Deformation Measurements
Deformationsmessungen
(Eds: I. Joó and Á. Detrekői)

PREPROCESSING OF DEFORMATION DATA BY MEANS OF DIGITAL FILTERING

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INTRODUCTION

In order to detect time-varying effects of man-made constructions and buildings deformation measurements have to be taken out, continously or discretely to different time epochs. For further investigations about the subject to be controlled the deformation data will be analysed to see whether or not any phenomena have been occurred. However, before such an analysis can be done one has to preprocess the data very often because of noisy observations or to eliminate some undesired informations whose frequencies are known a priori.

A mathematical tool for eliminating noise or any frequencies known within the observations is digital filtering, part of the digital signal processing discipline, a relatively new field for surveying engineers. The filter is realized by any filter coefficients, also called 'weights', to present the filtered version of the observations as simple weighted arithmetic mean with these filter coefficients in mind. All determinations of such coefficients we will call 'filter design' in contrast to the actual filter operation itself which is called 'filter implementation', whereby in filter design our objective is directed towards a finite number of coefficients to guarantee always stable solutions of observations being filtered.

The design problem may be solved by different techniques known in digital signal processing, generally to put into

- ad hoc methods
- optimization and approximation methods

where for example as ad hoc method might serve filter coefficients chosen arbitrarily or to multiply ideal filter coefficients with given functions as it is called 'windowing technique' and demonstrated in detail by J. F. Kaiser (1966). The disadvantage of filter coefficients chosen arbitrarily is that they could have some undesired influences on frequencies we are interested in; a difficulty for the latter method may be the computation of Fourier coefficients for given transfer functions being approximated, what is generally not trivial because one has to determine closed-form expressions for these coefficients.

Therefore the most flexible design methods are the optimization and approximation techniques known as the frequency sampling method (L. R. Rabiner/ R. W. Schafer, 1971, 1972) and the Chebyshev approximation method (L. R. Rabiner/B. Gold, 1975, p. 123); both methods are common optimization algorithms such as the SIMPLEX- and the REMEZ-algorithm, only scarcely or not at all available for surveying engineers.

However, in many cases, there is no need to use such algorithms because similar results will be obtained by a very familiar method in surveying engineering: the method of least squares, which can be optimized in the sense of a Chebyshev approximation by additional inequality constraints as demonstrated by D. Fritsch/B. Schaffrin (1980), D. Fritsch (1982).

The main subject of this paper will be to introduce the concepts of digital filtering as means for preprocessing of deformation data, therefore filter design is bounded on least squares approximations without inequality constraints. Procedures in filter implementation, where the user can choose among slow and fast convolution algorithms, are also considered. There is another bound in this paper in such way that one-dimensional filtering only will be regarded because in surveying engineering the observation of time-varying functions results frequently into time series, which are one-dimensional. Indeed, the concepts presented here are also applicable for two-dimensional digital filtering problems as it is subject of D. Fritsch (1982) and commonly for multi-dimensional digital filtering, but until now only one- and two-dimensional digital filtering is solved sufficiently.

The digital filtering concepts presented later on regard the filters as linear systems with given transfer functions in contrast to H. Pelzer (1976, 1977, 1978) and W. Möhlenbrink (1978), where the determination of system features have been treated. But once the transfer functions are known these digital filtering concepts might also be used to predict some deformations or take into consideration any system behaviours on future time functions to be measured.

DIGITAL SIGNALS AND TRANSFER FUNCTIONS

Let **x**(t) be a continous one-dimensional function as it might appear in surveying engineering as height, distance or angle etc. to observe continously in any time interval $t \in [t_o, t_{m-1}]$ (see Figure 1). This function contains analog data and therefore we will call it 'analog signal' which has to be digitized by any sampling process to get discrete data for easier data handling.

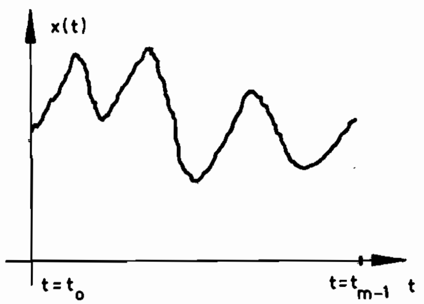

Figure 1: Analog signal

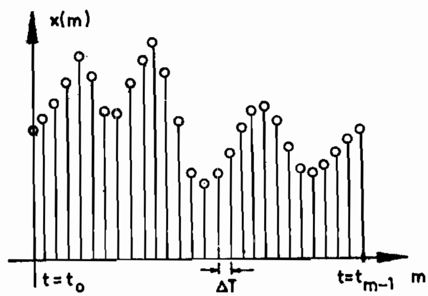

Figure 2: Digital signal

In surveying engineering the sampling process will take place by measurements to different time epochs $t = t_i$; as results of our measurements between $t = t_o$ and $t = t_{m-1}$ we get a sampled data set designated by $[x(t=t_o), x(t=t_1), \ldots, x(t=t_{m-1})]$ with M samples in all.

This data set represents a 'digital signal', which is causal and of finite length because of the definition of the time interval and to describe by

$$x(m) \quad \text{for } m \in \{0,1,2, \ldots, M-1\}$$

assumed, that equidistant data sampling will be used with ΔT as sampling interval (see Figure 2). For the choice of ΔT the sampling theorem has to be considered (S.D. Stearns, 1975, p.37), otherwise by non-equidistant sampling some interpolation methods will provide for equidistant sampling, but these methods will not be commented on in this paper.

With the digital unit sample (impulse) in mind, defined by

$$d(m) = \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases} \quad \text{and} \quad d(m-k) = \begin{cases} 1 & m = k \\ 0 & m \neq k \end{cases} \tag{1-1}$$

the signal x(m) can be presented as discrete convolution

$$x(m) = \sum_{k=0}^{M-1} x(k)\, d(m-k) = x(m) * d(m) \tag{1-2}$$

with x(k) as just the magnitude of the signal x(m); this convolution is usually symbolized by an asterisk (*).

Now, the digital filtering operation what we are searching for with result y(m) (see Figure 3), will be derived from the theory of linear,time-invariant systems(LTI systems)(L. R. Rabiner/B. Gold, 1975, p.13), because any digital filter can be seen as such a system as demonstrated in D. Fritsch (1982).

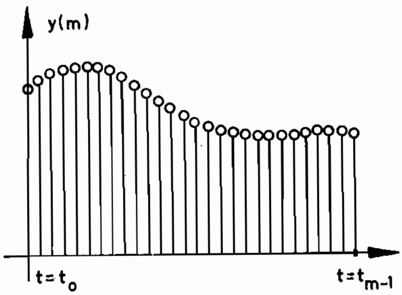

Figure 3: Filtered digital signal

Consider Ø as an arbitrary operator of a LTI system and x(m) as input signal, the output or filtered signal is functionally related to the input by

$$y(m) = \emptyset \left[x(m)\right] \qquad (1\text{-}3)$$

and for the input signal as just the unit sample d(m) the operator Ø characterizes the response of the LTI system, generally to denote with

$$h(m) = \emptyset \left[d(m)\right] \qquad (1\text{-}4)$$

wich is called 'unit impulse response' or shortly 'impulse response'.

Because of the assumption of linearity and time-invariance of our system the filtered signal y(m) is given by both the relation

$$y(m) = \emptyset\left[\sum_{k=0}^{M-1} x(k)\ d\ (m-k)\right]$$

$$= \sum_{k=0}^{M-1} x(k)\ \emptyset\ \left[d(m-k)\right]$$

$$= \sum_{k=0}^{M-1} x(k)\ h(m-k) = x(m) * h(m) \qquad (1\text{-}5a)$$

and by exchange of the variables with

$$y(m) = \sum_{k=0}^{M-1} h(k)\ x(m-k) = h(m) * x(m) \qquad (1\text{-}5b)$$

that is , the filtering operation is a convolution between the impulse response of the filter and the signal to be filtered.

An equivalent expression for (1-5) will be obtained by taking any unitary transform on both sides; in our case, where frequencies have to be considdered, an appropriate unitary transform is the Fourier transform, defined as pair for a finite sequence (L.R. Rabiner/B. Gold, 1975, p. 25)

$$X(e^{j\omega}) = \sum_{m=0}^{M-1} x(m)e^{-jm\omega} \tag{1-6a}$$

$$x(m) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})e^{jm\omega}d\omega \tag{1-6b}$$

with $j = \sqrt{-1}$ and the continous frequency variable $\omega \in [0,2\pi]$. By definition of the Eulerian notation $e^{\pm jm\omega} = \cos m\omega \pm j\sin m\omega$ the complex function $X(e^{j\omega})$ is a periodic complex function $X(e^{j\omega}) = X(e^{j\omega+2k\pi})$ for $k \in \mathbb{Z}$ and to express by both

$$X(e^{j\omega}) = X_{Re}(e^{j\omega}) + jX_{Im}(e^{j\omega}) \tag{1-7}$$

and

$$X(e^{j\omega}) = |X(e^{j\omega})| \; e^{j\arg X(e^{j\omega})} \tag{1-8}$$

with $X_{Re}(e^{j\omega})$ as real part and $X_{Im}(e^{j\omega})$ as imaginary part, also used in the second notation for the amplitude spectrum $|X(e^{j\omega})| = \sqrt{X^2_{Re}(e^{j\omega}) + X^2_{Im}(e^{j\omega})}$ and the phase spectrum $\arg X(e^{j\omega}) = \arctan \{X_{Im}(e^{j\omega})/X_{Re}(e^{j\omega})\}$. A further important property of the Fourier transform is its shifting theorem, which means, that the transform of any shifted sequence will be: $\sum_{m=0}^{M-1} x(m-k)e^{-jm\omega} = e^{-jk\omega}X(e^{j\omega})$ (V. Cappellini/A. G. Constantinides/P. Emiliani 1978, p. 16). With all these considerations in mind the Fourier transform of (1-5) is given by

$$\sum_{m=0}^{M=1} y(m)e^{-jm\omega} = \sum_{k=0}^{M-1} h(k) \left[\sum_{m=0}^{M-1} x(m-k)e^{-jm\omega} \right]$$

$$\Longrightarrow Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega}) \tag{1-9}$$

and it just shows that convolution in the time domain is converted to multiplication in the frequency domain. The function $H(e^{j\omega})$ is in general called 'transfer function', because the system (filter) behaviour is transferred into the variable ω; however since ω is frequency variable we will call it 'frequency response', then it is just the response of the LTI system or filter in the frequency domain.

In surveying engineering we can classify our measurements into deterministic signals or statistical signals, respectively; the first ones are to describe completely by sinusoidal functions (Fourier series) and the latter ones are only partly or not at all to describe by those functions but known in the statistical sense with given moments. Therefore one needs frequency responses which are taking into account such a separation, as result we have both: deterministic and statistical frequency responses. Another desired goal in the filtering process will be the preservation of the phase information of the signal to be filtered, what is resulting into pure real and even frequency responses.

At this point we are in a position to determine digital filters with ideal deterministic frequency responses directly; these are known as lowpass-, highpass-, bandpass- or bandstop-filters (see Figure 4), depending on the task the filter has to fulfill, namely to let unchanged that part of the amplitude spectrum of the signal to be filtered we are interested in.

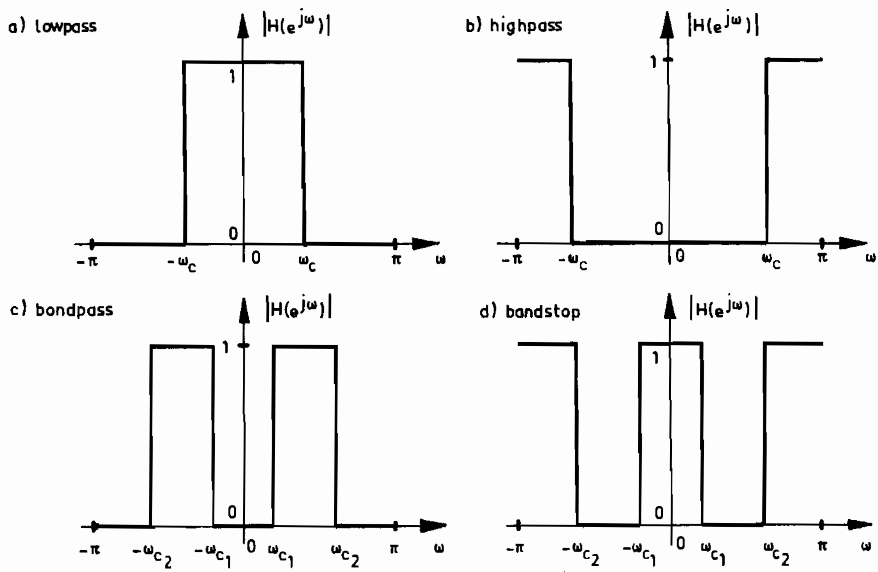

Figure 4: Ideal deterministic frequency responses of digital filters

There are also possibilities to have multiple bandpass- and bandstop filters as it is necessary to pass or stop different frequency bands in one filtering process. That means there exist passbands and stopbands bounded in each case by a cut-off frequency ω_{c_i} with the following constraints

$$\begin{aligned} &\text{passband: } |H(e^{j\omega})| = 1 \\ &\text{stopband: } |H(e^{j\omega})| = 0 \end{aligned} \tag{1-10}$$

For the derivation of statistical frequency responses we must have at first a look at the signal to be filtered, which will be

$$x(m) = y(m) + r(m) \tag{1-11}$$

with $x(m)$ which contains our measurements disturbed by any errors or noise $r(m)$, and $y(m)$ as faultless or true signal what we are searching for. In this case the digital filter has to provide for an estimate $\hat{y}(m)$ of $y(m)$, where the Wiener filter will be used; as optimality criterion it minimizes the variance σ^2 of the estimation error $\varepsilon(m) = y(m) - \hat{y}(m)$ defined with the expectation operator E as

$$\sigma^2 = E\left[\{\varepsilon(m) - E[\varepsilon(m)]\}^2\right] = \min \tag{1-12}$$

For stationary $x(m)$ and the mean values $E[y(m)] = 0$, $E[r(m)] = 0$ as well as $E[\varepsilon(m)] = 0$ the solution of (1-12) results into the discrete form of the famous Wiener-Hopf integral equation (D. Fritsch, 1982)

$$R_{yx}(\kappa) = \sum_{k=0}^{M-1} h(k)\, R_{xx}(\kappa-k) = h(\kappa) * R_{xx}(\kappa) \tag{1-13}$$

in which it is denoted by $R_{yx}(\kappa)$ the crosscorrelation function between $y(m)$ and $x(m)$ with correlation lag $\kappa \in \{0,1,2,....,M-1\}$, $R_{xx}(\kappa)$ the autocorrelation function of $x(m)$ and $h(\kappa)$ the impulse response of the Wiener filter. This convolution could be solved as linear equation system, but for larger data sets the inversion of the autocorrelation matrix $\underline{R}_{xx}$ may cause some trouble if it has not an special structure as Toeplitz matrices have. But by taking the Fourier transform on both sides of (1-13) the convolution sum is transferred into multiplication of

$$S_{yx}(e^{j\omega}) = H(e^{j\omega})\, S_{xx}(e^{j\omega}) \tag{1-14}$$

with $S_{yx}(e^{j\omega})$ and $S_{xx}(e^{j\omega})$ as just the power spectra and $H(e^{j\omega})$ the frequency response of the Wiener filter.

Are the signal sequence $y(m)$ and the noise sequence $r(m)$ not correlated with each other, as it can be usually presupposed in surveying engineering, the following relation for the frequency response $H(e^{j\omega})$ will be obtained

$$H(e^{j\omega}) = \frac{S_{yy}(e^{j\omega})}{S_{yy}(e^{j\omega}) + S_{rr}(e^{j\omega})} \tag{1-15}$$

because then simplifications are valid in such a way that $R_{yx}(\kappa) = R_{yy}(\kappa)$ and $R_{xx}(\kappa) = R_{yy}(\kappa) + R_{rr}(\kappa)$ with its appropriate power spectra $S_{yy}(e^{j\omega})$ and $S_{rr}(e^{j\omega})$.

With all these considerations in mind one has to decide about basic assumptions on necessary correlation functions by which the Wiener filter is defined. On the one hand it is likely realistic to consider our noise process as white noise because the measurement errors are independent from each other; but on the other hand one could also work with colored noise processes if this basic assumption is not true. Let us suppose a white noise process r(m), then its correlation function will be

$$R_{rr}(\kappa) = \sigma_r^2 d(\kappa) \tag{1-16}$$

with σ_r^2 as variance of r(m) and the unit sample $d(\kappa)$; the power spectra of this correlation function is constant over the whole frequency domain with

$$S_{rr}(e^{j\omega}) = \sigma_r^2. \tag{1-17}$$

For the true signal y(m) some correlation functions may be introduced with decreasing behaviours for increasing correlation lags. Such a correlation function was derived in D. Fritsch (1982)

$$R_{yy}(\kappa) = \sigma_y^2 \alpha^{|\kappa|} \quad \text{for } 0 \leq \alpha < 1 \tag{1-18}$$

just as result of a 1st. order Markov process for our true signal y(m); the power spectra of this correlation function is given by

$$S_{yy}(e^{j\omega}) = - \frac{2\sigma_y^2 \ln\alpha}{(\ln\alpha)^2 + \omega^2} \tag{1-19}$$

With the power spectra (1-17) and (1-19) the frequency response of the Wiener filter will be

$$H(e^{j\omega}) = \frac{1}{1 - \dfrac{\sigma_r^2\{(\ln\alpha)^2 + \omega^2\}}{2\sigma_y^2 \ln\alpha}} \qquad (1\text{-}20)$$

which can be interpreted as optimal lowpass filter for $\sigma_y^2 > \sigma_r^2$, what is predominantly the case in surveying engineering. Another digital filter for an optimal filtering of noisy observations was introduced by K. R. Koch (1975), where the digital version of the frequency response of a Butterworth filter was used to approximate an optimal filter.

All our optimal filters described here have to be fitted onto the data to be filtered and thus for given variances σ_y^2 and σ_r^2 one needs the interdependence of the signal y(m) which one can only estimate if y(m) has the ergodic property in addition to being stationary.

DIGITAL FILTER DESIGN

The design problem of digital filters consists of approximations of ideal frequency responses previously introduced, but before we will solve these approximation problems the functional relations between the impulse responses and the ideal frequency responses of digital filters must be given in more detail.

Since digital filters can be seen as LTI systems the following difference equation is valid

$$\sum_{k=0}^{K-1} a(k)x(m-k) = \sum_{l=0}^{L-1} b(l)y(m-l) \qquad (2\text{-}1)$$

it delivers with b(0) := 1 and b(k) := 0 for k $\neq$ 0 as well as for h(k) = a(k) the nonrecursive digital filtering algorithm or convolution

$$y(m) = \sum_{k=0}^{K-1} h(k)x(m-k) \qquad (2\text{-}2)$$

with K < M, and this equation just shows that the convolution process (1-5) is reduced into fewer summations. Because of this finite summation the digital filter is called 'finite impulse response filter' (FIR filter) in contrast to an 'infinite impulse response filter' (IIR filter), where the convolution sum tends to infinity and therefore filter design and implementation have to be done otherwise as demonstrated later on.

A further important point in filter design is the linear phase behaviour of the FIR filter, which means that for any frequency response $H(e^{j\omega})$, defined as

$$H(e^{j\omega}) = |H(e^{j\omega})| e^{-j\varphi\omega} = \sum_{k=0}^{K-1} h(k) e^{-jk\omega} \tag{2-3}$$

one is searching for the linear phase shift φ with result (D. Fritsch, 1982)

$$\varphi = \frac{K-1}{2} \tag{2-4}$$

$$h(k) = h(K-1-k) \tag{2-5}$$

that is the impulse response of a linear phase FIR filter has to be a symmetrical one around its center $\varphi = \frac{K-1}{2}$.

With these two constraints in mind as relations between the impulse response and the frequency response of a FIR filter with linear phase will be obtained (L. R. Rabiner/B. Gold, 1975, E. U. Fischer/H. Friedsam, 1977) in the K odd case

$$H(e^{j\omega}) = e^{-j\left(\frac{K-1}{2}\right)\omega} \left\{ \sum_{k=0}^{(K-1)/2} a(k) \cos(k\omega) \right\} \tag{2-6a}$$

or in the K even case

$$H(e^{j\omega}) = e^{-j\left(\frac{K-1}{2}\right)\omega} \left\{ \sum_{k=1}^{K/2} b(k) \cos\left[\left(K-\frac{1}{2}\right)\omega\right] \right\} \tag{2-6b}$$

with the simple substitutions a(k) and b(k) for the impulse response h(k) in such a way that it is holding

$$a(0) = h\left(\frac{K-1}{2}\right), \quad a(k) = 2h\left(\frac{K-1}{2} \pm k\right) \quad \text{for } k \in \{0,1,\ldots,\frac{K-1}{2}\} \tag{2-7a}$$

$$b(k) = 2h\left(\frac{K}{2} - k\right) \qquad \text{for } k \in \{1,2,\ldots,\frac{K}{2}\} \; . \tag{2-7b}$$

As it can be seen by (2-3) and (2-4) the bracket terms in (2-6) are the relations what we were searching for, so that for the pure real and even ideal frequency responses previously mentioned ω can be bounded on $\omega \in [0,\pi]$.

For the approximation of ideal deterministic frequency responses there is a need to have so-called 'transition bands' because of the Gibbs phenomenon, by which it is stated, that truncation of Fourier series for a rectangular function leads to a fixed-percentage overshoot in the approximation. That is the reason for the following constraints

$$\text{passband: } |H(e^{j\omega})| = 1$$
$$\text{stopband: } |H(e^{j\omega})| = 0 \qquad (2\text{-}8)$$

transition band: increasing (decreasing) behaviour

in contrast to the statistical frequency responses which are being approximated directly.

For a least squares approximation we have to minimize the total mean square error σ_t^2, defined by

$$\sigma_t^2 = \int_0^{\pi} [\,|H(e^{j\omega})| - |H^*(e^{j\omega})|\,]^2 d\omega = \min \qquad (2\text{-}9)$$

with $|H(e^{j\omega})|$ as ideal and $|H^*(e^{j\omega})|$ as approximated frequency response; the numerical evaluation can only be done by sampling of the frequency domain. This sampling process for deterministic frequency responses must take place in the pass- and stopbands only, whereas for statistical frequency responses the whole frequency domain has to be sampled (see Figure 5).

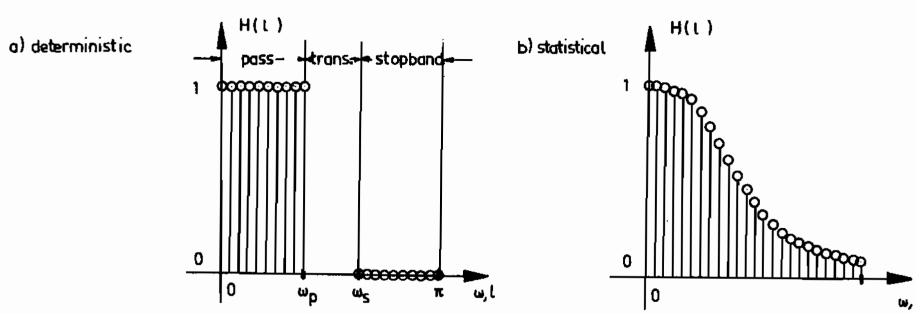

Figure 5: Sampling of lowpass-filters in the frequency domain

Let H(l) for $l \in \{0,1,2,\ldots,L-1\}$ be the sampled value of the frequency response for the sampling point $\omega_1 = 1\omega/(L-1)$ and $H^*(1)$ the value being approximated, as approximation error we can define

$$e(1) = H(1) - H^*(1) \tag{2-10}$$

so that with the functional relations (2-6) in mind the following model is valid for all sampling points in the frequency domain

$$\underline{e} = \underline{y} - \underline{X}\,\underline{\beta} \tag{2-11}$$

in which it is denoted by the Lxu Matrix $\underline{X}$ with $rk(\underline{X}) = u$ all cosine coefficients given on the sampling points, the ux1 vector $\underline{\beta}$ contains the unknown filter coefficients a(k) or b(k), respectively, the Lx1 vector y is standing for all values of the frequency response being sampled and $\underline{e}$ is the Lx1 vector of approximation errors.

The minimization of the quadratic form of $\underline{e}$ leads to the well-known normal equations in least-squares

$$\underline{X}'\underline{X}\,\hat{\underline{\beta}} = \underline{X}'\underline{y} \tag{2-12}$$

with its estimates $\hat{\underline{\beta}}$ of the unknown filter coefficients and the vector $\hat{\underline{e}}$ of approximation errors given by

$$\hat{\underline{\beta}} = (\underline{X}'\underline{X})^{-1}\underline{X}'\underline{y} \tag{2-13}$$

$$\hat{\underline{e}} = \underline{y} - \underline{X}\,\hat{\underline{\beta}} \tag{2-14}$$

which are influenced unimportantly by the density of our sampling points. As goodness of fit for the approximation we should use the maximum approximation error to define for both the passbands and stopbands in deterministic filter design as $\hat{\delta}_P = \max |\hat{e}_{P_i}|$ and $\hat{\delta}_S = \max |\hat{e}_{S_i}|$, whereby $\hat{\underline{e}}$ is to be separated into $\hat{\underline{e}}_P$ and $\hat{\underline{e}}_S$ or in statistical filter design the maximum approximation error will be : $\hat{\delta} = \max |\hat{e}_i|$, instead of the mean square error as it is usually the case in surveying engineering.

Maximum values $\hat{\delta}_P$ and $\hat{\delta}_S$ we will have at the edges of the passbands and stopbands, so that on the one hand wider transition bands will provide for an improvement of these values or on the other hand more filter coefficients also results into better approximations, but one has to keep in mind that the longer the impulse response will be the longer computation time is needed for our filtering process.

DIGITAL FILTER IMPLEMENTATION

In order to carry out the actual filtering operation itself let us take the convolution sum (2-2), which defines the filter, so that with the informations (2-5) about the symmetries of the impulse response the following convolutions will be obtained, both for the K odd case

$$y(m) = h\left(\frac{K-1}{2}\right)x\left(m-\frac{K-1}{2}\right) + \sum_{k=1}^{K-1/2} h\left(\frac{K-1}{2} \pm k\right)\left[x\left(m-\frac{K-1}{2}-k\right)+x\left(m-\frac{K-1}{2}+k\right)\right] \quad (4\text{-}1)$$

and for K even

$$y(m) = \sum_{k=o}^{K/2-1} h(k)\left[x(m-k) + x(m-K+1+k)\right]. \quad (4\text{-}2)$$

Now, let us see what happens, if these convolution sums define also the filtering process. With our pure real frequency responses previously introduced, which are to describe with $H(e^{j\omega}) = \bar{H}(\omega)$, the linear phase behaviour of a FIR filter is given by

$$H(e^{j\omega}) = \bar{H}(\omega)e^{-j\left(\frac{K-1}{2}\right)\omega} \quad (4\text{-}3)$$

and with (1-9) the filtering operation in the frequency domain will be

$$\begin{aligned} Y(e^{j\omega}) &= e^{-j\left(\frac{K-1}{2}\right)\omega}\bar{H}(\omega)X(e^{j\omega}) \\ &= e^{-j\left(\frac{K-1}{2}\right)\omega}\bar{Y}(e^{j\omega}) \end{aligned} \quad (4\text{-}4)$$

whereby $\bar{Y}(e^{j\omega})$ is the Fourier transform of those filtered values what we are searching for. Equation (4-4) shows that the linear phase FIR filter provides for a phase shift in the frequency domain and with the shifting theorem of the Fourier fransform an equivalent expression of (4-4) in the time domain is given by

$$y(m) = \bar{y}\left(m - \frac{K-1}{2}\right) \qquad (4\text{-}5)$$

with $\bar{y}(m)$ as zero phase shift filtered value; it also shows that zero phase will be obtained by shifting of all our filtered values y(m) by (K-1)/2 samples. Furthermore, as one can easily see, an even length of the impulse response of the filter delivers zero phase shift filtered values not defined on the sampling points but in the midst of it.

By substitution for k=k-(K-1)/2 the zero phase shift impulse response $\bar{h}(k)$ is obtained, since the following relation is valid

$$H(e^{j\omega}) = H(\omega)e^{-j\left(\frac{K-1}{2}\right)\omega} = \sum_{k=-(K-1)/2}^{(K-1)/2} h(k)e^{-j\left(k+\frac{K-1}{2}\right)\omega}$$

$$= e^{-j\left(\frac{K-1}{2}\right)\omega} \sum_{k=-(K-1)/2}^{(K-1)/2} \bar{h}(k)e^{-jk\omega} \qquad (4\text{-}6)$$

and for this zero phase shift impulse response the convolution sum (4-1) can be rewritten in

$$\bar{y}(m) = \bar{h}(0)x(m) + \sum_{k=1}^{(K-1)/2} \bar{h}(k)[x(m-k) + x(m+k)] \qquad (4\text{-}7)$$

which shows, that the filter needs (K-1)/2 samples in order to swing in and out at the beginning and the end, respectively, of the data definition.

For longer sequences to be filtered there is another implementation method, called 'fast convolution', in contrast to the convolution sums previously defined, which are called 'slow convolution' or 'direct convolution'. The fast convolution algorithms are based on the presentation of the filtering operation in the frequency domain, therefore (1-9) has to be rewritten into a numerical form

$$Y(k) = H(k)X(k) \qquad \text{for } k \in \{0,1,2,\dots,M-1\} \qquad (4\text{-}8)$$

whereby these discrete sequences will be obtained by taking the discrete Fourier transform (DFT) of the sequences y(m), h(m) and x(m), which still has to be defined.

Let x(m) for $m \in \{0,1,2,\dots,M-1\}$ be a discrete sequence, then its DFT is defined by

$$X(k) = \sum_{m=0}^{M-1} x(m) e^{-j2\pi km/M} \qquad \text{for } k \in \{0,1,2,\ldots,M-1\} \tag{4-9a}$$

and also the inverse DFT (IDFT) can be given as

$$x(m) = \frac{1}{M} \sum_{k=0}^{M-1} X(k) e^{j2\pi km/M} \tag{4-9b}$$

(L. R. Rabiner/B. Gold, 1978, p. 448), in which the frequency variable ω is sampled at $\omega_k = 2\pi k/M$.

However, corresponding to the definition of (4-8) all discrete frequency sequences have to be of the same length, but unfortunately this is not the case for our data sequences, where $k \in \{0,1,2,\ldots,K-1\}$ and $m \in \{0,1,2,\ldots,M-1\}$ with $K < M$. This problem is to be overcomed by adding of zeros to the original data sequences so that we will have sequences of length

$$\begin{aligned}
\tilde{h}(m) &:= h(k) && \text{for } m \in \{0,1,2,\ldots,K-1\} \\
\tilde{h}(m) &:= 0 && \text{for } m \in \{K,K+1,\ldots,K+M-2\} \\
\tilde{x}(m) &:= x(m) && \text{for } m \in \{0,1,2,\ldots,M-1\} \\
\tilde{x}(m) &:= 0 && \text{for } m \in \{M,M+1,\ldots,K+M-2\}.
\end{aligned} \tag{4-10}$$

The computation of the DFT of $\tilde{h}(m)$ and $\tilde{x}(m)$ can be reduced significantly if the length of both sequences is a power of 2, because then algorithms of the fast Fourier transform (FFT) can be used (S. D. Stearns, 1975, p. 74); that is the reason why it is called 'fast convolution' and therefore the length has to be : $L = 2^l \geq K+M-1$. The filtered sequence $y(m)$ is contained in the sequence $\tilde{y}(m)$ and to recover by

$$y(m) := \tilde{y}(m) \qquad \text{for } m \in \{0,1,2,\ldots,K+M-2\} \tag{4-11}$$

where $\tilde{y}(m)$ is being contained by the IDFT of $Y(k)$.

EXAMPLE

For the control of a brown coal opencast mining distances have been measured to different time epochs between fixed points and endpoints of slopes, where changes are to be expected. These measurements were performed in different time intervals, but for most of the measurements an interval of 7 [days] was used. In Figure 6a the measurements of one distance are sketched; these measurements have been interpolated for a digital filtering operation to a regular data sequence at a sampling interval of 7 [days] (see Figure 6b).

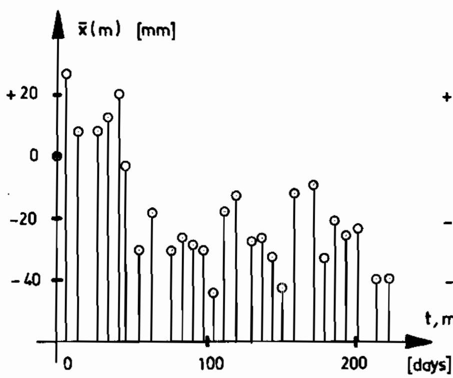

Figure 6a: Original data sequence $\bar{x}(m)$

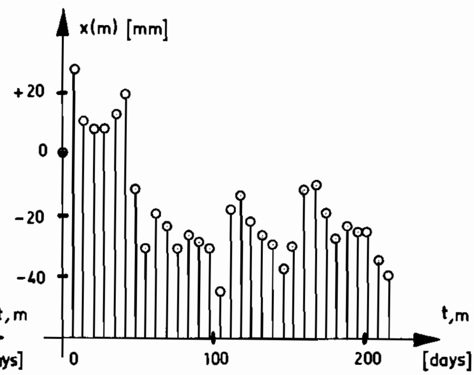

Figure 6b: Equidistant data sequence $x(m)$

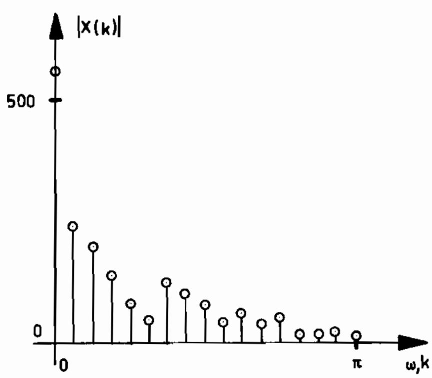

Figure 7: Spectral sequence X(k)

The DFT of the sequence $x(m)$ delivers the spectral sequence $|X(k)|$, where because of the periodicity only the interval $\omega \in [o,\pi]$ has to be considered (see Figure 7).

It is desired to eliminate all higher fluctuations within the data sequence $x(m)$ with frequencies $\omega \geq 0{,}33\pi$. For solving of this problem a lowpass filter has been designed with specifications : $\omega_P = 0{,}26\pi$, $\omega_S = 0{,}40\pi$ and $N = 9$, so that the results of the filtering process obtained by the slow convolution can be seen in Figure 8.

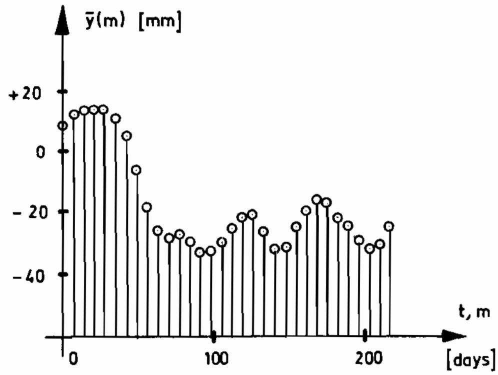

Figure 8: Zero phase filtered data sequence $\bar{y}(m)$

CONCLUSIONS

In this paper the concepts of one-dimensional digital filtering were introduced and explained in detail. It was shown that digital filtering is an appropriate tool to eliminate any frequencies within our data sequences we are not interested in or to eliminate any noise within our observations. Furthermore the digital filtering concepts were demonstrated by an example of real deformation data. A recommendation for all users of digital filtering processes in surveying engineering may be to perform the measurements in equidistant time intervals, if it is possible, in order to prevent any interpolation method.

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SUMMARY

In this paper the concepts of one-dimensional digital filtering are treated. After the introduction of frequency responses for deterministic and statistical signals the design problem is solved by the method of least squares, widely used in surveying engineering for any estimations of parameters and approximations. Also the filter implementation is described with special emphasis on the discrete Fourier transform, which is also needed for informations about frequency contents of signals to be filtered. An example of a digital filtering operation of real deformation data shows the ability of these concepts.

ZUSAMMENFASSUNG

In diesem Beitrag werden die Verfahren des digitalen Filterns aufgezeigt. Nach der Einführung von Frequenzantworten für deterministische und statistische Signale wird das Entwurfsproblem unter Zuhilfenahme der Methode der kleinsten Quadrate gelöst, die in der Ingenieurvermessung vielfach für Parameterschätzungen und Approximationen Verwendung findet. Des weiteren ist die eigentliche Filteroperation beschrieben, bekannt als 'Implementierung'; besonderer Wert wird hier der diskreten Fouriertransformation beigemessen, da diese ebenso den Frequenzinhalt des zu filternden Signals aufzeigt. Ein Beispiel der Filterung von reellen Deformationsdaten soll die Fähigkeit dieser Konzepte demonstrieren.