PHOTOGRAMMETRY AS A TOOL FOR DETECTING RECENT CRUSTAL MOVEMENTS

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ABSTRACT

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Progress in model building and in advanced exposure and evaluation methods is leading photogrammetry up into the field of precise point determination, up to now solved by classical geodetic triangulation and/or trilateration. In combination with the statistical test theory, photogrammetry can be used as a tool to detect recent crustal movements or deformations of man-made constructions, such as in coal mining or oil and gas exploitation.

For these reasons, this paper deals with model building for modern aerial triangulation with respect to hypothesis tests to detect point displacements greater than the measurement noise. An example from brown-coal mining demonstrates accuracies and the application of some statistical tests shows the efficiency of advanced aerial photogrammetry.

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INTRODUCTION

The application of aerial triangulation dates back more than 50 years. Although its conventional scope was medium-scale topographic mapping, progress in model building within the last two decades has brought photogrammetry up into the field of precise point determination. In the beginning, analytical solutions in aerial photogrammetry reached the 10 (cm) level and somewhat better (Ackermann, 1968), but nowadays analytical bundle adjustments with additional parameters could increase the accuracy to the 1 (cm) level (Grün, 1982), a constant image scale of 1 : 5000 being supposed. Future model refinements will also provide for an increase of accuracy, so that the precision of aerial photogrammetry may be sufficient to determine recent crustal movements or movements in anthropogenously influenced areas, e.g. in mining or oil and gas exploitation areas.

The progress in accuracy has been achieved primarily by the introduction of additional parameters to compensate systematic errors, also called "self-calibration" (Ebner, 1976), and secondly, by changing the model for block adjustment, resulting in bundle adjustment with self-calibration. Obviously, it is not so simple to make a proper choice of additional parameters for self-calibration that are purely deterministic; therefore, one has to think of a combination between deterministic and random parameters (Koch, 1983a) or of refinements of dispersion components by estimating variance–covariance components (Förstner and Schroth, 1982), supplemental to a few deterministic parameters.

In order to detect displacements of points in moving areas, photogrammetric images or photograms fix the area being controlled to a certain epoch i, with i as repetition number going from $i \in \{1, 2, \ldots\}$. The bundle adjustment estimates coordinates of the points to be analysed for every epoch i and delivers, if necessary, estimates for dispersion components of the estimated quantities. Let us consider the estimated coordinates as signals and dispersion components as noise, the decision about displacements depends on the signal/noise ratio (SNR) in connection with given statistics, which means percentage points or fractils of any distribution. The following approach to detecting displacements of points by means of photogrammetry will first introduce the models and secondly the statistical tests prepared especially for photogrammetric point determination.

MODELS FOR PHOTOGRAMMETRIC POINT DETERMINATION

Let us start from the central-perspective image transformation between the space of photograms and the object space given by Schwidefsky and Ackermann (1976):

$$\begin{bmatrix} x_i - x_0 \\ y_i - y_0 \\ -c \end{bmatrix} = s\boldsymbol{R} \begin{bmatrix} X_i - X_0 \\ Y_i - Y_0 \\ Z_i - Z_0 \end{bmatrix} \tag{1}$$

with x_i, y_i $\forall i = 1, 2, \ldots$ as coordinates of the photogram for point P_i, c as calibrated focal length and x_0, y_0 as coordinates of the principal point; the right-hand side consists of scale factor s, the rotation matrix $\boldsymbol{R}$ and the coordinates X_i, Y_i, Z_i $\forall i = 1, 2, \ldots$ of the object points P_i, as well as the coordinates X_0, Y_0, Z_0 of the projection centre. If the scale factor s is resolved and substituted, we get the well-known observation functionals:

$$\left.\begin{aligned} x_i - x_0 &= -c\frac{r_{11}(X_i - X_0) + r_{12}(Y_i - Y_0) + r_{13}(Z_i - Z_0)}{r_{31}(X_i - X_0) + r_{32}(Y_i - Y_0) + r_{33}(Z_i - Z_0)} \\ y_i - y_0 &= -c\frac{r_{21}(X_i - X_0) + r_{22}(Y_i - Y_0) + r_{23}(Z_i - Z_0)}{r_{31}(X_i - X_0) + r_{32}(Y_i - Y_0) + r_{33}(Z_i - Z_0)} \end{aligned}\right| \tag{2}$$

The rotation matrix $\boldsymbol{R} = (r_{ij})$ $\forall i, j = 1, 2, 3$ may be the Rodriques representation $\boldsymbol{R} = f(q_0, q_1, q_2, q_3)$ with q_0, q_1, q_2, q_3 as coordinates of a quaternion (Grafarend, 1983) or consisting of the traditionally used rotations by the Cardan axes ϕ, ω, κ.

Let us assume a given interior orientation c, x_0, y_0, so that the parameters of the exterior orientation X_0, Y_0, Z_0, ϕ, ω, κ, as well as the object coordinates X_i, Y_i, Z_i, have to be estimated by the observation functionals (2). These observation functionals are invariant against S-transformations within the $\mathscr{R}^3$, which means the photogrammetric point manifold can be translated, rotated and scaled without changing the observations. The number of non-estimatable parameters will be 7, i.e., 3 translations, 3 rotations and 1 scale factor, which define the datum of the point manifold. Traditionally, the datum problem is solved in photogrammetry with the introduction of coordinates of control points, whose number is greater than the rank or datum deficiency. But in detecting recent crustal movements, the datum should be defined by the statistical analysis of the data, because also control points themselves may have moved.

For that reason, the following models will consider the requirements of modern parameter estimation to define also the final datum for fixing the point manifold within the parameter space. The model building will be restricted to be static and not kinematic between the exposure epochs. Furthermore, for reasons of simplicity, only models for pairwise different epochs will be given.

(i) Let us assume linearization of the observation functionals for the introduction of the first linear model:

$$E(y_{ij}) := E\left(\begin{bmatrix} y_i \\ y_j \end{bmatrix}\right) = \begin{bmatrix} Z_i & 0 \\ 0 & Z_j \end{bmatrix}\begin{bmatrix} x_i \\ x_j \end{bmatrix} =: Z_{ij}x_{ij} \tag{3a}$$

$$D(y_{ij}) := D\left(\begin{bmatrix} y_i \\ y_j \end{bmatrix}\right) = \begin{bmatrix} \sigma_i^2 & 0 \\ 0 & \sigma_j^2 \end{bmatrix} \boxdot \begin{bmatrix} Q_{ii} & 0 \\ 0 & Q_{jj} \end{bmatrix} =$$

$$= \begin{bmatrix} \sigma_i^2 Q_{ii} & 0 \\ 0 & \sigma_j^2 Q_{jj} \end{bmatrix} =: \Sigma_{ij} \boxdot Q_{ij} \tag{3b}$$

which we will call a sequential univariate Gauss-Markov model; for photogrammetric purposes the first-order model will be split up into (Fritsch et al., 1984):

$$Z_k := [A_1, A_2, A_3] \tag{4a}$$

$$x'_k = [x'_1, x'_2, x'_3] \qquad \forall k = i, j \tag{4b}$$

with the $r \times 1$ vector x_1 of unknown parameters of points being analysed, the $s \times 1$ vector x_2 of exterior orientation parameters and the $t \times 1$ vector x_3 of additional parameters for self-calibration or other information to be considered within the parameter estimation. The $n_k \times u_k$ matrix of coefficients Z_k is not of full rank, this means rk $Z_k = q_k < u_k$, if no control points are introduced. The operators E and D characterize expectation and dispersion, respectively, of the observation vector y_k; its second-order model will be described by the unknown variance component of unit weight σ_k^2 in connection with the $n_k \times u_k$ positive definite cofactor or weight matrix Q_{kk} of the observations, represented as a Block-Hadamard product between the 2×2 matrix Σ_{ij} and Q_{ii} as well as Q_{jj}, whereas the Block-Hadamard product is commonly defined as

$$\begin{bmatrix} G_{11} & \cdots & G_{1k} \\ & \vdots & \\ G_{j1} & \cdots & G_{jk} \end{bmatrix} \boxdot \begin{bmatrix} H_{11} & \cdots & H_{1k} \\ & \vdots & \\ H_{j1} & \cdots & H_{jk} \end{bmatrix} := \begin{bmatrix} G_{11} \otimes H_{11} & \cdots & G_{1k} \otimes H_{1k} \\ & \vdots & \\ G_{j1} \otimes H_{j1} & \cdots & G_{jk} \otimes H_{jk} \end{bmatrix} \tag{5}$$

with $\otimes$ as Kronecker product:

$$G \otimes H := \left[g_{ij} H\right] \tag{6}$$

The general solution by least squares leads to partly MINOLESS-type estimates (MInimum NOrm LEast Squares Solution) (Schaffrin, 1975; Koch, 1980):

$$\hat{x}_k = \left(Z_k' Q_{kk}^{-1} Z_k\right)^{-} Z_k' Q_{kk}^{-1} y_k \tag{7a}$$

$$D(\hat{x}_k) = \sigma_k^2 \left(Z_k' Q_{kk}^{-1} Z_k\right)_{rs}^{-} \tag{7b}$$

$$\hat{\sigma}^2 = \hat{v}_k' Q_{kk}^{-1} \hat{v}_k / (n_k - q_k) \tag{7c}$$

with v_k as the least squares residual vector. Like (3), the model with its solutions (7) is currently in use in photogrammetry, however, in fixing the datum a priori by introduction of coordinates of control points as observational equations, whose number is greater than the datum deficiency. Thus, there exists no possibility of varying the datum a posteriori, which should also be done in detecting displacements of control points.

(ii) In order to detect more sensitive displacements, Koch (1980, 1983a) introduces the following model:

$$E(y_{ij}) := D\left(\begin{bmatrix} y_i \\ y_j \end{bmatrix}\right) = \begin{bmatrix} Z & 0 \\ 0 & Z \end{bmatrix} \begin{bmatrix} x_i \\ x_j \end{bmatrix} =: Z_{ij} x_{ij} \tag{8a}$$

$$D(y_{ij}) := D\left(\begin{bmatrix} y_i \\ y_j \end{bmatrix}\right) = \begin{bmatrix} \sigma_i^2 & \sigma_{ij} \\ \sigma_{ij} & \sigma_j^2 \end{bmatrix} \Box \begin{bmatrix} Q & Q \\ Q & Q \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_i^2 Q & \sigma_{ij} Q \\ \sigma_{ij} Q & \sigma_j^2 Q \end{bmatrix} =: \Sigma_{ij} \Box \theta_{ij} \tag{8b}$$

which we will call a simple multivariate Gauss-Markov model, also to be written with $y_{ij} =: \mathrm{vec}\, Y$ and $x_{ij} =: \mathrm{vec}\, X$ as follows

$$E(Y) = ZX, \qquad D(\mathrm{vec}\, Y) = \textstyle\sum_{ij} \otimes Q \tag{9}$$

The $n \times u$ coefficient matrix Z must be the same for every exposure epoch; the same holds for the weight matrix Q but with the advantage of estimating the and the 3×1 vector of unknown variance-covariance components

$$\tau_{ij} := \left[\sigma_i^2, \sigma_{ij}, \sigma_j^2\right]' \tag{14}$$

we convert model (11) into the model of variance–covariance components:

$$E(y_{ij}) := Z_{ij} x_{ij}, \quad \mathrm{vec}\, D(y_{ij}) \doteq V_{ij} \tau_{ij} \tag{15}$$

where we can first estimate the variance-covariance components and secondly the unknown parameters.

After Schaffrin (1983), Fritsch and Schaffrin (1986), we have to solve the nonlinear equations:

$$T_{ij}\hat{\tau}_{ij} = q_{ij} \tag{16}$$

with:

$$T_{ij} := V'_{ij}\left(\hat{W}_{ij} \otimes \hat{W}_{ij}\right)V_{ij} \tag{17a}$$

$$q_{ij} := V'_{ij}\left(\hat{W}_{ij}P_{ij}y_{ij} \otimes \hat{W}_{ij}P_{ij}y_{ij}\right) \tag{17b}$$

$$P_{ij} := I - Z_{ij}\left(Z'_{ij}Z_{ij}\right)^{-}Z'_{ij} \tag{17c}$$

$$\hat{W}_{ij} := \hat{\hat{\theta}}_{ij}^{-1} - \hat{\hat{\theta}}_{ij}^{-1}Z_{ij}\left(Z'_{ij}\hat{\hat{\theta}}_{ij}^{-1}Z_{ij}\right)Z'_{ij}\hat{\hat{\theta}}_{ij}^{-1} = W'_{ij} \tag{17d}$$

and arive at the estimation:

$$\hat{\tau}_{ij} = T_{ij}^{-}q_{ij} = \left[V'_{ij}\left(\hat{W}_{ij} \otimes \hat{W}_{ij}\right)V_{ij}\right]^{-} V'_{ij}\left(\hat{W}_{ij}P_{ij}y_{ij} \otimes \hat{W}_{ij}P_{ij}y_{ij}\right) \tag{18}$$

which is called repro-BIQUAMBE (reproducing Best Invariant QUAdratic Minimum Biased Estimation (Schaffrin, 1983) if (18) is iterated onto $\hat{\tau}_{n+1} = \hat{\tau}_n$.

After the estimation of the variance–covariance components, follows the partly MINOLESS for the unknown parameters x_{ij}:

$$\hat{x}_{ij} = \left(Z'_{ij}\hat{\hat{\theta}}_{ij}^{-1}Z_{ij}\right)^{-} Z'_{ij}\hat{\hat{\theta}}_{ij}^{-1}y_{ij} \tag{19}$$

dependent on the previously estimated dispersion components. The generalization of the model (11) has been achieved at the cost of much computational expenditure in solving (16) to (18), therefore, further numerical experience is necessary for practical application of that model, especially in photogrammetry with its large number of observations.

DEFINITION OF THE DATUM

In detecting displacements of points in moving areas, only those points should define the datum which did not move significantly, i.e., whose movements have been caused by the measurement noise alone.

Traditionally in photogrammetry, the datum is defined by the coordinates of control points introduced as random information into the adjustment procedure covariance component between the epochs i and j. This covariance component is a number for homogeneous data acquisition in both observation epochs, i.e., the same flight disposition, the same region being controlled, the same device for obtaining photogram coordinates, and so on.

The partly MINOLESS estimate within the simple multivariate Gauss-Markov Model reads as follows:

$$\hat{X} = (Z'Q^{-1}Z)^{-}Z'Q^{-1}Y \tag{10a}$$

with its dispersion matrix:

$$D(\text{vec}\,\hat{X}) = \Sigma_{ij} \otimes (Z'QZ)^{-}_{rs} \tag{10b}$$

whose components can easily be estimated by:

$$\hat{\sigma}_k^2 = \hat{v}_k' Q^{-1}\hat{v}_k/(n-q) \qquad \forall k = i,\ j \tag{10c}$$

$$\hat{\sigma}_{ij} = \hat{v}_i' Q^{-1}\hat{v}_j/(n-q) \tag{10d}$$

The estimated parameters $\hat{X}$ are independent of the covariance component σ_{ij}, contrary to the following.

(iii) The more generalized model for the evaluation of repeated observations, has been introduced by Schaffrin (1981):

$$E(y_{ij}) := E\left(\begin{bmatrix} y_i \\ y_j \end{bmatrix}\right) = \begin{bmatrix} Z_i & 0 \\ 0 & Z_j \end{bmatrix}\begin{bmatrix} x_i \\ x_j \end{bmatrix} =: Z_{ij}x_{ij} \tag{11a}$$

$$D(y_{ij}) := D\left(\begin{bmatrix} y_i \\ y_j \end{bmatrix}\right) = \begin{bmatrix} \sigma_i^2 & \sigma_{ij} \\ \sigma_{ij} & \sigma_j^2 \end{bmatrix} \boxdot \begin{bmatrix} Q_{ii} & Q_{ij} \\ Q_{ij}' & Q_{jj} \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_i^2 Q_{ii} & \sigma_{ij}Q_{ij} \\ \sigma_{ij}Q_{ij} & \sigma_j^2 Q_{jj} \end{bmatrix} = \Sigma_{ij} \boxdot \theta_{ij} =: \bar{\theta}_{ij} \tag{11b}$$

It is called the incomplete multivariate Gauss-Markov model, and considers different point locations and cofactor matrices for epochs i and j, contrary to the above mentioned simple multivariate Gauss-Markov model.

By means of:

$$\text{vec}(\Sigma_{ij} \boxdot \theta_{ij}) = V_{ij}\tau_{ij} \tag{12}$$

with the $(n_i + n_j) \times 3$ matrices V_{ij} defined as follows:

$$V_{ij} := \left[\text{vec}\begin{bmatrix} Q_{ii} & 0 \\ 0 & 0 \end{bmatrix},\ \text{vec}\begin{bmatrix} 0 & Q_{ij} \\ Q_{ij}' & 0 \end{bmatrix},\ \text{vec}\begin{bmatrix} 0 & 0 \\ 0 & Q_{jj} \end{bmatrix}\right] \tag{13}$$

(Schmid, 1980) which is not the general solution for MINOLESS estimates. For this reason, the given information will be introduced as approximate values, that provide also for the determination of approximate values for the object coordinates being estimated. The final datum will be derived from a combination of some control points in connection with object points, whose movements have been statistically checked to be non-significant.

For presentation of the choice of the datum in photogrammetry, let us introduce the normal equation system resulting from (4) for the least-squares solution within the sequential univariate Gauss-Markov model

$$\begin{bmatrix} \boldsymbol{N}_{11k} & \boldsymbol{N}_{12k} & \boldsymbol{N}_{13k} \\ \boldsymbol{N}_{21k} & \boldsymbol{N}_{22k} & \boldsymbol{N}_{23k} \\ \boldsymbol{N}_{31k} & \boldsymbol{N}_{32k} & \boldsymbol{N}_{33k} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{x}}_{1k} \\ \hat{\boldsymbol{x}}_{2k} \\ \hat{\boldsymbol{x}}_{3k} \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}_{1k} \\ \boldsymbol{b}_{2k} \\ \boldsymbol{b}_{3k} \end{bmatrix}, \qquad \text{epoch } k \tag{20}$$

with $\boldsymbol{N}_{ijk} := \boldsymbol{A}'_{ijk}\boldsymbol{Q}_{kk}^{-1}\boldsymbol{A}_{ijk}$ and $\boldsymbol{b}_{ik} := \boldsymbol{A}'_i\boldsymbol{Q}_{kk}^{-1}y_k$, whereby $\boldsymbol{N}_{11}$, $\boldsymbol{N}_{33}$ are matrices of full rank and N_{22} is rank deficient. By means of the constraints:

$$[\boldsymbol{B}_1, \boldsymbol{B}_2, \boldsymbol{B}_3] \begin{bmatrix} \boldsymbol{x}_{1k} \\ \boldsymbol{x}_{2k} \\ \boldsymbol{x}_{3k} \end{bmatrix} = \boldsymbol{c} \tag{21}$$

the datum could be fixed, if the number of rows of $\boldsymbol{B} := [\boldsymbol{B}_1, \boldsymbol{B}_2, \boldsymbol{B}_3]$ is in accordance with the rank deficiency. Let the datum be defined by the point parameters only (Fritsch and Schaffrin, 1982), i.e., by a subgroup of coordinates which did not move between the two observation epochs:

$$||\boldsymbol{x}_{1k}||^2_{sg} = \min \tag{22}$$

which results in (Meissl, 1969):

$$[\boldsymbol{B}_1, \boldsymbol{0}, \boldsymbol{0}] \begin{bmatrix} \boldsymbol{x}_{1k} \\ \boldsymbol{x}_{2k} \\ \boldsymbol{x}_{3k} \end{bmatrix} = \boldsymbol{0} \tag{23}$$

The ordering scheme of the point parameters should be such that control points are standing first. If the number of the datum deficiency is $u_k - q_k$, the initial datum can be fixed by setting the first $u_k - q_k$ parameters to zero. Nowadays, all bundle adjustments use the information on control points as observational equations with appropriate weights, so that the procedure above can be easily implemented by using as many observational equations as the rank deficiency will be, with appropriate weights going on infinity.

The final point parameters result from an S-transformation (Baarda, 1973):

$$\hat{\boldsymbol{x}}_{1kb} = \boldsymbol{T}_{1kb}\hat{\boldsymbol{x}}_{1kc} \tag{24}$$

with $\hat{\boldsymbol{x}}_{1kc}$ as initial solution of the point parameters and $\boldsymbol{T}_{1kb}$ as the upper symmetric part of order $s \times s$ from the S-transformation matrix:

$$\boldsymbol{T}_{kb} = \boldsymbol{I} - \begin{bmatrix} \boldsymbol{E}'_{1k} \\ \boldsymbol{E}'_{2k} \\ \boldsymbol{E}'_{3k} \end{bmatrix} \left([\boldsymbol{B}_1, \boldsymbol{0}, \boldsymbol{0}] \begin{bmatrix} \boldsymbol{E}'_{1k} \\ \boldsymbol{E}'_{2k} \\ \boldsymbol{E}'_{3k} \end{bmatrix} \right)^{-1} [\boldsymbol{B}_1, \boldsymbol{0}, \boldsymbol{0}] =$$

$$= \begin{bmatrix} \boldsymbol{I} - \boldsymbol{E}'_{1k} (\boldsymbol{B}_1\boldsymbol{E}'_{1k})^{-1}\boldsymbol{B}_1 & \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{E}'_{2k} (\boldsymbol{B}_1\boldsymbol{E}'_{1k})^{-1}\boldsymbol{B}_1 & \boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{E}'_{3k} (\boldsymbol{B}_1\boldsymbol{E}'_{1k})^{-1}\boldsymbol{B}_1 & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} = \begin{bmatrix} \boldsymbol{T}_{1bk} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{T}_{2bk} & \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{T}_{3bk} & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \tag{25}$$

and $\boldsymbol{E}_{ik} := \mathcal{N}(Z_{ik})$ defined as null space of the coefficient matrices $\boldsymbol{Z}_{ik}$ $\forall i = 1, 2, 3$, given, for instance, for the point parameters by:

$$\boldsymbol{E}_{1k} := \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & \dots & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & \dots & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & \dots & 0 & 0 & 1 \\ 0 & -z_{10} & y_{10} & 0 & -z_{20} & y_{20} & \dots & 0 & -z_{s0} & y_{s0} \\ z_{10} & 0 & -x_{10} & z_{20} & 0 & -x_{20} & \dots & z_{s0} & 0 & -x_{s0} \\ -y_{10} & x_{10} & 0 & -y_{20} & x_{20} & 0 & \dots & -y_{s0} & x_{s0} & 0 \\ x_{10} & y_{10} & z_{10} & x_{20} & y_{20} & z_{20} & \dots & x_{s0} & y_{s0} & z_{s0} \end{bmatrix} \tag{26}$$

if a full rank deficiency of 7 has been supposed. The matrix $\boldsymbol{B}_1$ is obtained from $\boldsymbol{E}_{1k}$ by cancellation of any columns of $\boldsymbol{E}_{1k}$, i.e., it contains only coefficients x_{i0}, y_{i0}, z_{i0} and 1's of points i contributing to the datum of the photogrammetric network. It should be noted, that x_{i0}, y_{i0} and z_{i0} are approximate coordinates with regard to the center of gravity of the network.

The transformation of the cofactor matrix is somewhat more complicated. Let us introduce the symmetrical reflexive generalized inverse:

$$\boldsymbol{Q}_{kc} = \begin{bmatrix} \boldsymbol{0}_{u_k - q_k} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{N}_k^{-1} \end{bmatrix} \tag{27}$$

with $\boldsymbol{Q}_{kc}$ as inverse of the normal equation matrix (20), where the first $u_k - q_k$ parameters had been set to zero. After Pope (1973) and Koch (1983b), the transformation of the cofactor matrix:

$$Q_{kb} = T_{kb} Q_{kc} T'_{kb} \tag{28}$$

can be rewritten to:

$$Q_{kb} = Q_{kc} - Q_{kc} B'(E_k B')^{-1} E_k - E'_k (BE'_k)^{-1} BQ_c + E'_k (BE'_k)^{-1} BQ_c B'(E_k B')^{-1} E_k \tag{29}$$

and with (28) also as:

$$(BE'_k)^{-1} B = (B_1 E'_{1k})^{-1} B_1 = [H_1, \; F_1] \tag{30}$$

with $(u_k - q_k) \times (n_k - q_k)$ matrix H_1 and $(u_k - q_k) \times (s - u_k + q_k)$ matrix F_1, we arrive at the final expression:

$$Q_{kb} = Q_{kc} - \begin{bmatrix} 0 \\ N_k^{-1} F'_k \end{bmatrix} E_k - E'_k \left[0, \; F_1 N_k^{-1} \right] + E'_k F_1 N_k^{-1} F'_1 E_k \tag{31}$$

For computation of Q_{kc} and $N_k^{-1} F'_1$, the normal equations (20) will be extended by the matrix F'_1 as follows:

$$\begin{bmatrix} N_{11k}^{(0)} & N_{11k}^{(0)} & N_{12k}^{(0)} & N_{13k}^{(0)} \\ N_{11k}^{(0)} & N_{11k}^{(r)} & N_{12k}^{(r)} & N_{13k}^{(r)} \\ N_{21k}^{(0)} & N_{21k}^{(r)} & N_{22k} & N_{23k} \\ N_{31k}^{(0)} & N_{31k}^{(r)} & N_{32k} & N_{33k} \end{bmatrix} \begin{bmatrix} \hat{x}_{1k}^{(0)} \\ \hat{x}_{1k}^{(r)} \\ \hat{x}_{2k} \\ \hat{x}_{3k} \end{bmatrix} = \begin{bmatrix} b_{1k}^{(0)} & 0 \\ b_{1k}^{(r)} & F'_1 \\ b_{2k} & 0 \\ b_{3k} & 0 \end{bmatrix} \tag{32}$$

whereby the point parameters $\hat{x}_{1k}$ can be eliminated:

$$\begin{bmatrix} N_{22k} - N_{21k} N_{11k}^{-1} N_{12k}, & N_{23k} - N_{21k} N_{11k}^{-1} N_{13k} \\ N_{32k} - N_{31k} N_{11k}^{-1} N_{12k}, & N_{33k} - N_{31k} N_{11k}^{-1} N_{13k} \end{bmatrix} \begin{bmatrix} \hat{x}_{2k} \\ \hat{x}_{3k} \end{bmatrix} =$$

$$= \begin{bmatrix} b_{2k} - N_{21k} N_{11k}^{-1} b_{1k}, & -N_{21k} N_{11k}^{-1} F'_1 \\ b_{3k} - N_{31k} N_{11k}^{-1} b_{1k}, & -N_{31k} N_{11k}^{-1} F'_1 \end{bmatrix} \Rightarrow \tilde{N} \tilde{x} = [\tilde{b}, \; \tilde{F}'] \tag{33}$$

and it results by back substitution with $\tilde{x} = \tilde{N}^{-1} \tilde{b}$ that:

$$\hat{x}_{1k} = N_{11k}^{-1} b_{1k} - N_{12k} \hat{x}_{2k} - N_{13} \hat{x}_{3k} \tag{34}$$

and its corresponding cofactor matrix is given by:

$$Q_{11kc} = \begin{bmatrix} 0 & & 0 \\ 0 & N_{11}^{-1} \{ I + [N_{12}, \; N_{13}] \tilde{N}^{-1} & [N_{12}, \; N_{13}]' N_{11}^{-1} \} \end{bmatrix} \tag{35}$$

and thirdly we get from (33):

$$N_k F_1' = \tilde{N}^{-1} \tilde{F}' \tag{36}$$

The matrix Q_{11kc} need not be known for all elements, but should be given pointwise; this reduces the computational expenditure in photogrammetry drastically. By means of (31), the final elements of Q_{11kb} can be obtained:

$$Q_{11kb} = Q_{11kc} - \begin{bmatrix} 0 \\ N_k^{-1} F_1' \end{bmatrix} E_{1k} - E_{1k}' \left[0,\ F_1 N_k^{-1}\right] + E_{1k}' F_1 N_k^{-1} F_1' E_{1k} \tag{37}$$

We arrive at the final datum in successive stages. First, all these control points could be introduced as datum information, which is available to both observation epochs. Secondly, one introduces minimum datum information with those coordinates of control points whose studentized displacements are minimum. All other control points common to both epochs will be tested by means of hypothesis tests on significant displacements. If the test is negative, this means that there is no significant displacement for the point or any individual coordinate, or this point or coordinate may now additionally contribute to definition of the datum. Thus, all control points could be checked on displacements, and moreover object points will also be tested on movements in order to contribute to the datum, if they are common to both epochs and have non-significant movements. In this way, the final datum will be derived from control points and object points, which will not move significantly between the two observation epochs.

HYPOTHESIS TESTING

For separation of the set of control points and object points into two disjunctive sets of datum points and variable points, hypotheses will be introduced based on the assumption that certain points are fixed points. Because of the different models, hypothesis testing methods will differ somewhat from each other, as can be seen in the following.

(i) The hypothesis test within the sequential univariate Gauss-Markov model presupposes any test on homogeneity of the observations between the two epochs; this can be stated as follows:

$$H_0\colon \sigma_i^2 = \sigma_j^2 \text{ against } H_1\colon \sigma_i^2 \neq \sigma_j^2 \tag{38}$$

its acceptance or rejection depends on the variance ratio (Wolf, 1975):

$$\sigma_i^2 / \sigma_j^2 \sim F(u_i - q_i,\ u_j - q_j) \tag{39}$$

If the hypothesis (38) has been accepted, we can proceed with formulations on hypothesis testing for point movements. For this reason, let us introduce the hypothesis:

$$H_0\colon\ |x_{1ib}^{(P_i)} - x_{1jb}^{(P_i)}| = \mathbf{0} \text{ against } H_1\colon\ |x_{1ib}^{(P_i)} - x_{1jb}^{(P_i)}| \neq \mathbf{0} \tag{40}$$

for pointwise hypothesis testing, where individual coordinates or all coordinates of point P_i can be tested. Its test statistic is given by Koch (1985):

$$T = \frac{1}{r\left(\hat{\sigma}_i^2 + \hat{\sigma}_j^2\right)}\left(\hat{x}_{1ib}^{(P_i)} - \hat{x}_{1jb}^{(P_i)}\right)'\left[\left((Z_i'Q_{ii}^{-1}Z_i)_{rs}^{-}\right)_{P_i} + \left((Z_j'Q_{jj}^{-1}Z_j)_{rs}^{-}\right)_{P_i}\right]^{-1} \times\left(\hat{x}_{1ib}^{(P_i)} - \hat{x}_{1jb}^{(P_i)}\right) \tag{41}$$

and decides on acceptance if $T < F_{1-\alpha;r,n_{ij}-q_{ij}}$ or the percentage point

$$\alpha_T = \int_T^{\infty} F(r,\ n_{ij} - q_{ij})\ \mathrm{d}T > \alpha \tag{42}$$

with α as level of significance.

(ii) Let hypothesis (40) within the simple multivariate Gauss-Markov model be defined. The following test value decides on acceptance or rejection:

$$T = \frac{1}{r\left(\hat{\sigma}_i^2 - 2\hat{\sigma}_{ij} + \hat{\sigma}_j^2\right)}\left(\hat{x}_{1ib}^{(P_i)} - \hat{x}_{1jb}^{(P_i)}\right)'\left[(Z'Q^{-1}Z)_{rs}\right]_{P_i}^{-1}\left(\hat{x}_{1ib}^{(P_i)} - \hat{x}_{1jb}^{(P_i)}\right) \tag{43}$$

which considers the covariance between epoch i and epoch j.

(iii) The hypothesis (40) is also valid for testing within the incomplete multivariate Gauss-Markov model, where the test value will be:

$$T = \frac{1}{r\left(\hat{\sigma}_i^2 - 2\hat{\sigma}_{ij} + \hat{\sigma}_j^2\right)}\left(\hat{x}_{1ib}^{(P_i)} - \hat{x}_{1jb}^{(P_i)}\right)'\left[\left(Z_{ij}'\hat{\bar{\theta}}_{ij}^{-1}Z_{ij}\right)_{rs}^{-}\right]_{P_i}^{-1}\left(\hat{x}_{1ib}^{(P_i)} - \hat{x}_{1jb}^{(P_i)}\right) \tag{44}$$

which can be used approximately, because the unknown parameters should be seen independently from the variance–covariance components, although they are dependent.

EXAMPLE

Let us consider an example coming from brown-coal mining, where an area of opencast mining has been controlled for movements caused by working of the coal. The area contains many control points and additional points, which we will call quality points, determined by classical geodetic measurements with an accuracy of about $\sigma_{x,y} = \pm 10$ (mm), to derive a quality control of the photogrammetric point determination (see Fig. 1).

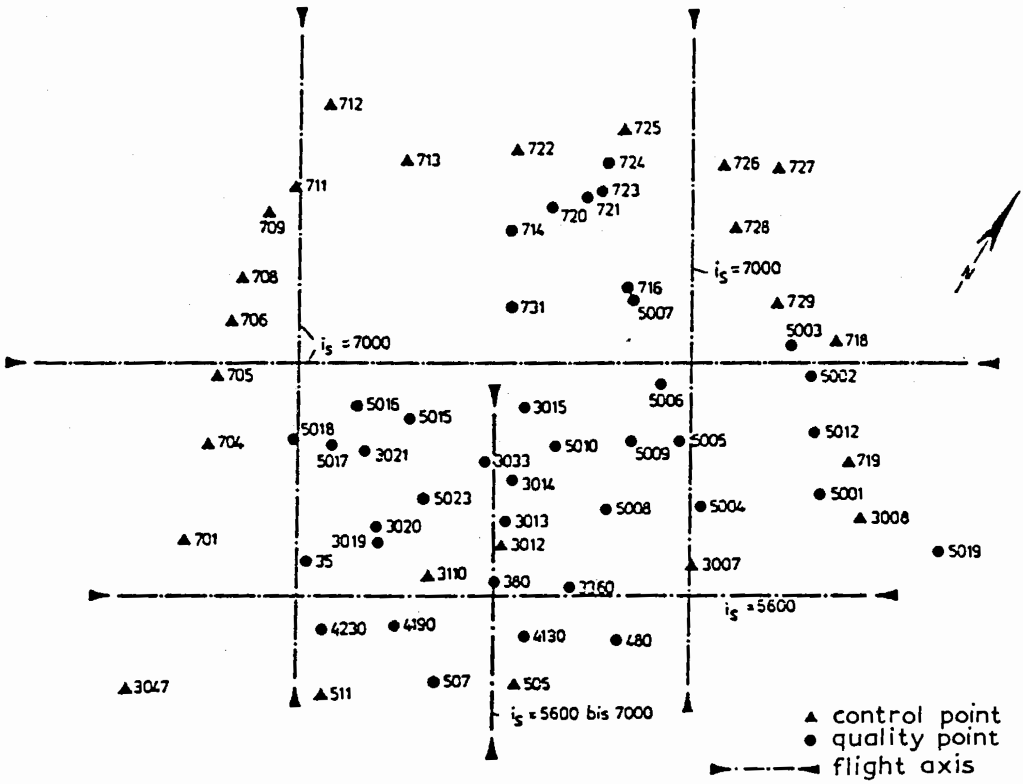

Fig. 1. Observation scheme and control points.

TABLE 1

Information on exposures and image evaluations

Date of exposure	Image scale	Camera	Comparator	Evaluation programme
1.11.80	7000 5600	Zeiss 15/23 AIV	Zeiss PSK 1	PAT-B * with self calibration
16.11.81	7000 5600	Zeiss 15/23 AIV	Zeiss PSK 1	PAT-B * with self calibration

* Bundle Adjustment within sequential Gauss-Markov models with control points as random information.

All control points and quality points geodetically determined have been signalized for photogrammetric evaluation. The whole area has been flown periodically since 1978 (Reichenbach, 1981), but let us consider only two photogrammetric evaluations for demonstrating the accuracies being achieved (see Tables 1 and 2).

The values σ_x and σ_y demonstrate rms-deviations for control points and quality points between geodetic and photogrammetric point determination, whereby $\epsilon_{x\max}$ and $\epsilon_{y\max}$ are numbers for maximum deviations of control points only. The interior accuracy of the photogrammetric point determination can be given to 5.4 (mm) $\leqq \sigma_x$, $\sigma_y \leqq 16.5$ (mm) with $\sigma_{\text{rms}} = \pm 7.9$ (mm) (Deutsch. Braunkohlen-Industrie-Verein e.V., 1983).

For a demonstration of hypothesis testing, let us take some samples of the point manifold determined photogrammetrically. Because of the availability of variances, the tests are restricted to single coordinates only and not groups of coordinates. The test on homogeneity delivers $\sigma_1^2/\sigma_1^2 = 1.0$ and will be accepted for redundancy $f = 1250$ and $\alpha = 0.05$. The points tested with test values (41) are represented in Table 3.

As we can see from Table 3, for example, it is not allowed to use the $x-y$ coordinates of control point 713 as datum information in the bundle adjustment.

TABLE 2

Absolute accuracies

Date of exposure	Point type	σ_0 (μm)	σ_x (mm)	σ_y (mm)	$\epsilon_{x\max}$ (mm)	$\epsilon_{y\max}$ (mm)
1.11.80	control point	3.4	11	10	–	–
	quality point		12	19	28	38
16.11.81	control point	3.4	6	10	–	–
	quality point		6	10	15	20

TABLE 3

Hypothesis tests for $\sigma_{x,y} = \pm 8$ (mm) and $\sigma_z = \pm 16$ (mm)

Point	Coordinates	T	Movements ($\alpha = 0.05$)	
			Yes	No
713	x	1.56	×	
	y	6.25	×	
	z	0		×
3015	x	0.19		×
	y	4.25	×	
	z	151.6	×	
3021	x	3.29	×	
	y	0.06		×
	z	1.0		×

Therefore, it is indispensable to check the control points in moving areas on movements, too, to arrive at a definite datum definition.

CONCLUSIONS

Present-day photogrammetry is an efficient tool for making precise point determination and controlling points for displacements, if repeated observations are available. It is, therefore, an alternative to classical geodetic observation techniques, which may in some cases, especially in developing countries, be very expensive, so that photogrammetry has the additional advantage of being more economical.

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