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SECOND ORDER DESIGN OF GEODETIC NETWORKS: PROBLEMS AND EXAMPLES

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ABSTRACT

For second order design of geodetic networks some problems arise. One has, for instance, to take into account the accuracy ratio between direction and distance measurements, furthermore the problem of orientatien unknowns and to fulfil some cost functions, to name but a few. Moreover for free networks, idealized regular criterion matrices must be transformed into singular ones. In this paper there are some proposals to solve these problems.The weight matrix for an observation plan is obtained by means of a linear complementarity problem. This solution improves the simple least-squares solution by add of inequalities to fulfil some requirements such as positive weights, accuracy ratios and cost functions. To demonstrate the design method, examples of real geodetic networks are presented, where some desired constraints are introduced. Numerical and software problems will also be discussed.

INTRODUCTION

In second order design (SOD) of geodetic networks the problem is to determine weights of the observations. It is a desired goal to plan the number of observations at home, because only on this way one can optimize the use of its staff and instruments to save time and money. For this reason some restrictions are to fulfil. On the one hand the observations ought to be uncorrelated, which leads to weights greater or equal zero, on the other hand, there must be considered cost functions, accuracy ratios for heterogenious observations etc. One problem is to pretend the criterions, which have to be approximated by SOD-solutions. Many geodesists have worked onto this field, such as F.R. Helmert, O.Schreiber, H.Bruns, W.Jordan, I. Jung, A. Tarczy-Hornoch, to name but a few. The starting point was to minimize some functions, described by scalar values, but since W. Baarda and E. Grafarend, which introduced criterion matrices, much work was done about the approximation of these matrices. E. Grafarend/B. Schaffrin (1979) derived such idealized matrices in a pure analytical way, they are called 'Taylor-Karmann structered criterion matrices'. These matrices are always regular, so they have to be transformed into singular ones for free networks. One transformation will be treated in this paper, because of its less numerical expenditure. Another problem is to find a solution, which bears in mind the previously mentioned restrictions on the observation weights. Schaffrin et al. (1980) have shown, that this can be done by means of the linear complementarity problem. (LCP). For its solution algorithms exist, especially Lemke's and the Cottle-

Dantzig algorithm; both base upon complementary pivot strategie. To use such an algorithm the desired restrictions have to be formulated as inequality constraints, whereby the number of the constraints is bounded only by size of the computer the algorithm is implemented. Thus there are many possibilities for the user to introduce restrictions to see which observation plan is 'best' in some sense. Also reliability aspects might be formulated as inequality constraints (J. v. Mierlo, 1981). Furthermore, B. Schaffrin (1981 a, 1981 b) has shown, that attenuation of the inequality constraints leads to a parameterized linear complementarity problem (PLCP), to solve with the same algorithms available for LCP. In the following, after a short description of the solution method for SOD, the main objective of this paper will be to show the efficiency of such an algorithm, namely the Cottle-Dantzig algorithm, available as FORTRAN program written by C.R. Liew/J.K. Shim (1978). Some restrictions on the observation weights are introduced and demonstrated by examples.

CRITERION MATRICES

In SOD with criterion matrices the fundamental equation is (K.R.Koch, 1980:153)

$$D(\hat{\underline{x}})=\sigma^2\underline{C}_x=\sigma^2(\underline{A}'\underline{P}\underline{A})^{-1} \tag{1.1}$$

derived from the Gauss-Markoff-model of full rank

$$E(\underline{y})=\underline{A}\underline{x} \quad \text{with} \quad D(\underline{y})=\sigma^2\cdot\underline{P}^{-1} \tag{1.2}$$

The operators E and D denotes the expectation and dispersion, whereby the nxu matrix $\underline{A}$ contains given coefficients with $rk(\underline{A})=u<n$, $\underline{x}$ is the ux1 vector of unknowns, $\underline{y}$ the nx1 observation vector, σ^2 the variance of the weight unit and $\underline{P}$ the nxn weight matrix of the observations. For the uxu matrix $\underline{C}_x$ some desired criterions may be introduced, such as error circles for points, whose coordinates are to be estimated in an evaluation afterwards. Thus this matrix contains such criterions in an idealized form, which the weights to be realized can approximate only. Some proposals were made to construct such matrices (W. Baarda, 1973, E. Grafarend, 1970, 1972, F. Sárközy, 1979, E. Grafarend/B. Schaffrin, 1979). For the examples in this paper criterion matrices after E. Grafarend/B. Schaffrin are used,which have Taylor-Karman-structure, they are to construct very easy by means of computers. In a free network, where $rk(\underline{A})=q<u<n$ the idealized criterion matrix has to be transformed into a singular one, because the unbiased estimable parameters are projected parameters (K.R. Koch, 1980 : 169). A useful projection is

$$\underline{R}=(\underline{A}'\underline{P}\underline{A})^-_m\underline{A}'\underline{P}\underline{A} \tag{1.3}$$

(C.R. Rao/S.K. Mitra, 1971:51) with $(\underline{A}'\underline{P}\underline{A})^-_m$ as g-inverse for minimum norm least-squares solutions. The matrix $\underline{R}$ is symmetric and idempotent because of the definition of this g-inverse. For $\tilde{\underline{x}}=(\underline{A}'\underline{P}\underline{A})^-_m\underline{A}'\underline{P}\underline{A}\,\underline{x}$ the dispersion matrix for the projected parameters is the Moore-Penrose inverse or pseudoinverse

$$\begin{aligned} D(\tilde{\underline{x}}) &=\sigma^2(\underline{A}'\underline{P}\underline{A})^-_m\underline{A}'\underline{P}\underline{A}(\underline{A}'\underline{P}\underline{A})^-_{rs}[(\underline{A}'\underline{P}\underline{A})^-_m\underline{A}'\underline{P}\underline{A}]' \\ &=\sigma^2(\underline{A}'\underline{P}\underline{A})^+ \end{aligned} \tag{1.4}$$

Thus, for the g-inverse in Eq. (1.3) the Moore- Penrose inverse will be introduced such that

$$\underline{R}=(\underline{A}'\underline{P}\underline{A})^{+}\underline{A}'\underline{P}\underline{A} \tag{1.5}$$

An efficient formula for calculating $\underline{R}$ in Eq. (1.5) is described by K.R. Koch (1980:59) by means of the matrix $\underline{E}$ as basis for $N(\underline{A})$ and which is therefore called 'null-space basis'. The projection matrix $\underline{R}$ is obtained as

$$\underline{R}=\underline{I}-\underline{E}'(\underline{E}\underline{E}')^{-1}\underline{E} \tag{1.6}$$

which also W. Baarda used as 'S_m-transformation', because it provides for minimum trace of the dispersion matrix for the estimable parameters (J.v. Mierlo, 1978, 1980). Eq. (1.6) is valid for arbitrary regular $\underline{P}$ matrices, then $\underline{R}$ may be rewritten as

$$\underline{R}=(\underline{U}'\underline{U})^{+}\underline{U}'\underline{U} \quad \text{with} \quad \underline{U}=\underline{S}\underline{A} \quad \text{and} \quad \underline{P}=\underline{S}'\underline{S} \tag{1.7}$$

Because of $\underline{E}:=N(\underline{A})$ and therefore $\underline{A}\underline{E}'=\underline{O}$, $\underline{E}$ is null basis of $\underline{U}$, too, then $\underline{S}\underline{A}\underline{E}'=\underline{O}$. The matrix $\underline{E}$ can generally be given by purely geometrical considerations since it contains the changes the unknown parameters x_i can undergo without affecting the observations y_i (P. Meissl, 1969, A.J. Pope, 1973). In combined plane networks (directions and distances are measured) the matrix $\underline{E}$ will be

$$\underline{E} = \begin{vmatrix} 1 & 0 & 1 & 0 & \dots & 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 1 & \dots & 0 & 1 & 0 & \dots & 0 \\ -\overset{\circ}{y}_1 & \overset{\circ}{x}_1 & -\overset{\circ}{y}_2 & \overset{\circ}{x}_2 & \dots & -\overset{\circ}{y}_m & \overset{\circ}{x}_m & \rho & \dots & \rho \end{vmatrix} \tag{1.8}$$

because the networks may undergo translations and a differential rotation. The first columns are responsible for coordinate parameters x_i, y_i, $i\in\{1,2...m\}$, the last columns relate to the orientation parameters o_i. Coefficients in the last row are the approximate coordinates and a factor ρ for dimensions (is the measurement taken as dimension : gon, ρ will be $200/\pi$). For these networks, the Gauss-Markoff-model can be written as

$$E(\underline{y})=\begin{vmatrix} \underline{A}_1, \underline{A}_2 \end{vmatrix} \begin{vmatrix} \underline{x}_1 \\ \underline{x}_2 \end{vmatrix} \quad \text{with} \quad D(\underline{y})=\sigma^2 . \underline{P}^{-1} \tag{1.9}$$

which leads to the normal equations

$$\begin{vmatrix} \underline{A}_1'\underline{P}\underline{A}_1 & \underline{A}_1'\underline{P}\underline{A}_2 \\ \underline{A}_2'\underline{P}\underline{A}_1 & \underline{A}_2'\underline{P}\underline{A}_2 \end{vmatrix} \begin{vmatrix} \hat{\underline{x}}_1 \\ \hat{\underline{x}}_2 \end{vmatrix} = \begin{vmatrix} \underline{A}_1'\underline{P}\underline{y} \\ \underline{A}_2'\underline{P}\underline{y} \end{vmatrix} \tag{1.10}$$

with $\underline{x}_1$ as the u_1x1 coordinate vector and $\underline{x}_2$ as u_2x1 vector of the orientation unknowns.

To approximate by SOD is now

$$\begin{vmatrix} \underline{A}_1'\underline{P}\underline{A}_1 & \underline{A}_1'\underline{P}\underline{A}_2 \\ \underline{A}_2'\underline{P}\underline{A}_1 & \underline{A}_2'\underline{P}\underline{A}_2 \end{vmatrix} = \begin{vmatrix} \underline{C}_{x_1x_1} & \underline{C}_{x_1x_2} \\ \underline{C}_{x_2x_1} & \underline{C}_{x_2x_2} \end{vmatrix}_{rs}^{-} = \begin{vmatrix} \underline{C}_{x_1x_1}^{+} & \underline{M}_{12} \\ \underline{M}_{21} & \underline{M}_{22} \end{vmatrix} \tag{1.11}$$

with $\underline{C}_{x_ix_j}$ as criterion matrix for the coordinates and orientation parameters. In this paper only the upper left halve will be approximated, because much work has to be done to built up combined criterion matrices and about the choice of the g-inverse, respectively. For the conditions of the g-inverse in Eq. (1.11) see the paper of B. Schaffrin (1981 c). For the rank deficiency of $\underline{A}_1$, an idealized regular criterion matrix has to be transformed into a singular one, which should have the same deficiency as $\underline{A}_1$. This transformation will be done by the previously introduced projection matrix $\underline{R}$, such that

$$\underline{C}_{x_1x_1} = \underline{R}\underline{C}_{TK}\underline{R}' \tag{1.12}$$

In this case, if $\underline{C}_{TK}$ has Taylor-Karman structure, $\underline{C}_{x_1x_1}$ will have'derived Taylor-Karman structure', with $rk(\underline{C}_{x_1x_1})=rk(\underline{A}_1)=q_1 < u_1 < u$. In combined plane networks, the deficiency of $\underline{A}_1$ will only be $u_1-q_1=2$, a result, which may surprise some readers because $\underline{A}_2$ has full rank and the total rank deficiency of $\underline{A}$ is u-q=3. The matrix $\underline{R}$ for this case can be written as

$$\underline{R} = \begin{vmatrix} 1-\frac{1}{m} & 0 & -\frac{1}{m} \cdots\cdots & -\frac{1}{m} & 0 \\ 0 & 1-\frac{1}{m} & 0 \cdots\cdots & 0 & -\frac{1}{m} \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ -\frac{1}{m} & 0 \cdots\cdots\cdots\cdots & & 1-\frac{1}{m} & 0 \\ 0 & -\frac{1}{m}\cdots\cdots\cdots\cdots & & 0 & 1-\frac{1}{m} \end{vmatrix} \tag{1.13}$$

with m as number of the netpoints, because the networks can only undergo translations, that means, only the two first rows without columns for orientation parameters of $\underline{E}$ must be considered. More informations about the rank deficiency of these networks provides the contribution of D. Fritsch/B. Schaffrin,(1981).

LEAST-SQUARES SOLUTION WITH INEQUALITY CONSTRAINTS FOR SECOND ORDER DESIGN PROBLEMS

The SOD problem is to solve the matrix equation

$$\underline{Q} := \underline{C}^{-} = \underline{A}'\underline{P}\underline{A} \tag{2.1}$$

which leads to the linear equations

$$\underline{q} = (\underline{A}'\otimes\underline{A}')\underline{p} \tag{2.2}$$

with $\underline{q}$:= vec $\underline{Q}$ and $\underline{p}$:= vec $\underline{P}$, the vectors of the matrices $\underline{Q}$ and $\underline{P}$, columnwise built one vector after another, wheras $\otimes$ denotes the Kronecker product. A solution of Eq. (2.2) delivers generally correlated observations which are not realizable (J.Bossler et al.,1973).Hence due to a proposal of B. Schaffrin (1977), the following reformulation of Eq. (2.2) has to be solved

$$(\underline{A}'\odot\ \underline{A}')\underline{p}=\underline{g} \tag{2.3}$$

which constrains a weight matrix $\underline{P}:=\text{diag}\ \underline{p}$ and is called 'Diagonal Second Order Design Problem'. The symbol $\odot$ denotes the Khatri-Rao product, here defined as

$$\underline{A}_1'\odot\underline{A}_1' := |\underline{a}_1'\otimes\underline{a}_1', \ \ldots\ldots, \underline{a}_n'\otimes\underline{a}_n'| \tag{2.4}$$

A solution of the in general inconsistent equation system (2.3) can be found by means of least-squares and will perhaps lead to some negative weights (G. Schmitt et al. 1978, G. Schmitt, 1980). The minimization of

$$\underline{p}'(\underline{A}'\odot\underline{A}')'(\underline{A}'\odot\underline{A}')\underline{p}-2\underline{p}'(\underline{A}'\odot\underline{A}')\underline{g}+\underline{g}'\underline{g} \tag{2.5}$$

results into the normal equation system

$$(\underline{A}'\odot\underline{A}')'(\underline{A}'\odot\underline{A}')\tilde{\underline{p}}=(\underline{A}'\odot\underline{A}')'\underline{g} \tag{2.6}$$

which can be rewritten to

$$(\underline{A}\underline{A}'\boxdot\underline{A}\underline{A}')\tilde{\underline{p}}=(\underline{A}'\odot\underline{A}')\underline{g} \tag{2.7}$$

with $\boxdot$ as Hadamard product, in this case defined as

$$(\underline{A}\underline{A}'\boxdot\underline{A}\underline{A}')=\begin{vmatrix} \underline{a}_1\underline{a}_1'\otimes\underline{a}_1\underline{a}_1' & \ldots\ldots & \underline{a}_1\underline{a}_n'\otimes\underline{a}_1\underline{a}_n' \\ \vdots & & \vdots \\ \underline{a}_n\underline{a}_1'\otimes\underline{a}_n\underline{a}_1' & \ldots\ldots & \underline{a}_n\underline{a}_n'\otimes\underline{a}_1\underline{a}_n' \end{vmatrix} \tag{2.8}$$

To fulfil some requirements, such as positive weights and accuracy ratios , let us now introduce the inequalities $\underline{H}\underline{p} \geq \underline{c}$. To solve is then

$$(\underline{A}'\odot\underline{A}')\underline{p}=\underline{g} \quad \text{subject to} \quad \underline{H}\underline{p} \geq \underline{c} \tag{2.9}$$

with known rxu matrix $\underline{H}$ and known rx1 vector $\underline{c}$. The least-squares solution of Eq. (2.9) leads to the quadratic program

$$\begin{aligned} &\text{minimize} \quad \underline{p}'(\underline{A}'\odot\underline{A}')'(\underline{A}'\odot\underline{A}')\underline{p}-2\underline{p}'(\underline{A}'\odot\underline{A}')\underline{g}+\underline{g}'\underline{g} \\ &\text{subject to} \quad \underline{H}\underline{p} \geq \underline{c} \end{aligned} \tag{2.10}$$

By means of the vector $\underline{v}$ of slack variables, defined as $\underline{v}':=|v_1^2,v_2^2,\ldots.v_r^2|\geq\underline{0}$, the inequalities may be reformulated as equalities

$$\underline{H}\underline{p}-\underline{v}=\underline{c} \tag{2.11}$$

The minimization of the Lagrangian function
$L(p,\lambda,v):=p'(A'\odot A')'(A'\odot A')p-2p'(A'\odot A')'q+q'q-2\lambda'(Hp-v-c)$ delivers the well-known Kuhn-Tucker conditions

$$
\begin{aligned}
&(A'\odot A')'(A'\odot A')\hat{p}-(A'\odot A')'q-H'\lambda &&= 0\\
&H\hat{p}\quad -v\quad -c &&= 0 \qquad\qquad (2.12)\\
&v'\lambda &&= 0
\end{aligned}
$$

with $v,\lambda\geq 0$, which in quadratic programming are known as 'linear complementarity problem (LCP)'

$$v=W\lambda+z,\quad v'\lambda=0 \quad\text{and}\quad v,\lambda \geq 0 \qquad (2.13)$$

This can be solved by means of complementarity algorithms. In Eq. (2.13) the rxr matrix W and the rx1 vector z are substituted for

$$
\begin{aligned}
&W=H(AA'\boxdot AA')^{-1}H' \\
&z=H(AA'\boxdot AA')^{-1}(A'\odot A')'q-c=H\tilde{p}-c
\end{aligned}
\qquad (2.14)
$$

with $\tilde{p}$ as simple least-squares estimation, obtained by Eq.(2.7). The final observation weights will be got by

$$\hat{p}=\tilde{p}+(AA'\boxdot AA')^{-1}H'\lambda \qquad (2.15)$$

which shows, that the simple least-squares estimation $\tilde{p}$ is improved to fulfil the inequality constraints. We will call this solution 'Inequality Constraint Least-Squares Solution (ICLS)' in contrast to the 'Simple Least-Squares Solution (SLS)'.

WEIGHT CONSTRAINTS FOR THE DIAGONAL SECOND ORDER DESIGN PROBLEM

As solution of the LCP one gets weights which fulfil some inequality constraints. The problem is to formulate all desired restrictions on the observation weights as inequalities. One important restriction is that the estimated weights are always greater or equal zero, also

$$\hat{p}\geq 0 \qquad (3.1)$$

because negative weights give no sense. Another restriction may be the idealized accuracy structure contained in the criterion matrix to improve on the average (B. Schaffrin 1980, 1981 b). This will results into the inequalities

$$(Z'\boxdot Z')\hat{p}\geq 1 \qquad (3.2)$$

with $Z:=AU$, $Q=ULU'$ and $L=\text{diag }1$. The matrix U contains the eigenvectors of the inverse criterion matrix and the matrix L the latent roots, respectively.
B. Schaffrin et al. (1977), used this singular value decomposition also to reformulate the diagonal SOD and it is called 'Canonical formulation of diagonal SOD'.

Furthermore, J. van Mierlo (1981) has shown that reliability aspects will be considered as

$$\hat{\underline{p}} \leq \underline{\alpha}(\alpha_o, \beta_o, \bar{\delta}_o,) \tag{3.3}$$

where $\underline{\alpha}$ is a nx1 constant vector derived from the level of significance α_o, the power β_o and a 'measure' $\bar{\delta}_o$ as value for the external reliability of a network.

For networks, where directions and distances can be measured, the available instruments may be bounded, so one has to consider the highest attainable accuracy. This means, that $D(\underline{y})_{max} = \sigma^2\underline{P}^{-1}$ has to be reformulated into inequalities. The result is

$$\hat{\underline{p}} \leq \sigma^2 | \sigma^{-2}_{1max}, \sigma^{-2}_{2max}, \ldots, \sigma^{-2}_{imax}, \ldots |' \tag{3.4}$$

which is very important, because it contains also the accuracy ratio between the direction and distance measurements. Furthermore, by these inequalities, the attainable absolute point accuracy can be calculated, expressible as error ellipses. This will be demonstrated by the examples later. The values σ^2_{imax} are ' a priori variances for the measurements, which might be individual perhaps for considerations of length dependence accuracies.
Moreover, if directions are measured, it is desired to have same accuracy for every direction observed on one station. This leads to group weights, here espressable as

$$\hat{p}_{ij} = \hat{p}_{ik} \qquad i \in \{1,2,,,,m\} \tag{3.5}$$

with i as index for the number of observation station and j,k are target points. Eq.(3.5) changed in inequalities gives

$$\begin{aligned} \underline{F}\hat{\underline{p}} &\geq \underline{d} \\ -\underline{F}\hat{\underline{p}} &\geq -\underline{d} \end{aligned} \tag{3.6}$$

however with known vector $\underline{d}$. The consideration of these inequalities can be done by two steps: at first one gets individual weights by a first ICLS solution and introduced the maximum value $\hat{p}_i$ as elements d_i, secondly, a further ICLS solution will provide for group weights. G. Schmitt (1979) has shown experiences with individual and group weights obtained by SLS solutions.
Last but not least perhaps cost functions are to taken into account, because only a fixed amount for execution the measurements is available. A simple formulation is

$$\underline{c}'\hat{\underline{p}} \leq k \tag{3.7}$$

where a scalar value k is the given cost constraint. It should be noticed, that this manner of ICLS solutions cannot minimize some cost functions in contrast to

W. Augath (1976) and F. Sárközy (1979), to name only two. It is the accuracy of the points contained in the criterion matrix, which will be approximated subject to some restrictions. The inequalities formulated previously can be inconsistent in some cases, so they have to be attenuated to lead to a solution. This can be done by means of a ' parameterized LCP (PLCP) ', introduced in SOD by B. Schaffrin (1981 b), but will not commented on in this paper.

EXAMPLES

In the following two examples are considered, a very simple network (only a triangle) and a real network to establish for detecting recent crustal movements in Turkey. Both examples are combined plane free networks, where the main objective will be to fulfil accuracy requirements only. At first, the solutions for the triangle will be discussed. The ICLS1 solution contains the inequalities Eq.(3.1) and Eq.(3.2). As shown by Table 1 and Table 2 the negative weight for the distance 2-3 is vanished and the error ellipses are always smaller than the projected ones. For $\sigma^2 = 1$ the variances of the directions would be ≈ 0.03 [mgon] which cannot be realized. As highest attainable accuracy was introduced : $\sigma_{dir} = 0.1$ [mgon] and $\sigma_{dis} = 5$ [mm]. The including of Eq.(3.4) to Eq.(3.1) and Eq.(3.2) led to none solution, because Eq.(3.2) was to strong. A second ICLS solution, called ICLS2, without Eq.(3.2) shows a better shape of the error ellipses, thougt the desired accuracy structure is unimportant worse. Also this solution provided for group weights, which needed not to formulate as inequalities especially. The error ellipsesare figured in Fig.1 and Fig.2 for both solutions.
For the turkish network also group weights for the directions were desired subject to the highest accuracies: $\sigma_{dir} = 0.1$ [mgon] and $\sigma_{dis} = 10$ [mm]. In Table 4 the estimated weights by means of ICLS solution are shown. For $\sigma^2 = 4$, the constraints are fulfilled and the dimension for the error ellipses is fixed. As one can seen in Table 5 the error ellipses of the ICLS solution are always smaller than the desired ones, thougt inequalities of Eq.(3.2) were not considered. Also the shape of the error ellipses is better, because of the same accuracy ratio of the direction and distance measurements.

CONCLUSION

In this paper was shown, that the problem of ' diagonal SOD ' can be solved by means of an ICLS solution, which one gets by complementarity algorithms. For the examples previously the Cottle-Dantzig algorithm was used. The FORTRAN version by C.K. Liew/J.K. Shim of this algorithm is easy to implement because it is built up modular and consists of six subroutines, the largest one has 44 statements. If there is no complementarity solution, the program will terminate his run with ' ray termination ', that means, the inequalities are too strong and the user must perhaps reformulate his problem. Thus, for the two main problems in SOD, namely the set-up of criterions and the approximation of these criterions, the last one can be solved very well with such algorithms. But for the first problem much work has to be done, particularly for combined networks, where one has to consider coordinate and orientation unknowns. Moreover in free combined networks investigations must be made about the choice of g-inverses, because the criterion matrix for coordinate parameters only might be given, which was the case for this paper.

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Table 1: SOD solutions for the triangle

from	to	SLS	ICLS1	ICLS2
1	2	2.575	3.111	1.000
	3	2.579	3.324	1.000
2	3	2.866	3.479	1.000
1	2	C.170	C.197	0.756
	3	0.037	0.427	0.609
2	3	-0.031	0.534	0.466

Table 2: Error ellipses for the triangle (solution 1)

point	coordinates x [m]	y	error ellipses projected A [cm]	B	Φ[gon]	ICLS1 A [cm]	B	Φ[gon]
1	3000	4000	0.56	0.40	5.13	0.55	0.33	8.72
2	7000	8000	0.59	0.40	91.04	0.59	0.32	87.59
3	9000	1000	0.65	0.43	139.55	0.61	0.31	140.28

Table 3: Error ellipses for the triangle (solution 2)

point	coordinates x [m]	y	error ellipses projected A [cm]	B	Φ[gon]	ICLS2 A [cm]	B	Φ[gon]
1	3000	4000	0.56	0.40	5.13	0.53	0.49	13.70
2	7000	8000	0.59	0.40	91.04	0.57	0.50	88.16
3	9000	1000	0.65	0.43	139.55	0.62	0.53	134.57

Table 4: SOD solutions for the turkish network

from	to	SLS	ICLS
1	2	5.513	4.000
	3	9.512	4.000
	4	15.260	4.000
	5	8.738	4.000
	6	5.340	4.000
2	3	14.455	4.000
	4	7.721	4.000
	5	7.431	4.000
	6	6.152	4.000
3	4	6.449	4.000
	5	6.779	4.000
	6	7.578	4.000
4	5	6.845	4.000
	6	6.857	4.000
5	6	4.792	4.000
1	2	-0.864	0
	3	0.222	0
	4	-0.787	0.309
	5	1.197	1.567
	6	-0.276	0.415
2	3	-0.937	0.295
	4	0.172	0
	5	2.036	2.310
	6	-0.257	0.419
3	4	-1.041	0
	5	-0.314	0.636
	6	1.816	2.198
4	5	-0.412	0.327
	6	2.233	2.481
5	6	0.114	1.736

Table 5: Error ellipses for the turkish network

point	coordinates x [m]	y	error ellipses projected A [cm]	B	Φ[gon]	ICLS A [cm]	B	Φ[gon]
1	4500	6500	1.48	1.04	16.71	1.08	0.92	0.96
2	18500	1500	1.40	1.00	111.85	0.98	0.82	149.05
3	25000	18500	1.38	1.00	21.47	0.96	0.82	197.12
4	13000	23500	1.38	0.99	109.43	0.97	0.81	143.67
5	13500	6000	1.05	0.82	89.13	0.72	0.64	176.91
6	17500	18500	1.02	0.78	88.49	0.69	0.63	194.49

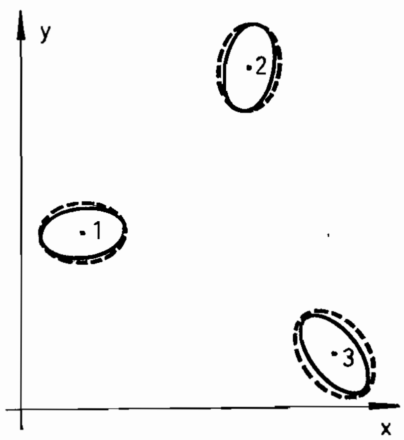

Fig. 1: Solution 1 for the triangle

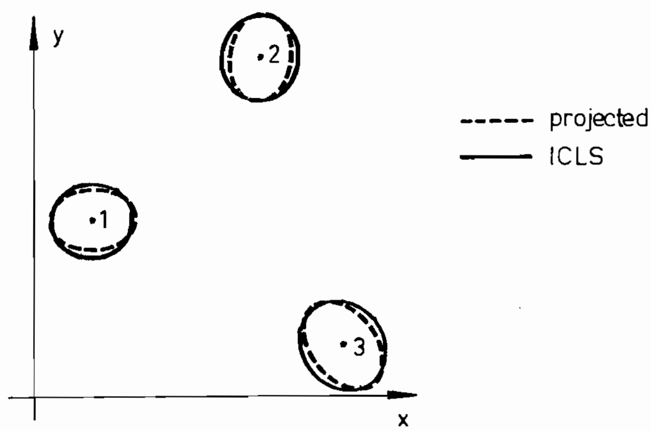

Fig. 2: Solution 2 for the triangle

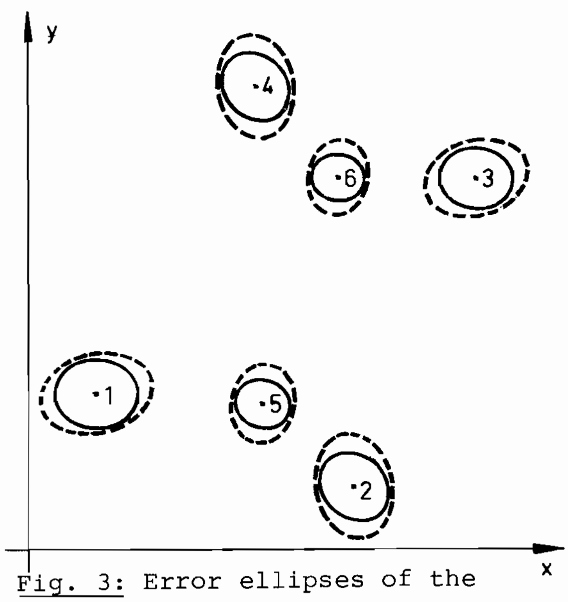

Fig. 3: Error ellipses of the turkish network

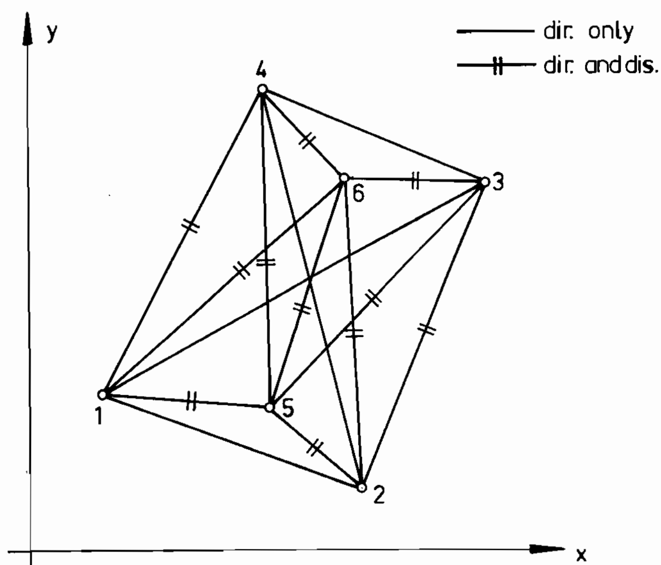

Fig. 4: Final observation plan of the turkish network