squares: minimum function, influence function and weight function

Logically erroneous observations have to be handled with reduced weights and can not be treated with the same weights as errorfree data. All the observations must be introduced into the adjustment with weights chosen in correspondence with their errors. The problem of locating gross errors is therefore identical with the determination of proper weights for the observations.

An alternative to least squares is the minimum norm method (Huber, 1981). Thereby the weights of the observations are progressively determined in an iterative process. After each iteration step new weights for the observations are calculated as a function of the residuals with $P(v) = \frac{1}{|v|}$. The influence function shows, that after convergency of the procedure the influence of all the observations onto the result is equal. Observations with gross errors have the same influence onto the result as errorfree data. This is better than with least squares but still not sufficient.

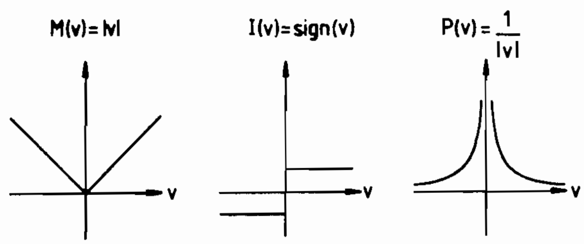

Fig. 2. Minimum Norm: minimum function, influence function and weight function

An adjustment procedure which uses weight functions for complete elimination of the influence of gross errors is the so-called method of robust estimators (robust against the influence of gross errors) (Krarup, 1980; Kubik, 1984). After convergency of the iterative process proper weights are determined for all observations and erroneous data will get weights approximately equal to zero and thus will have no influence at all onto the result of the adjustment. Their residuals will show the true errors. The method of robust estimators can be interpreted as an a posteriori estimation of the variances. Many simple weight functions can be found which meet the conditions of robust estimators, but most of them cover only a small range of gross errors and will fail with the variety of gross errors occuring in practical cases. The reason for the failure in these cases is the assumption of linearity by the robust estimators (Huber, 1981).

4. Weight function for PAT-M43:

Thus, a lot of research was necessary to find a weight function and to develop a procedure which covers the wide range of gross errors, their multiple combinations and the varying geometry of photogrammetric blocks (Werner, 1984). Because of their effectiveness the following hyperbolic weight function was chosen for the blockadjustment program PAT-M43:

$$P = P_i \cdot F(v_i, \sigma_{v_i}, Q)$$

$$= P_i \cdot \frac{1}{1+(\alpha_i \cdot |v_i|)^d} \qquad (1)$$

in which:

$$\alpha_i = \frac{1}{1.4 \cdot \hat{\sigma}_{v_i}} = \frac{\sqrt{P_i}}{1.4 \cdot \sqrt{r_i} \cdot \hat{\sigma}_0} \qquad (2)$$

$$d = 3.5 + \frac{82}{81 + Q^4} \qquad (3)$$

$$Q = \frac{\hat{\sigma}_0}{\sigma_{a\ priori}} \qquad (4)$$

v_i = residual of observation i

P_i = a priori weight of observation i

r_i = local redundancy of observation i

$\hat{\sigma}_{v_i}$ = estimated sigma of the residual v_i

$\hat{\sigma}_0$ = estimated sigma-naught

worth mentioning are two attributes of the weight function expanding the range of gross errors locatable with this function.

The first is the dependence on Q (see formula 3 and 4). At the start of the iteration process the value of Q is relatively big and it will become smaller with convergency. At the end of the procedure Q will reach approximately the value one. Thus the curve of the weight function is flat in the beginning and will become steeper and steeper with the disappearing influence of the gross errors and the final orientation of the models. This attribute of the weight function allows the correction of wrong decisions caused by false 0-approximations of the residuals at the beginning and makes it easier to distinguish between errorfree and erroneous observations at the end of the iteration process.

The second attribute is the dependence on the estimated standard deviation of the particular residual $\hat{\sigma}v_i$ (see formula 2). Even with the simplification of using the value one as local redundancy for all the observations this feature allows the determination of small gross errors in the critical range of localization.

Without any further modifications the localization of locatable gross errors up to $50 \cdot \sigma_0$ causes no problems, even with geometrically very weak configurations, as long as there are still errorfree redundant observations.

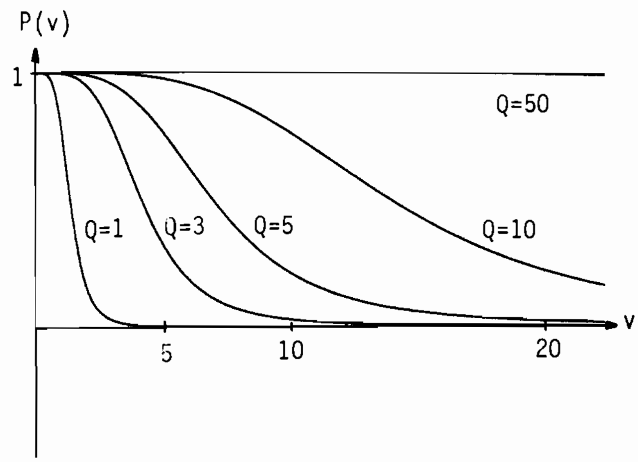

Fig. 3. PAT-M43: weight function

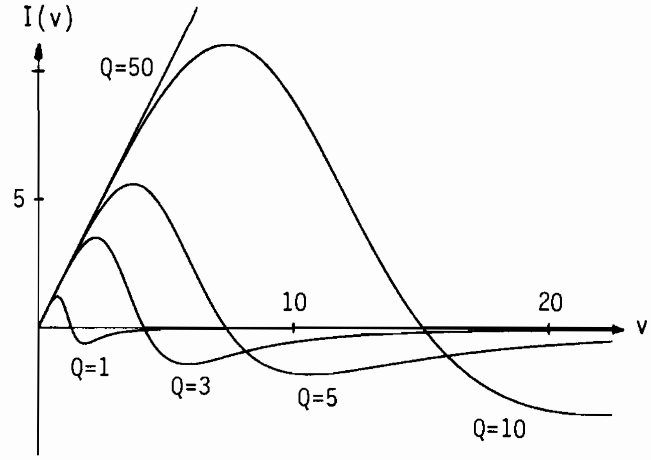

Fig. 4. PAT-M43: influence function

with $P_i=1; r_i=1; \sigma_{a\ priori}=1$ and Q=1;3;5;10;50

5. New structure of the program:

If no initial values of the orientation parameters are available the program begins with a least squares horizontal adjustment that does not require approximate values. The resulting transformed model coordinates enter into a vertical least squares adjustment using a shift in z only. Thus big gross errors in height do not disturb the orientation of the models too much.
After these first two iteration steps initial 0-approximations for the residuals are calculated, needed to start consecutive robust iteration steps. Robust estimators can relatively easy be realized using the least squares algorithm and modifying the weights after each iteration step by means of the weight function. Robust iteration steps are repeated until sufficient convergence is reached. The convergence is quite good but highly correlated to the number and the size of gross errors and to the geometric stability of the block configuration. Thus the number of iteration steps may differ from 6 to 20. If the change of $\hat{\sigma}_0^2$ between two iteration steps becomes less than $2 \cdot \hat{\sigma}(\hat{\sigma}_0^2)$ in planimetry or height after a corresponding iteration step the final elimination of erroneous observations is performed. All observations being used in that iteration step and getting $F(v_i, \hat{\sigma}_{v_i}, Q) < 0.01$ (see formula 1) will be marked as erroneous observations and will get an infinitely small weight. The others receive their original a priori weight. Some least squares iteration steps complete the procedure to reach the final result.

Treating errorfree data in a least squares adjustment the favourable sequence of iteration steps is a consecutive alteration between planimetry and height. Handling erroneous data the succession of iteration steps depends on the existing gross errors. That means the succession is data dependent and therefore must be directed by the program itself. The robust iteration steps always begin with horizontal adjustments in order to reduce the influence of the very big gross errors in planimetry because big gross errors in the planimetric coordinates would disturb the levelling of the models completely. Due to the same effect the first robust iteration step in height applies only a shift in z. The sequence of all further iteration steps is chosen properly in order to keep the reduced influence of gross errors in planimetry and height approximately on the same level because of the mutual interference of erroneous data.

6. Classification of gross errors:

Regarding the different effects of gross errors related to their size we can group them into 3 different classes:

1. small gross errors
2. medium-size gross errors
3. large gross errors

The classification bounds are not fixed, they depend on the geometry and may vary for different photogrammetric blocks.
All gross errors greater than $4 \cdot \sigma$ and less than $50 \cdot \sigma$ can be designated as small gross errors. They have no significant influence onto the orientation of the models and do not disturb the domain of linearity of the adjustment. Gross errors of the stochastical model and systematic errors are not taken into account but can be considered as small gross errors. Errors less than $4 \cdot \sigma$ are integrated with the random errors.
All errors between $50 \cdot \sigma$ and 2-3 base lengths belong to the medium-size gross errors. They have no big influence onto the geometry of the photogrammetric block and don't disturb the convergence of the adjustment but they are not within the range of the linearization and the solution may tend to a different 0-point. Errors bigger than 3 base lengths are named large gross errors. They change the geometry of the block severely and cause poor convergence or even divergence. Especially for blocks with bad geometry the adjustment must be stopped before reaching the point of convergence.

7. Location of small gross errors:

The location of small gross errors poses no problems for the robust adjustment with the chosen weight function. Even small gross errors at the limit of possible location are detected as long as the observations are sufficiently well distributed within the models.
Only for really bad distributions the consideration of the local redundancy (see formula 2) would improve the effectiveness of the procedure. The check for the inherent limit of localization can be performed only with artificial data. Example 1 shows that the introduced errors greater than the lower limit of 5σ are located without any wrong decision. This lower limit is even better than the theoretical expectation for the statistical test (R. Schroth, 1980).
Example 2 shows a practical photogrammetric block and is demonstrating the effectiveness of the robust estimators. At first data cleaning has been performed manually and the cleaned data have been submitted to the automatic procedure. Although the residuals after the manual procedure did not indicate remaining gross errors, the automatic procedure located further ones.

8. Modifications of the procedure with respect to medium-size and large gross errors:

Medium-size gross errors and all larger gross errors do not belong to any normal distribution of observations, they are independent from the a priori weights introduced into the adjustment. Thus as long as large gross errors have still an influence onto the adjustment all photogrammetric observations are treated with the starting weight 1, used as a priori weight in the weight function. This starting weight tends to the introduced specific a priori weight in dependency on the value of Q by a weight function. The same is true for all non-photogrammetric observations, but for them the starting weight 1/100 is used. The weight function is as follows:

Weight function for modified a priori weights:

$$P = SW + (P_i - SW) \cdot \frac{37}{36 + (Q-1)^2}$$

in which:

SW = starting weight
P_i = a priori weight
P = modified a priori weight

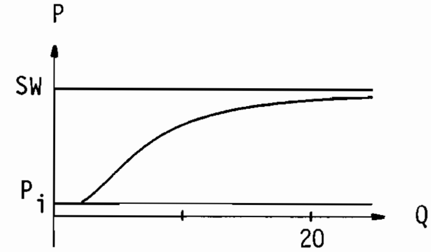

Fig. 5. Modification of a priori weights: with SW=1; P_i=0.1

The ratio of the two starting weights has the effect to reduce the influence of gross errors of photogrammetric observations always a little earlier than for control. This supports the location of gross errors at control points in case of weak control point distributions.

As long as there is no bad accumulation of medium-size gross errors in relation to the geometry their location is no problem. But there are two effects to be avoided:
The bigger the gross errors the more falsified are the 0-approximations of the residuals. Sometimes this results in so-called "swimming" models. By means of false 0-approximations the weights for all the observations of a model will be reduced too much in spite of a flat weight function and the model will not be able to get oriented. Nevertheless the calculated weights point to the biggest gross errors.

An other effect is, that after location of the medium-size gross errors the adjustment approaches a different 0-point and the location of the small gross errors will not be correct. The preelimination of large and medium-size gross errors will solve this problem. As soon as the value of F $(v_i, \hat{\sigma}_{v_i}, Q)$ reaches a certain lower limit the corresponding observation will get the minimum weight for elimination, all other observations receive their a priori weights to start a new robust adjustment. The lower limit of the weight function for preelimination starts with 10^{-18} and is increased for each iteration step by a factor 10 up to the value 10^{-9}. This modification results in a step by step elimination of all large errors down to gross errors of approximately $50 \cdot \sigma$. Thus "swimming" models will be reincluded into the block and linearity for the final elimination of small gross errors in provided.

Large gross errors may disturb the geometry of the block completely. Already the 0-approximations of the residuals after the starting least squares iteration are false to the extent that the point of convergence will not be reached. Therefore large gross errors have to be introduced with already reduced weights into the starting least squares iteration step. The problem can be solved by calculating a center point for each model and the distances to this point for all observations. The ratio of distance and mean distance is used in order to reduce weights by a weight function.

The coordinates of the center point are the arithmetic mean of the coordinates of observations, as long as there are more than 5 observations, otherwise the median is used. The same is true for the mean distance, but the median is used already for 20 and less observations. The calculations are done seperately for planimetry and height. The weight functions used are as follows:

planimetry: $$P = P_i \cdot \frac{256}{256 + R_i^8}$$

height: $$P = P_i \cdot \frac{81}{81 + R_i^4}$$

in which:

P_i = a priori weight of observation i
$R_i = D_i/D$
D_i = distance of observation i from center point
D = mean distance

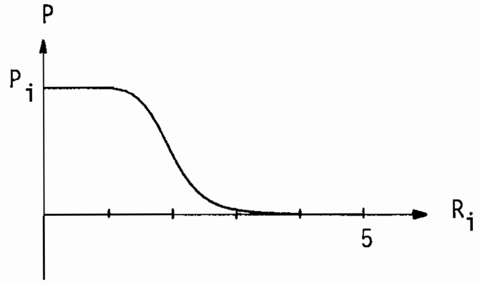

Fig. 6. Modification of weights for the starting least squares planimetric iteration step

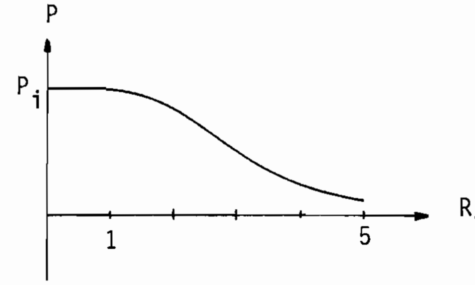

Fig. 7. Modification of weights for the starting least squares height iteration step

The effectiveness of the modifications related to medium size and large gross errors is shown in Example 3. With the relatively poor geometry the gross error of three base lengths at point 10201 would not be locatable in planimetry without the reduction of the weight in the first least squares iteration step.

9. Reinsertion of observations:

In two cases it is required to reinsert already eliminated observations. Due to falsified 0-approximations it may happen that an observation is wrongly eliminated. After orientation of the model the residuals of this observation will become small and a reinsertion is advisable.

Secondly the result of a least squares adjustment differs from the result of an adjustment with robust estimators in the range of 1-2σ. After the final elimination of the small gross errors at the end of the robust adjustment some iteration steps with least squares are performed and small gross errors just at the limit of localization will tend to move to the class of random errors in the least squares adjustment and thus should be reinserted.

Therefore the weight function (formula 1) is used in the final least squares iteration steps to check for reinsertion of eliminated observations.

During the whole procedure of adjustment, as soon as the value of $F(v_i, \hat{\sigma}_{v_i}, Q)$ becomes larger than the value 0.01, used for elimination, an already eliminated observation will be reinserted in order to stabilize the geometry of the block and to contribute to a final result of adjustment.

10. Conclusion:

The above described procedure of automatic gross error localization is a specific development for the blockadjustment by the method of independent models and is not transferable to other problems without modifications.

The procedure covers the full range of occuring gross errors from the small ones, just at the limit of localization, up to the big ones with several base lengths and shows the power of robust estimators.

When the worst comes to the worst the procedure results in the elimination of a complete model or in the elimination of observations up to the point where no redundancy is remaining in a model. Then the user has to analyse the observations of the specific model and to take a decision.

In most cases the result of the procedure will be only a proposal, although a very good one, and the person in charge of the project has to judge the proposal and to decide about the final corrections of the gross errors.

The program is in an operational stage and the automatic error detection procedure is easy to handle. No parameters with respect to the procedure have to be changed by the operator, except for the decision whether the adjustment shall be performed with or without automatic error detection.

References

Ackermann, Ebner/Klein (1970): Ein Programmpaket für die Aerotriangulation mit unabhängigen Modellen, Bildmessung und Luftbildwesen 4/1970.

Ackermann, Ebner/Klein (1972): Combined Block Adjustment of APR Data and Independent Photogrammetric Models. The canadian surveyor-Vol 26, pp. 384-396.

Förstner, W. (1978): Die Suche nach groben Fehlern in photogrammetrischen Lageblöcken DGK-C-240, München, 1978.

Förstner, W. (1982): Results of Test I on Gross Error Detection of ISP WG III/1 and OEEPE. Proceedings of ISPRS-Symposium (Commission III), Helsinki 1982.

Hampel, F.R.(1973): "Robust Estimation: A condensed Partial Survey". Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete, 1973.

Huber, P.J. (1981): "Robust Statistics", New York, 1981.

Krarup/Juhl/Kubik (1980): "Götterdämmerung over Least Squares Adjustment" IAP-Vol. 23-33, Hamburg 1980, pp. 369-378.

Kubik, K. (1984): Gross Data Errors and Robust Estimation. IAP-Vol. 24-III, Rio de Janeiro 1984.

Schroth, R. (1980): Innere Zuverlässigkeit von photogrammetrischen Blöcken. Schriftenreihe des Instituts für Photogrammetrie der Universität Stuttgart, Heft 7, pp. 41-58.

Werner, H. (1984): Automatic Gross Error Detection by Robust Estimators, IAP-Vol. 25-III, Rio de Janeiro 1984.

EXAMPLE 1:
****

EXTRACT FROM PRINTOUT
ARTIFICIAL BLOCK WITH 32 MODELS
4 STRIPS; ~ 6 POINTS PER MODEL
20% SIDELAP
SCALE= 1/10000
SIGMA= 10 MICRON
MODELS IN MICRON
CONTROL IN METER

SUPERPOSED GROSS ERRORS IN MICRON RESP. METER:

MODEL-NO.	POINT-NO.	DX	DY	DZ
101	20203	80.000	0.000	-80.000
102	10203	0.000	-100.000	100.000
107	30803	30.000	0.000	30.000
304	50503	0.000	-30.000	-30.000
402	80203	-70.000	70.000	100.000
404	80503	-40.000	0.000	-40.000
407	80703	0.000	50.000	50.000
407	70803	30.000	0.000	30.000
HC	10103	-1.000	0.000	
HC	50103	-0.300	0.000	
HC	50903	0.000	-0.500	
HC	90903	0.500	0.500	
VC	10103			-1.000
VC	50103			-0.300
VC	50903			0.500
VC	90903			0.500

END OF ERROR DETECTION IN ELEVATION
SIGMA REACHED = 8.6602

MODEL	101	POINT	10103	HV 1	VZ=	0.955	ELIMINATED IN HEIGHT
MODEL	401	POINT	80203	TP 2	VZ=	0.469	ELIMINATED IN HEIGHT
MODEL	102	POINT	10203	TP 2	VZ=	0.799	ELIMINATED IN HEIGHT
MODEL	102	POINT	20203	TP 2	VZ=	0.392	ELIMINATED IN HEIGHT
MODEL	207	POINT	30701	TP 4	VZ=	0.417	ELIMINATED IN HEIGHT
MODEL	208	POINT	50903	HV 2	VZ=	0.467	ELIMINATED IN HEIGHT
MODEL	308	POINT	50903	HV 2	VZ=	0.451	ELIMINATED IN HEIGHT
MODEL	408	POINT	90903	HV 1	VZ=	0.521	ELIMINATED IN HEIGHT

END OF ERROR DETECTION IN PLANIMETRY
SIGMA REACHED = 8.8794

MODEL	101	POINT	10103	HV 1	VXY=	1.052			ELIMINATED IN PLANIMETRY
MODEL	401	POINT	80203	TP 2	VXY=	0.898			ELIMINATED IN PLANIMETRY
MODEL	102	POINT	10203	TP 2	VXY=	0.900			ELIMINATED IN PLANIMETRY
MODEL	102	POINT	20203	TP 2	VXY=	0.686			ELIMINATED IN PLANIMETRY
MODEL	207	POINT	30701	TP 4			VZ=	0.421	RE-INSERTED IN HEIGHT
MODEL	208	POINT	50903	HV 2			VZ=	0.470	RE-INSERTED IN HEIGHT
MODEL	308	POINT	50903	HV 2			VZ=	0.454	RE-INSERTED IN HEIGHT
MODEL	408	POINT	90903	HV 1	VXY=	0.702			ELIMINATED IN PLANIMETRY

TRANSFORMED PHOTOGRAMMETRIC MODEL COORDINATES AND RESIDUALS
****
(IN UNITS OF THE TERRAIN SYSTEM)

MODEL NUMBER 101 SC= 99.99699

10100	-0.210	899.974	1500.073	PC 1				0	
10101	0.083	-0.092	-0.040	HV 1	-0.041	0.023	0.037	0	. . .
10103	0.083	-0.092	-0.040	HV 1 -> HV 0/ 0	-1.052*	0.029*	-0.953*	30	
10200	900.252	899.988	1500.079	PC 2	-0.037	-0.017	0.024	0	. . .
10201	900.042	-0.028	0.230	TP 2	0.022	0.026	0.009	0	. . .
10203	900.042	-0.028	0.230	TP 2 -> SP 1/ 1				0	
20101	0.044	899.990	0.038	SP 1				0	
20201	900.188	900.180	0.286	TP 2	0.051	-0.097	-0.109	0	. . .
20203	900.988	900.180	-0.514	TP 2 -> SP 1/ 1				0	
30101	-0.057	1800.031	-0.027	VE 2	0.040	-0.011	-0.022	0	. . .
30201	900.113	1799.923	0.076	TP 4	-0.071	0.058	0.061	0	. . .

MODEL NUMBER 401 SC= 100.00412

40100	-0.165	6300.204	1499.784	PC 1				0	
40200	899.850	6299.856	1499.855	PC 2	-0.034	0.144	0.091	0	. . .
70101	-0.137	5400.061	0.069	VE 2	-0.054	0.030	-0.005	0	. . .
70201	899.856	5400.138	-0.156	TP 4	0.056	0.021	-0.003	0	. . .
80101	-0.246	6299.994	-0.187	SP 1				0	
80201	899.826	6300.090	0.027	TP 2	-0.017	-0.054	-0.135	0	. . .
30203	899.826	6300.090	0.027	TP 2 -> SP 1/ 1	-0.735*	0.593*	0.730*	30	
90101	-0.029	7200.133	-0.054	HV 1	0.007	-0.042	0.019	0	. . .
90201	899.822	7200.103	-0.234	TP 2	0.008	0.045	0.033	0	. . .

MODEL NUMBER 102 SC= 100.00261

10200	900.178	899.954	1500.127	PC 2	0.037	0.017	-0.024	0	. . .
10201	900.087	0.024	0.246	TP 2	-0.022	-0.026	-0.008	0	. . .
10203	900.087	-0.976	1.246	TP 2 -> SP 1/ 1	-0.045*	0.948*	-1.017*	30	
10300	1800.161	900.038	1500.126	PC 2	-0.077	-0.093	-0.006	0	. . .
10301	1800.117	0.103	-0.022	TP 2	0.033	-0.006	0.079	0	. . .
20201	900.289	899.987	0.069	TP 2	-0.051	0.096	0.109	0	. . .
20203	900.289	899.987	0.069	TP 2 -> SP 1/ 1	0.699*	0.193*	-0.583*	30	
20301	1800.147	899.834	0.412	TP 2	0.007	0.028	-0.188	0	. . .
30201	900.028	1800.073	0.196	TP 4	0.013	-0.092	-0.059	0	. . .
30301	1800.119	1799.971	0.034	TP 4	0.019	0.000	0.098	0	. . .

MODEL NUMBER 408 SC= 99.99940

40800	6299.539	6299.904	1499.887	PC 2	0.142	0.147	-0.020	0	. . .
40900	7199.497	6299.810	1500.097	PC 1				0	
70801	6300.100	5399.757	-0.070	TP 4	-0.013	0.036	-0.058	0	. . .
70803	6300.100	5399.757	-0.070	TP 4	0.062	0.036	0.017	0	. . .
70901	7200.106	5399.878	0.023	VE 2	0.013	-0.065	0.030	0	. . .
80801	6300.189	6299.996	-0.238	TP 2	-0.068	-0.022	-0.019	0	. . .
80901	7200.172	6299.611	0.050	SP 1				0	
90501	6300.195	7199.889	-0.429	TP 2	0.027	0.008	0.021	0	. . .
90901	7200.136	7199.988	-0.071	HV 1	-0.022	0.007	0.029	0	. . .
90903	7200.136	7199.988	-0.071	HV 1 -> HV 0/ 0	0.473*	0.509*	0.537*	30	

EXAMPLE 2: EXTRACT FROM PRINTOUT:
****

PRACTICAL BLOCK WITH 32 MODELS
4 STRIPS; ~18 POINTS PER MODEL
SCALE= 1/28000; 20% SIDELAP
SIGMA PLANIMETRY = 5.6 MICRON
SIGMA HEIGHT = 9.3 MICRON
MODELS IN 1/100 MM
CONTROL IN METER

ADJUSTMENT WITH AUTOMATIC ERROR DETECTION AFTER MANUAL DATA CLEANING

ITERATION STEP 9.....VERTICAL ADJUSTMENT

ITERATION STEP FOR ERROR DETECTION

END OF ERROR DETECTION IN ELEVATION
SIGMA REACHED = 0.5511

VERTICAL CONTROL POINT			15401	HV 2			VZ=	0.554	ELIMINATED
VERTICAL CONTROL POINT			16202	HV 2			VZ=	0.405	ELIMINATED
VERTICAL CONTROL POINT			21101	HV 1			VZ=	0.544	ELIMINATED
VERTICAL CONTROL POINT			21201	HV 2			VZ=	1.494	ELIMINATED
VERTICAL CONTROL POINT			31201	HV 4			VZ=	0.435	ELIMINATED
VERTICAL CONTROL POINT			36201	HV 2			VZ=	0.921	ELIMINATED
MODEL	112111	POINT	6211	TP 2			VZ=	0.881	ELIMINATED IN HEIGHT
MODEL	212211	POINT	16111	TP 4			VZ=	0.852	ELIMINATED IN HEIGHT
MODEL	212211	POINT	26111	TP 4			VZ=	0.799	ELIMINATED IN HEIGHT
MODEL	212211	POINT	26112	TP 4			VZ=	1.008	ELIMINATED IN HEIGHT
MODEL	212211	POINT	26211	TP 4			VZ=	0.844	ELIMINATED IN HEIGHT
MODEL	212211	POINT	26212	TP 4			VZ=	1.074	ELIMINATED IN HEIGHT
MODEL	312311	POINT	36212	TP 4			VZ=	0.897	ELIMINATED IN HEIGHT
MODEL	211210	POINT	21001	HV 2			VZ=	0.863	ELIMINATED IN HEIGHT
MODEL	411410	POINT	41000	PC 2	VXY=	1.865			ELIMINATED IN PLANIMETRY
MODEL	411410	POINT	46101	HO 2			VZ=	0.945	ELIMINATED IN HEIGHT
MODEL	310309	POINT	26011	TP 4			VZ=	0.850	ELIMINATED IN HEIGHT
MODEL	410409	POINT	30802	HV 4			VZ=	0.882	ELIMINATED IN HEIGHT
MODEL	109108	POINT	10800	PC 2	VXY=	1.709			ELIMINATED IN PLANIMETRY
MODEL	109108	POINT	10900	PC 2	VXY=	1.658			ELIMINATED IN PLANIMETRY
MODEL	207206	POINT	20612	TP 2			VZ=	0.786	ELIMINATED IN HEIGHT
MODEL	306305	POINT	35611	TP 4			VZ=	0.905	ELIMINATED IN HEIGHT

ITERATION STEP 10.....HORIZONTAL ADJUSTMENT

ITERATION STEP FOR ERROR DETECTION

END OF ERROR DETECTION IN PLANIMETRY
SIGMA REACHED = 0.3618

HORIZONTAL CONTROL POINT			46101	HO 2	VXY=	0.911			ELIMINATED
VERTICAL CONTROL POINT			31201	HV 4			VZ=	0.446	RE-INSERTED
MODEL	213212	POINT	16311	TP 2	VXY=	1.511			ELIMINATED IN PLANIMETRY
MODEL	213212	POINT	16312	TP 2	VXY=	1.648			ELIMINATED IN PLANIMETRY
MODEL	112111	POINT	6211	TP 2			VZ=	0.895	RE-INSERTED IN HEIGHT
MODEL	112111	POINT	16101	HV 2	VXY=	0.715			ELIMINATED IN PLANIMETRY
MODEL	212211	POINT	16111	TP 4			VZ=	0.888	RE-INSERTED IN HEIGHT
MODEL	212211	POINT	16201	HV 3	VXY=	0.793			ELIMINATED IN PLANIMETRY
MODEL	212211	POINT	16211	TP 4	VXY=	0.803			ELIMINATED IN PLANIMETRY
MODEL	212211	POINT	16212	TP 4	VXY=	0.808			ELIMINATED IN PLANIMETRY
MODEL	212211	POINT	26111	TP 4			VZ=	0.942	RE-INSERTED IN HEIGHT
MODEL	212211	POINT	26112	TP 4			VZ=	1.008	RE-INSERTED IN HEIGHT
MODEL	212211	POINT	26211	TP 4			VZ=	0.820	RE-INSERTED IN HEIGHT
MODEL	212211	POINT	26212	TP 4			VZ=	0.962	RE-INSERTED IN HEIGHT
MODEL	212211	POINT	26212	TP 4	VXY=	0.760			ELIMINATED IN PLANIMETRY
MODEL	312311	POINT	36212	TP 4			VZ=	0.899	RE-INSERTED IN HEIGHT
MODEL	211210	POINT	21001	HV 2			VZ=	0.665	RE-INSERTED IN HEIGHT
MODEL	411410	POINT	46101	HO 2			VZ=	0.955	RE-INSERTED IN HEIGHT
MODEL	310309	POINT	26011	TP 4			VZ=	0.663	RE-INSERTED IN HEIGHT
MODEL	310309	POINT	26011	TP 4	VXY=	0.984			ELIMINATED IN PLANIMETRY
MODEL	310309	POINT	26012	TP 4	VXY=	0.862			ELIMINATED IN PLANIMETRY
MODEL	410409	POINT	30802	HV 4			VZ=	0.885	RE-INSERTED IN HEIGHT
MODEL	308307	POINT	25711	TP 4	VXY=	0.842			ELIMINATED IN PLANIMETRY
MODEL	207206	POINT	20612	TP 2			VZ=	0.873	RE-INSERTED IN HEIGHT
MODEL	207206	POINT	25711	TP 4	VXY=	0.922			ELIMINATED IN PLANIMETRY
MODEL	207206	POINT	25712	TP 3	VXY=	0.990			ELIMINATED IN PLANIMETRY
MODEL	306305	POINT	35611	TP 4			VZ=	0.908	RE-INSERTED IN HEIGHT

ITERATION STEP 11.....VERTICAL ADJUSTMENT

HORIZONTAL CONTROL POINT			46101	HO 2	VXY=	0.933			RE-INSERTED
VERTICAL CONTROL POINT			21101	HV 1			VZ=	0.348	RE-INSERTED
MODEL	112111	POINT	16101	HV 2	VXY=	0.719			RE-INSERTED IN PLANIMETRY
MODEL	212211	POINT	16201	HV 3	VXY=	0.894			RE-INSERTED IN PLANIMETRY
MODEL	212211	POINT	16211	TP 4	VXY=	0.736			RE-INSERTED IN PLANIMETRY
MODEL	212211	POINT	16212	TP 4	VXY=	0.809			RE-INSERTED IN PLANIMETRY
MODEL	212211	POINT	26212	TP 4	VXY=	0.762			RE-INSERTED IN PLANIMETRY
MODEL	308307	POINT	25711	TP 4	VXY=	0.845			RE-INSERTED IN PLANIMETRY

ITERATION STEP 12.....HORIZONTAL ADJUSTMENT

VERTICAL CONTROL POINT			15401	HV 2			VZ=	0.521	RE-INSERTED
MODEL	207206	POINT	25711	TP 4	VXY=	0.556			RE-INSERTED IN PLANIMETRY
MODEL	207206	POINT	25712	TP 3	VXY=	0.611			RE-INSERTED IN PLANIMETRY
MODEL	310309	POINT	26011	TP 4	VXY=	0.865			RE-INSERTED IN PLANIMETRY
MODEL	310309	POINT	26012	TP 4	VXY=	0.672			RE-INSERTED IN PLANIMETRY

TRANSFORMED PHOTOGRAMMETRIC MODEL COORDINATES AND RESIDUALS
****
(IN UNITS OF THE TERRAIN SYSTEM)

MODEL NUMBER 213212 SC= 3.53452

16201	19985.406	41662.499	657.137	HV 3		-0.001	-0.076	0.295	0	. . .
16202	20170.998	43717.534	658.197	HV 2	-> HO 2/ 2	-0.205	0.001	-0.148	0	. . .
16211	20987.462	41518.197	650.274	TP 4		0.005	-0.157	-0.087	0	. . .
16212	20957.464	41519.737	650.688	TP 4		0.127	0.037	-0.156	0	. . .
16311	18519.591	41480.567	629.558	TP 2	-> TP 1/ 2	-1.079*	0.266*	0.018	20	.
16312	18489.311	41482.252	632.152	TP 2	-> TP 1/ 2	-1.212*	0.253*	-0.152	20	.
21200	21100.479	43821.375	4981.772	PC 2		0.309	0.113	-0.002	0	. . .
21201	19044.651	46404.112	633.600	HV 2	-> HO 2/ 2	-0.016	-0.013	-0.112	0	. . .
21211	21052.156	43987.740	624.532	TP 2		0.004	0.083	0.194	0	. . .
21212	21022.216	43989.129	625.909	TP 2		0.015	0.049	0.060	0	. . .
21300	18512.329	43865.191	4962.960	PC 1					0	
21311	18457.043	43995.038	650.887	SP 1					0	
21312	18427.154	44000.347	650.543	SP 1					0	
26211	21062.232	46492.155	600.110	TP 4		-0.156	0.014	0.099	0	. . .
26212	21032.230	46492.295	601.323	TP 4		-0.100	-0.025	0.095	0	. . .
26311	18507.653	46547.861	621.303	TP 2		0.111	0.055	-0.100	0	. . .
26312	18477.646	46549.505	621.239	TP 2		0.217	0.033	-0.003	0	. . .

MODEL NUMBER 411410 SC= 3.46565

31101	23425.562	50895.996	597.317	HO 4		-0.079	-0.024	0.023	0	. . .
36001	25689.497	52144.309	606.611	HV 4		0.065	0.123	-0.083	0	. . .
36011	26016.967	51446.019	602.409	TP 4		0.178	0.220	0.083	0	. . .
36012	25986.920	51446.510	602.578	TP 4		0.110	0.054	-0.072	0	. . .
36101	23350.184	53669.629	570.693	HO 2		-0.066	-0.102	-0.363	0	. . .
36111	23503.741	51336.075	592.963	TP 4		-0.170	0.029	0.320	0	. . .
36112	23473.658	51337.630	593.553	TP 4		-0.125	0.057	0.245	0	. . .
36113	23570.230	51613.751	598.551	TP 4		-0.218	-0.338	-0.361	0	. . .
36114	23559.386	51641.645	598.430	TP 4		-0.158	-0.013	0.045	0	. . .
41000	26066.920	53965.705	5040.819	PC 2	-> PC 1/ 2	-0.097*	1.903*	0.025	20	.
41001	24822.185	56248.214	545.618	HV 1		0.135	-0.080	-0.095	0	. . .
41011	26011.589	54037.733	591.393	TP 2		0.024	0.234	0.052	0	. . .
41012	25981.642	54033.412	590.659	TP 2		0.037	0.187	0.075	0	. . .
41100	23577.763	53996.041	5039.784	PC 2		-0.091	-0.046	0.051	0	. . .
41111	23505.941	54014.470	570.090	TP 2		0.043	-0.029	-0.066	0	. . .
41112	23477.258	54023.282	570.016	TP 2		-0.020	-0.081	-0.070	0	. . .
46011	26045.236	56582.862	547.764	TP 2		0.010	0.024	-0.116	0	. . .
46012	26043.799	56612.918	547.338	TP 2		-0.001	-0.062	0.080	0	. . .
46101	23057.228	56908.700	608.497	HO 2		0.119	-0.404	0.411	0	. 1 .
46131	23790.150	56277.211	597.472	TP 2		0.021	0.112	-0.128	0	. . .
46132	23709.480	56328.343	601.201	TP 2		0.097	0.093	-0.057	0	. . .

MODEL NUMBER 109108 SC= 3.51248

5701	31761.875	36832.748	583.670	HV 2		0.003	0.252	0.131	0	. . .
5702	31224.845	38422.509	581.233	HV 2		-0.038	0.031	-0.119	0	. . .
5801	30639.872	36334.185	587.957	HV 2		0.065	0.080	0.179	0	. . .
5811	30992.108	36477.780	579.263	TP 2		0.043	-0.024	-0.126	0	. . .
5812	30962.101	36479.483	580.450	TP 2		0.069	-0.103	-0.112	0	. . .
5911	28582.380	36477.794	588.840	TP 2		-0.138	-0.235	-0.270	0	. . .
5912	28553.043	36480.071	589.198	TP 2		-0.067	-0.136	0.093	0	. . .
10800	31120.157	38970.670	4989.630	PC 2	-> PC 1/ 2	0.255*	2.045*	-0.095	20	.
10801	29397.694	39347.598	592.489	HV 1		0.034	0.096	0.248	0	. . .
10811	30977.247	39028.096	569.533	TP 2		0.049	0.169	0.105	0	. . .
10900	28607.830	38919.488	4986.889	PC 2	-> PC 1/ 2	2.613*	0.722*	0.011	20	.
10901	28475.439	40359.040	609.879	HV 2		-0.095	0.249	-0.328	0	. . .
10911	28831.187	38879.800	595.718	TP 2		-0.095	0.024	0.261	0	. . .
15801	29874.812	41539.893	589.158	HV 2		0.096	-0.122	0.095	0	. . .
15811	31031.536	41533.327	561.291	TP 4		-0.053	-0.049	-0.363	0	. . .
15812	31001.966	41537.827	561.320	TP 4		-0.040	-0.294	0.229	0	. . .
15911	28492.602	41207.073	566.629	TP 4		0.106	-0.057	0.010	0	. . .
15912	28495.158	41236.860	567.405	TP 4		0.061	0.079	0.048	0	. . .

VERTICAL CONTROL POINTS

5701	583.800	HV 2		0.001	2	.
5702	581.100	HV 2		0.065	2	.
5801	588.200	HV 2		-0.064	2	.
5901	602.800	HV 1		0.062	2	.
10401	548.900	HV 1		0.023	2	.
10501	592.800	HV 1		-0.062	2	.
10901	609.600	HV 2		-0.048	2	.
10902	583.400	HV 2		-0.063	2	.
15703	587.400	HV 1		-0.015	2	.
15801	589.300	HV 2		-0.046	2	.
15901	578.600	HV 4		-0.055	2	.
16102	623.900	HV 1		0.081	2	.
16201	657.600	HV 3		-0.118	2	.
16202	658.600	HV 2	-> HO 2/ 2	-0.551*	12	
20501	572.200	HV 1		0.087	2	.
20901	592.800	HV 2		0.154	2	.
21001	588.900	HV 2		-0.008	2	.
21101	629.800	HV 1		-0.026	2	.
21201	635.100	HV 2	-> HO 2/ 2	-1.612*	12	
25501	553.900	HV 2		-0.093	2	.
25801	609.400	HV 4		-0.051	2	.
35901	587.700	HV 2		0.016	2	.
36001	606.600	HV 4		-0.072	2	.
36201	577.300	HV 2	-> HO 2/ 2	-1.034*	12	
40501	541.000	HV 1		-0.028	2	.
40502	565.900	HV 2		-0.038	2	.
40601	572.200	HV 2		0.044	2	.

EXAMPLE 3: EXTRACT FROM PRINTOUT:
****

ARTIFICIAL BLOCK WITH 32 MODELS
4 STRIPS; 6 POINTS PER MODEL
SCALE= 1/10000; 20% SIDELAP
SIGMA= 10 MICRON
MODELS IN MICRON
CONTROL IN METER

SUPERPOSED GROSS ERRORS IN BASELENGTH:

MODEL-NO.	POINT-NO.	DXY	DZ
101	10201	3	3
406	90701	3	3
HVC	10901	3	3

ITERATION STEP 5.....HORIZONTAL ADJUSTMENT

ITERATION STEP FOR ERROR DETECTION

HORIZONTAL CONTROL POINT			10901	HV 1	VXY=	2699.649	ELIMINATED
MODEL	101	POINT	10201	TP 2	VXY=	2704.637	ELIMINATED IN PLANIMETRY
MODEL	406	POINT	90701	TP 2	VXY=	2702.264	ELIMINATED IN PLANIMETRY

ITERATION STEP 5.....VERTICAL ADJUSTMENT

ITERATION STEP FOR ERROR DETECTION

VERTICAL CONTROL POINT			10901	HV 1	VZ=	2699.824	ELIMINATED
MODEL	101	POINT	10201	TP 2	VZ=	2695.110	ELIMINATED IN HEIGHT
MODEL	406	POINT	90701	TP 2	VZ=	2701.269	ELIMINATED IN HEIGHT

ITERATION STEP 10.....VERTICAL ADJUSTMENT

ITERATION STEP FOR ERROR DETECTION

END OF ERROR DETECTION IN ELEVATION
SIGMA REACHED = 9.3376

MODEL	101	POINT	30201	TP 4	VZ=	0.537	ELIMINATED IN HEIGHT
MODEL	102	POINT	10200	PC 2	VZ=	0.632	ELIMINATED IN HEIGHT
MODEL	102	POINT	20201	TP 2	VZ=	0.806	ELIMINATED IN HEIGHT

ITERATION STEP 12.....VERTICAL ADJUSTMENT

MODEL	101	POINT	30201	TP 4	VZ=	0.346	RE-INSERTED IN HEIGHT
MODEL	102	POINT	10200	PC 2	VZ=	0.405	RE-INSERTED IN HEIGHT
MODEL	102	POINT	20201	TP 2	VZ=	0.286	RE-INSERTED IN HEIGHT

TRANSFORMED PHOTOGRAMMETRIC MODEL COORDINATES AND RESIDUALS
****
(IN UNITS OF THE TERRAIN SYSTEM)

MODEL NUMBER 101 SC= 100.00530

10100	0.251	899.692	1499.976	PC 1				0	
10101	-0.012	-0.356	-0.031	HV 1	0.036	0.010	0.018	0	. . .
10200	900.763	900.162	1499.852	PC 2	-0.083	0.042	0.035	0	. . .
10201	2809.769	1909.167	2699.508	TP 2 -> SP 1/ 1	-1909.911*	-1909.243*	-2699.317*	30	
20101	0.215	900.059	0.054	SP 1				0	
20201	900.054	899.941	-0.126	TP 2	-0.010	-0.008	-0.058	0	. . .
30101	0.356	1800.024	-0.062	VE 2	-0.043	0.051	0.016	0	. . .
30201	900.113	1800.225	-0.176	TP 4	0.023	-0.054	0.019	0	. . .

MODEL NUMBER 406 SC= 100.00953

40600	4500.055	6299.907	1500.089	PC 2	-0.025	0.159	0.061	0	. . .
40700	5399.963	6300.152	1500.020	PC 2	0.051	0.006	0.096	0	. . .
70601	4500.007	5400.016	0.144	TP 4	-0.043	0.059	-0.059	0	. . .
70701	5399.796	5400.076	0.074	TP 4	0.091	-0.058	-0.006	0	. . .
80601	4500.043	6299.955	0.223	TP 2	-0.003	0.014	-0.030	0	. . .
80701	5399.300	6299.942	0.239	TP 2	-0.049	0.025	-0.067	0	. . .
90601	4499.923	7200.099	0.127	TP 2	0.009	-0.040	0.005	0	. . .
90701	7309.120	9108.984	2699.622	TP 2 -> SP 1/ 1	-1909.235*	-1908.934*	-2699.713*	30	

CONTROL POINT COORDINATES AND RESIDUALS
****
(IN UNITS OF THE TERRAIN SYSTEM)

HORIZONTAL CONTROL POINTS

10101	0.033	-0.045	HV 1	-0.009	-0.003	2	. .
10501	3600.080	-0.022	HV 2	-0.007	0.026	2	. .
10901	9109.957	1909.005	HV 1 -> SP 1/ 1	-1910.117*	-1908.763*	22	
50101	-0.037	3600.061	HV 2	-0.009	-0.031	2	. .
50901	7200.012	3600.000	HV 2	0.016	0.002	2	. .
90101	-0.091	7199.952	HV 1	0.012	0.005	2	. .
90501	3599.979	7200.067	HV 2	-0.001	0.007	2	. .
90901	7200.001	7199.981	HV 1	-0.005	-0.007	2	. .

VERTICAL CONTROL POINTS

10101	-0.009	HV 1	-0.004	2	.
10501	0.027	HV 2	0.004	2	.
10901	2700.038	HV 1 -> SP 1/ 1	-2700.158*	12	
30101	-0.054	VE 2	0.009	2	.
30501	0.024	VE 4	-0.001	2	.
30901	0.003	VE 2	-0.007	2	.
50101	-0.018	HV 2	0.003	2	.
50501	-0.072	VE 4	-0.012	2	.
50901	0.065	HV 2	0.003	2	.
70101	0.032	VE 2	-0.010	2	.
70501	0.000	VE 4	0.007	2	.