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TWO-DIMENSIONAL FINITE IMPULSE RESPONSE (FIR) LINEAR SYSTEMS IN DIGITAL PHOTOGRAMMETRY

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SUMMARY

Recent developments in photogrammetry and remote sensing in doing image transformations and enhancements pure digitally require for theoretical secured operators than working with empirical ones. On the one hand concepts are needed to classify the underlying methods from a mathematical point of view, on the other hand these concepts should easily be realized for applications.

For these reasons the paper will give some insights into digital two-dimensional linear system theory as well as acts as intermediary for applications of these systems in photogrammetry. Working with digital images however ask for simple operators, easy to design and also to implement, what can be done quite well by finite impulse response (FIR) linear time-invariant systems closer classified in this paper.

I. INTRODUCTION

Interest in digital image processing dates back to the 1920's, when the first digitized images where transmitted between London and New York by submarine cable. From the beginning there was request on improvement of processing methods, but could not done efficiently during the next 35 years. The advent of developments in large-scale computing and the US space program started 1964 the break-through in modern digital image processing techniques, when pictures of the moon transmitted by Ranger 7 were processed by a computer at the Jet Propulsion Laboratory (JPL), Pasadena, California. Although at this time closed theoretical foundations were missing, the techniques applied served as a basis for later on improved methods used in the enhancement and restoration of images resulting from similar space programs.

Just as the digital signal processing discipline has been grown, whose development is strongly associated with the development of large scale integrated circuits, the ability for classification of digital image processing techniques were given. Nowadays, there exist closed theoretical concepts in digital signal processing, which can be applied to digital image processing not only in disciplines such as archeology, astronomy, biology, industrial applications and physics, but in digital image transformations for photogrammetric purposes, too. Here we have to eliminate some noise of scanned image rows or to interpolate 'white' pixels or whole 'white' rows to form a digital image data base for further processing, to name only two common problems arising during the data acquisition process (P. Nowak, 1978). The image might also be blurred by any motion, what means, the motion of the sensor has to be cancelled or the sensor was defocused during the exposure resulting in an unsharpened picture what has to be sharpened.

All these operations ask for computer-aided image processing techniques to improve the image quality; there are also possibilities to derive photogrammetric products without using any analytical plotter if one is thinking on preliminary situation maps for cartographic purposes. Therefore the aim of this paper will be to show up the performance of digital two-dimensional linear time-invariant systems in solving all the problems above. These systems are bounded only on enhancement and restoration of the digital image without totally change of the image geometry as it is the case in image rectifications. Also image correlation techniques will be excluded from linear time-invariant system (LTI system) theory in this paper at this time.

II. LINEAR TIME - INVARIANT SYSTEM THEORY

Let be given the digital image as spatially sampled light-intensity function denoted by $x(m,n)\ \forall\, m,n \in \mathbb{Z}\ (\mathbb{Z}_0^+)$. The array element $x(m,n)$ for fixed m,n is commonly called 'picture element' or shortly 'pixel' or 'pel' and indicates the gray-level quantization of the light intensity function. The image enhancement or restoration can be written as

$$y(m,n) = \Phi\left[x(m,n)\right] \tag{1}$$

whereby Φ characterises the desired image processing; it is proposed to be linear and time-invariant what means

$$\Phi\left[\sum_{i=-\infty}^{\infty} a_i x_i(m,n)\right] = \sum_{i=-\infty}^{\infty} a_i \Phi\left[x_i(m,n)\right] \tag{2a}$$

$$y(m-k,n-l) = \Phi\left[x(m-k,n-l)\right] \tag{2b}$$

For example, if the gray-level has to be raised or reduced, the operation of the LTI-system can be written as

$$y(m,n) = x(m,n) \pm b \qquad , \quad b \in \mathbb{Z} \tag{3}$$

and will not be considered furthermore because it is the trivial case for LTI systems.

The important class of LTI systems will be described not only in the time domain but also in the frequency domain. Let be the input of the LTI system the unit sample $d(m,n)$ with response of the system

$$h(m,n) = \Phi\left[d(m,n)\right] \tag{4}$$

which is called impulse response or point spread function. The z-transform of $h(m,n)$ will be (V.Capellini et al, 1978)

$$H(z_1,z_2) = \sum_{k=-\infty}^{\infty}\ \sum_{l=-\infty}^{\infty} h(k,l) z_1^{-k} z_2^{-l} \tag{5}$$

and delivers for stable LTI systems (D. Fritsch, 1982) with $z_1=e^{j\omega_1}$ and $z_2=e^{j\omega_2}$ the continuous in general complex frequency response

$$H(e^{j\omega_1},e^{j\omega_2}) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} h(k,l)e^{-j\omega_1}e^{-j\omega_2} \qquad (6)$$

that means $H(e^{j\omega_1},e^{j\omega_2})$ is the Fourier transform of the impulse response $h(m,n)$ with frequency variables $\omega_1,\omega_2 \in [0,2\pi]$.

In general the output $y(m,n)$ and the input $x(m,n)$ are functionally related by the linear inhomogenious difference equation (S.K. Mitra / M.P. Ekstrom, 1978)

$$\sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} a(k,l)x(m-k,n-l) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} b(i,j)y(m-i,n-j) \qquad (7)$$

Taking the z-transform of both sides delivers

$$\begin{aligned} A(z_1,z_2)X(z_1,z_2) &= B(z_1,z_2)Y(z_1,z_2) \\ \Longrightarrow \quad Y(z_1,z_2) &= H(z_1,z_2)X(z_1,z_2) \end{aligned} \qquad (8)$$

with $H(z_1,z_2):=A(z_1,z_2)/B(z_1,z_2)$. In dealing with finite impulse response LTI systems let $b(0,0):=1$ and $b(i,j):=0 \ \forall i,j \in \mathbb{Z}\setminus\mathbb{Z}_o$ as well as the summation indices be finite so that (7) can be written as

$$\begin{aligned} y(m,n) &= \sum_{h=0}^{K-1} \sum_{l=0}^{L-1} h(k,l)x(m-k,n-l) \\ &= h(m,n) * x(m,n) \end{aligned} \qquad (9)$$

with $h(k,l):=a(k,l)$. Under the assumptions above the z-transform of $b(i,j)$ will be constant with $B(z_1,z_2)=1 \ \forall \ z_1,z_2$ and $H(z_1,z_2)=A(z_1,z_2)$; the convolution (9) in the time domain results into multiplication in the z-domain or frequency domain depending on the transform variables.

III. FREQUENCY RESPONSES FOR IMAGE ENHANCEMENT AND IMAGE RESTORATION

All the image processing techniques by means of LTI systems have to be considered in the frequency domain, where different image models are supposed to be valid.

(i) Let us at first introduce the illumination-reflectance model

$$x(m,n) = i(m,n)\ r(m,n) \qquad (10)$$

with $0<i(m,n)<\infty$ as illumination array and $0<r(m,n)<1$ the reflectance array, respectively. The nature of $i(m,n)$ is determined by the light source, while $r(m,n)$ is given by the characteristics of the scene objects. Practically the values of $i(m,n)$ are finite, so that $x(m,n) \in [G_{min}, G_{max}]$ or $x(m,n) \in [0,G]$ with $x(m,n)=0$ is considered black and $x(m,n) = G$ is considered white on the scale. The illumination component varies slowly, that means it can be represented by low frequencies, whereby reflectance tends to vary abruptly particularly at dissimilar scene objects and is therefore represented by the higher frequencies in the frequency domain.

Let (10) be written into

$$v(m,n) = \ln[i(m,n)] + \ln[r(m,n)] \tag{11}$$

and taking the z-transform of (11)

$$V(z_1,z_2) = I(z_1,z_2) + R(z_1,z_2) \tag{12}$$

any enhancement process can be written as

$$H(z_1,z_2)V(z_1,z_2) = H(z_1,z_2)I(z_1,z_2)+H(z_1,z_2)R(z_1,z_2) \tag{13}$$

or in frequency variables

$$U(e^{j\omega_1},e^{j\omega_2})=H(e^{j\omega_1},e^{j\omega_2})I(e^{j\omega_1},e^{j\omega_2})+H(e^{j\omega_1},e^{j\omega_2})R(e^{j\omega_1},e^{j\omega_2}) \tag{14}$$

with $U(e^{j\omega_1},e^{j\omega_2}) := H(e^{j\omega_1},e^{j\omega_2})\ V(e^{j\omega_1},e^{j\omega_2})$ as Fourier transform of the enhanced image and $H(e^{j\omega_1},e^{j\omega_2})$ as amplifier function depending on the frequencies to be raised or reduced; this process is also called 'homomorphic filtering' (T.G. Stockham, 1972). The frequency response of an amplifier will look like Fig. 1. The complete specification will be obtained by rotating the cross section 2π about the vertical axis and will decrease low frequencies and amplify the high frequencies to provide for simultaneous dynamic range compression and contrast enhancement.

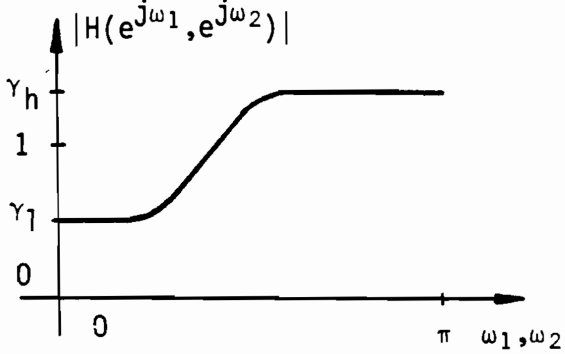

Fig. 1: Cross section of a circular symmetry amplifier frequency response function.

(ii) Secondly let us consider the signal-noise model

$$x(m,n) = y(m,n) + r(m,n) \tag{15}$$

where the image $x(m,n)$ is the sum of the signal $y(m,n)$ one is searching for and the noise component $r(m,n)$ to be removed during the image processing by means of an optimal filter function. The optimal filter will be derived by the minimum mean square error principle (MMSE principle) introduced by N. Wiener (1949)

$$\sigma^2 = E[\{y(m,n) - \sum_{k=0}^{K-1}\sum_{l=0}^{L-1} h(k,l)x(m-k,n-l)\}^2] = \min \tag{16}$$

with E as expectation on the stochastic process y(m,n) with its estimate $\hat{y}(m,n)$ being presented as convolution sum between the impulse response h(k,l) of the Wiener filter and the image x(m,n). In using orthogonality relations for (16) it can be represented as

$$E\left[\left\{y(m,n) - \sum_{k=0}^{K-1}\sum_{l=0}^{L-1} h(k,l)x(m-k,n-l)\right\}x(m-\kappa,n-\lambda)\right] = 0 \quad (17)$$

$$\kappa,\lambda \in \mathbb{Z}(\mathbb{Z}_o^+)$$

so that by taking the expectation on both terms the discretized Wiener-Hopf integral equation results

$$R_{yx}(\kappa,\lambda) = \sum_{h=0}^{K-1}\sum_{l=0}^{L-1} h(k,l)R_{xx}(\kappa-k,\lambda-l)$$

$$= h(\kappa,\lambda) * R_{xx}(\kappa,\lambda) \quad (18)$$

with cross-correlation array R_{yx} and auto-correlation array R_{xx} depending on the lags κ and λ. By means of the z-transform (18) delivers

$$H(z_1,z_2) = \frac{S_{yx}(z_1,z_2)}{S_{xx}(z_1,z_2)} \quad (19)$$

and with $E\,[y(m,n)r(m-\kappa,n-\lambda)] = 0$ the frequency response of the Wiener filter can be given

$$H(e^{j\omega_1},e^{j\omega_2}) = \frac{S_{yy}(e^{j\omega_1},e^{j\omega_2})}{S_{yy}(e^{j\omega_1},e^{j\omega_2})+S_{rr}(e^{j\omega_1},e^{j\omega_2})} \quad (20)$$

depending on the power spectra S_{yy} and S_{rr}. The frequency response for homogenious and isotropic stochastic processes will look like Fig.2 (D. Fritsch, 1982); the complete specification results by rotating 2π about the vertical axis and passes lower frequencies more or less unchanged, whereby the higher frequencies have continuously been attenuated to remove white or colored noise within the image. Also blurred digital images might be restored by means of Wiener filtering with similar frequency responses.

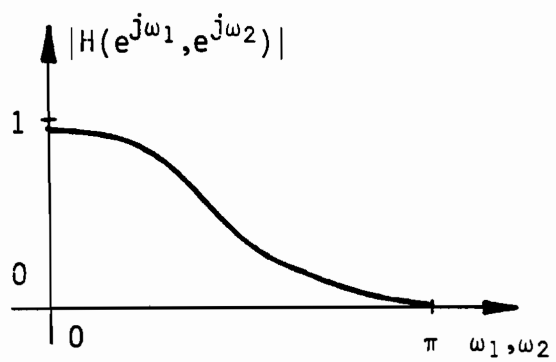

Fig.2: Cross section of circular symmetry Wiener filter frequency response function.

(iii) As next application image sharpening by highpass filtering and gradient techniques, respectively, will be considered. Let the image $x(m,n)$ be composed by the lower frequency array $y(m,n)$ and the higher frequency array $z(m,n)$.

$$x(m,n) = y(m,n) + z(m,n) \tag{21}$$

Taking the z transform of (21) delivers

$$X(z_1,z_2) = Y(z_1,z_2) + Z(z_1,z_2) \tag{22}$$

so that by substitution of $z_1=e^{j\omega_1}$ and $z_2=e^{j\omega_2}$ the Fourier transforms are given

$$X(e^{j\omega_1},e^{j\omega_2}) = Y(e^{j\omega_1},e^{j\omega_2}) + Z(e^{j\omega_1},e^{j\omega_2}) \tag{23}$$

within the frequency domain for $Y(e^{j\omega_1},e^{j\omega_2}) \in [0,\omega_c]$ and $Z(e^{j\omega_1},e^{j\omega_2}) \in [\omega_c,\pi]$, circular symmetry supposed to be valid. To filter out the lower frequency array let us introduce the filter function $H(e^{j\omega_1},e^{j\omega_2})$ of a highpass filter with zero phase spectrum $\arg H(e^{j\omega_1},e^{j\omega_2}):=0$ and its magnitude

$$|H(e^{j\omega_1},e^{j\omega_2})| = \begin{cases} 0 & \omega_1,\omega_2 \in [0,\omega_c] \\ 1 & \omega_1,\omega_2 \in [\omega_c,\pi] \end{cases} \tag{24}$$

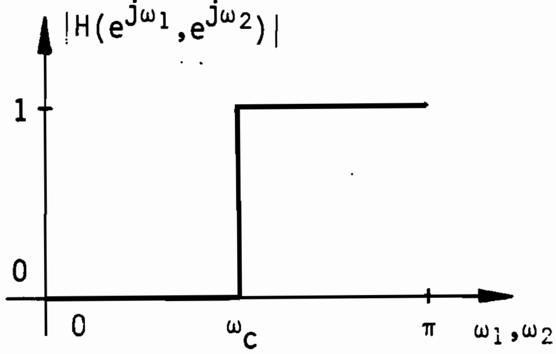

Fig.3: Cross section of the ideal frequency response of a circular symmetry highpass filter.

this filter function eliminates all lower frequencies and permits to pass all higher frequencies by unchanged phase spectra. The result can obviously be written as

$$|H(e^{j\omega_1},e^{j\omega_2})||X(e^{j\omega_1},e^{j\omega_2})| = |H(e^{j\omega_1},e^{j\omega_2})||Y(e^{j\omega_1},e^{j\omega_2})| + $$
$$+ |H(e^{j\omega_1},e^{j\omega_2})||Z(e^{j\omega_1},e^{j\omega_2})|$$

$$|H(e^{j\omega_1},e^{j\omega_2})||X(e^{j\omega_1},e^{j\omega_2})| = |Z(e^{j\omega_1},e^{j\omega_2})| \tag{25a}$$

$$\arg H(e^{j\omega_1},e^{j\omega_2}) + \arg X(e^{j\omega_1},e^{j\omega_2}) = \arg H(e^{j\omega_1},e^{j\omega_2}) + $$
$$+ \arg Z(e^{j\omega_1},e^{j\omega_2})$$

$$\arg X(e^{j\omega_1},e^{j\omega_2}) = \arg Z(e^{j\omega_1},e^{j\omega_2}) \quad (25b)$$

In using gradient techniques different procedures will reach the goal, which all are based on differentiation of the image. Let the light intensity function be written as $x(u,v)$ with its sampling $x(m,n)$. The gradient of $x(u,v)$ at coordinates (u,v) is defined as the vector

$$\underline{g}(u,v) := \text{grad } x(u,v) = \left[\frac{\partial x(u,v)}{\partial u} , \frac{\partial x(u,v)}{\partial v} \right]' \quad (26)$$

and points in the direction of maximum increasing rate of $x(u,v)$; its magnitude can be given as

$$g(u,v) := |\underline{g}(u,v)| = \left[\left(\frac{\partial x(u,v)}{\partial u}\right)^2 + \left(\frac{\partial x(u,v)}{\partial v}\right)^2 \right]^{1/2} \quad (27)$$

In digital image processing the derivatives of (26) will be approximated by differences

$$\frac{\partial x(u,v)}{\partial u} \simeq \Delta x_u(m,n) \quad , \quad \frac{\partial x(u,v)}{\partial v} \simeq \Delta x_v(m,n) \quad (28)$$

which are depending on the neighborhood being enclosed. If we look for example in the u direction to get differences like

$$\Delta x_u^1(m,n) = x(m,n) - x(m+1,n) \quad (29a)$$

or

$$\Delta x_u^2(m,n) = x(m-1,n) - x(m,n) \quad (29b)$$

or

$$\Delta x_u^3(m,n) = \frac{1}{2}(\Delta x_u^1+\Delta x_u^2) = \frac{1}{2}[x(m-1,n)-x(m+1,n)] \quad (29c)$$

the differences can be written as convolutions

$$\Delta x_u(m,n) = h_u(m) * x(m,n) \quad (30a)$$

$$\Delta x_v(m,n) = h_v(n) * x(m,n) \quad (30b)$$

with $h_u(m)$ and $h_v(n)$ as impulse response resulting from the purely complex frequency response (S.D. Stearns, 1975)

$$H(e^{j\omega}) = j\omega \quad (31)$$

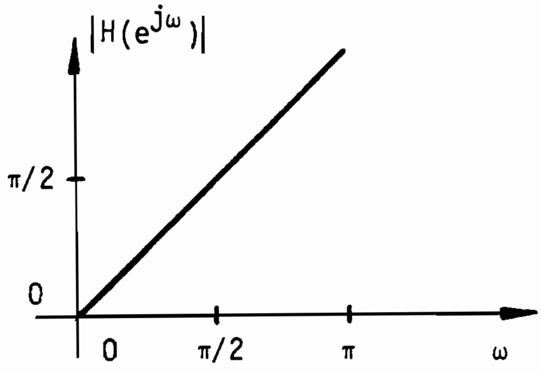

Fig.4: Frequency response of an ideal digital differentiator.

of the digital differentiator. Naturally the length of $h_u(m)$ and $h_v(n)$ will be bounded on the immediate surrounding of the sample $x(m,n)$.

The gradient array results then simply by substituting (27)

$$g(m,n) = \left[\Delta x_u(m,n)^2 + \Delta x_v(m,n)^2\right]^{1/2} \tag{32}$$

It should be noted the (32) is proportional to the difference in gray level between adjacent pixels, that means the gradient will be nearly zero for regions with nearly constant gray level and assumes large values for prominent image edges. For these reasons it may be advantegeous to interfere the gradient array with the original image

$$\tilde{g}(m,n) := \begin{cases} g(m,n) & \forall g(m,n) \geq T \\ x(m,n) & \text{otherwise} \end{cases} \tag{33}$$

to emphasize significant edges without destroying the characteristics of smooth backgrounds, where T is any nonnegative threshold. There are other variations possible depending on the task being solved.

IV. APPROXIMATIONS OF FREQUENCY RESPONSES

Once the frequency response of a LTI system is given the impulse response has to be calculated. Because of dealing with finite impulse responses for easier implementation and also inconsistencies within the frequency response this calculation, called 'design' (D. Fritsch, 1983), will be done by approximations not of ideal frequency responses but of frequency responses being realized (D. Fritsch, 1982).

A well known approximation procedure not only in photogrammetry as well as other engineering disciplines is the method of least-squares. Let the ideal frequency response be $H(e^{j\omega_1}, e^{j\omega_2})$ and the frequency response being realized $H^*(e^{j\omega_1}, e^{j\omega_2})$, then the method of least-squares minimises the total mean square error

$$\sigma_t^2 = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \left(|H(e^{j\omega_1}, e^{j\omega_2})| - |H^*(e^{j\omega_1}, e^{j\omega_2})|\right)^2 d\omega_1 d\omega_2 = \min \tag{34}$$

within the two-dimensional frequency space $\mathbb{C}^2$. (The one-dimensional case results from (34) by setting the frequency variable ω_2 to zero). To control the phase spectra of the original and the processed digital image the LTI system should have linear phase, from which zero phase can be derived (D. Fritsch, 1982). The following relations for even frequency response magnitudes of linear phase LTI systems are then true

$$H^*(e^{j\omega_1},e^{j\omega_2}) = e^{-j\{(\frac{K-1}{2})\omega_1+(\frac{L-1}{2})\omega_2\}} \sum_{k=0}^{(K-1)/2} \sum_{l=0}^{(L-1)/2} a(k,l)\cos(k\omega_1)\cos(l\omega_2) \tag{35}$$

with odd $K,L \in \mathbb{Z}_0^+$ to ensure the same pixel position after processing. The impulse response results by simple substitutions

$$\begin{aligned} a(0,0) &= h(\tfrac{K-1}{2},\tfrac{L-1}{2}) & a(0,l) &= 2h(\tfrac{K-1}{2},\tfrac{L-1}{2}-l) \\ a(k,0) &= 2h(\tfrac{K-1}{2}-k,\tfrac{L-1}{2}) & a(k,l) &= 4h(\tfrac{K-1}{2}-k,\tfrac{L-1}{2}-l) \end{aligned} \tag{36}$$

the relations for odd frequency response magnitudes of LTI systems e.g. the differentiator used in this paper can be given by (L.R. Rabiner / B. Gold, 1975) in $\mathbb{C}^1$

$$H^*(e^{j\omega}) = e^{-j(\frac{K-1}{2}\omega)} \; e^{j\pi/2} \sum_{k=1}^{(K-1)/2} c(k)\sin(\omega k) \tag{37}$$

where the impulse response results

$$c(k) = 2h(\tfrac{K-1}{2}-k) \quad , \quad h(\tfrac{K-1}{2}) := 0 \tag{38}$$

also here odd $K \in \mathbb{Z}_0^+$ has been supposed for the same reasons as above.

To evaluate (34) the continuous frequency domain $\mathbb{C}^2$ ($\mathbb{C}^1$) has to be sampled to transform the integral equation into numerical notation as it can be done with

$$\left\| \, |H(e^{j\omega_1},e^{j\omega_2})| - |H^*(e^{j\omega_1},e^{j\omega_2})| \, \right\|_2 = \min \tag{39}$$

here $\|\cdot\|_2$ denotes the l_2 norm. If the whole $\mathbb{C}^2$ ($\mathbb{C}^1$) has been equidistant sampled the following model can be given (D. Fritsch, 1982, 1983)

$$y - e = (A_1 \otimes A_2)x \tag{40}$$

with A_1 as coefficient matrix in the ω_1-direction, A_2 the coefficient matrix in the ω_2-direction and x the vector of the unknown impulse response coefficients. The vector y contains the values of the frequency response being realized and e is the vector of approximation errors, where $\otimes$ denotes the usual Kronecker product

$$(A_1 \otimes A_2) = \left[a^1_{ij} A_2\right] \tag{41}$$

For computational reasons the model (40) of vector equations will be transformed into the matrix equation model

$$Y - E = A_2 X A_1' \tag{42}$$

with y=:vecY, e=:vecE and x=:vecX in having now all the advantages of array algebra at first introduced and used by U.A. Rauhala (1980). The solution of (39) within (42) leads to

$$\tilde{X} = (A_2' A_2)^{-1} A_2' Y A_1 (A_1' A_1)^{-1} \tag{43}$$

with the estimates of the goodness of fit of the approximation

$$\tilde{E} = Y - A_2 \tilde{X} A_1' \tag{44}$$

To proof the approximation it may be advantegeous to transform $\tilde{Y} := A_2 \tilde{X} A_1'$ or $\tilde{E}$ on the log scale

$$\tilde{Y}_{\log_{10}} = (\tilde{y}_{ij\log_{10}}) = (2o \cdot \log_{10}(\tilde{y}_{ij})) \tag{45a}$$

$$\tilde{E}_{\log_{10}} = (\tilde{e}_{ij\log_{10}}) = (20 \log_{10}(\tilde{e}_{ij})) \tag{45b}$$

to see the approximation in dB as it is usually done in digital signal processing, because this presentation is much more sensitive as the linear scale.

V. CONCLUSIONS

This paper presents image enhancement and restoration as application of linear time invariant system theory. It was shown that important image processing products can be formulated by considering the desired processing within the frequency domain. The operators where derived as approximations on the frequency responses by the method of least-squares what could be improved by inequality restrictions within the solution space to search for a l_∞ approximation (D. Fritsch, 1983). The implementation of these operators can be done by the direct convolution sum, which was also introduced in the paper, because it seems reasonable in dealing with less coefficients providing for fast image processing.

VI. REFERENCES

CAPELLINI, V./A.G. CONSTANTINIDES/P. EMILIANI (1978): Digital Filters and their Applications. Academic Press, London.

FRITSCH, D. (1982): Entwurf digitaler zweidimensionaler nichtrekursiver Filter. German Geodetic Commission, Series C No. 275, Munich.

FRITSCH, D. (1983): Optimal Design of Two-Dimensional FIR-Filters. Proc. IEEE Int. Conf. Acoustics, Speech and Signal Processing, Boston, 1: pp.383-386.

GONZALEZ, R.C./P. WINTZ (1977): Digital Image Processing. Addison-Wesley Publ. Company, Reading, Mass.

MITRA, S.J./M.P. EKSTROM (1978): Two-Dimensional Digital Signal Processing. Dowden, Hutchinson and Ross, Stroudsbourg.

NOWAK, P. (1978): Ein Programmsystem zur Bildverbesserung an multispektralen Scannerdaten durch zweidimensionale digitale Filterung. In: Bildverarbeitung und Mustererkennung, Ed. E. Triendl, Springer, Berlin.

PAPOULIS, A. (1965): Probability, Random Variables and Stochastic Processes. McGraw-Hill, Inc. New York.

RABINER, L.R./B. GOLD (1975): Theory and Application of Digital Signal Processing. Prentice-Hall, Englewood Cliffs, New Jersey.

RAUHALA, U.A. (1980): Introduction to Array-Algebra. Photogram. Eng. Rem. Sens., 46: pp.177-192.

STEARNS, S.D. (1975): Digital Signal Analysis. Hayden Book, Rochelle Park, New Jersey.

STOCKHAM, T.G., Ir. (1972): Image Processing in the Context of a Visual Model. Proc. IEEE, 60, No.7: pp.828-842.

WIENER, N. (1949): Extrapolation, Interpolation and Smoothing of Stationary Time Series. J. Wiley, New York.