COMBINED PHOTOGRAMMETRIC-TERRESTRIAL CADASTRAL SURVEYING

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1. General

Speed and storage capacity of the third-generation computers permit the use of procedures of very high computational complexity; until recently, such procedures had been known only as to their theoretical foundation. Modern computer technology frequently even provides the impetus for the development of new methods. The final results of the computations can now be improved - without jeopardizing the economy of the process - by using nearly every available means of purely computational refinement. That is the point of view from which this paper should be considered.

Although photogrammetry generally does not quite achieve the high relative accuracy demanded by the specifications for cadastral surveys, photogrammetric cadastral surveying has nevertheless attained a certain significance, primarily with the property reallotment authorities [10]. In Baden-Württemberg [4], and in Bavaria [11], terrestrial inter-distances are still retained in photogrammetric cadastral surveys. Their function is twofold: on the one hand they are used to check the photogrammetric coordinates, and on the other they supplement authoritatively the documentary record of the property boundaries ([13], page 354). Other Länder of the Federal Republic of Germany rely exclusively on photogrammetric restitution, usually verified only by means of a second cover restitution of the same area.

In the " computer-compatible property cadastre "* of the future, property boundaries will be recorded primarily in the form of state coordinates, and the measured inter-distances will be resorted to only rarely.

For photogrammetric cadastral surveying, this means that the coordinates must satisfy the official error tolerances. The necessary augmentation of accuracy can be achieved easily by including the terrestrial survey elements in the automatic computation of coordinates. Simultaneous adjustment of all available parameters - standard procedure in solving many other geodetic problems - increases primarily the relative coordinate accuracy between adjacent points. Moreover, the final coordinates and the remaining corrected observations obtained from this type of adjustment constitute a property boundary record that is entirely free from discrepancies; this will greatly facilitate subsequent operations, most of all the revision surveys.

Since the inter-distances must be digitized in any event for the verification of the photogrammetric coordinates, these corrections are obtained by the methods used in Baden-Württemberg and Bavaria merely at the price of greater computational effort; but, as already mentioned, this extra effort is of no consequence. Besides serving as a check for the photogrammetric coordinates and supplementing the property boundary record, the terrestrial survey elements thus also assume the important function of increasing the accuracy of the coordinates to such an extent that they can be regarded as final coordinates in a "computer-compatible property cadastre". In this threefold function the inter-distances and the other terrestrial survey elements attain such extraordinary significance that they should be incorporated in every photogrammetric

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*A more appropriate name for "coordinate cadastre" [2] cadastral survey. Elimination of the second cover restitution, which is unresponsive to targeting errors and to a variety of identification mistakes anyway, would to some extent compensate for the additional effort required.

In this form, photogrammetric-terrestrial cadastral surveying was first employed at a modest scale in the Kelheim cadastral re-survey [3], [7]. This paper describes the fundamental concept and the efficiency of the new program for the Telefunken TR 4 computer; a report on extensive practical experience is given, and the theoretical foundations are expanded. This program takes over after the horizontal block adjustment [1] and is the conclusion of the entire numerical restitution process that leads from the machine coordinates of the independent models to the final state coordinates.

2. Theoretical foundations

The formulae on which the adjustment procedures are based have been published in two papers [6], [7]. For each terrestrial inter-distance and for each array of three points in a straight line, a condition equation is obtained. In addition, it is possible to "freeze" ground control points and tie points of existing surveyed areas by introducing zero corrections.

To satisfy the requirement for full verification of the results, each property boundary point must be witnessed in not less than two directions which should be perpendicular to one another. Figure 1 shows with a few typical examples how inter-distances and straight-line conditions can be combined efficiently for this purpose in order to keep the required field work to a minimum. Incorporation of these "control data" in the adjustment increases the accuracy of the coordinates significantly, particularly the interconnected longitudinal and transverse inter-distances on roads and in subdivisions [7].

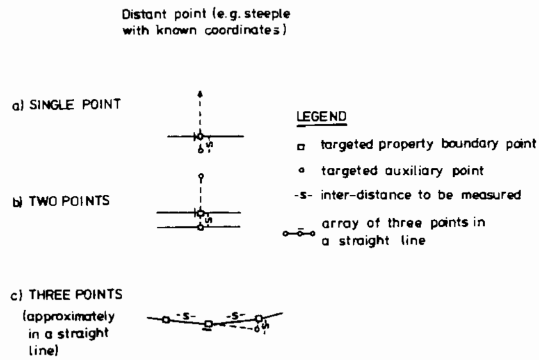

Figure 1: Examples for simple witnessing by means of straight-line conditions and inter-distances

To achieve the highest possible degree of automation, the electronic computer must be able to separate blunders from accidental errors. These criteria (= error tolerances) remain still to be developed for this procedure. The differences between the inter-distances measured and those computed from photogrammetric coordinates - the discrepancies of the adjustment - offer themselves for interrogation. Based on three times the standard error, we obtain the error tolerance

$$M_s = 3\sqrt{2\,m_k^2 + m_s^2}\,. \qquad (1)$$

The standard coordinate error m_k applicable here has been determined empirically as approximately ± 6 μm at the scale of the picture, and the standard distance error m_s can be computed with sufficient accuracy from the official error tolerance Δs for

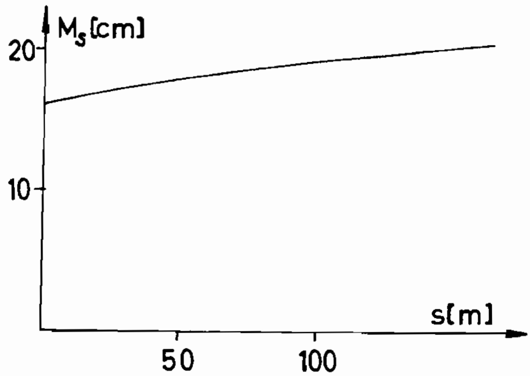

Figure 2: Error limit for the acceptable discrepancies in the adjustment of terrestrial inter-distances

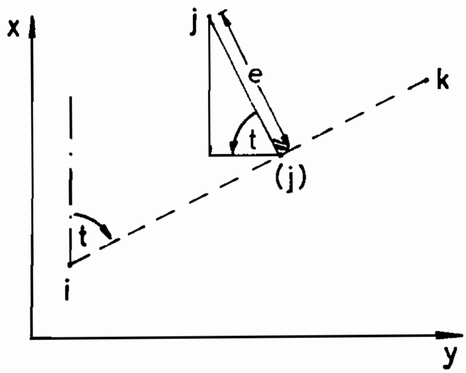

Figure 3: Discrepancy e in the straight-line condition

the verification of the inter-distances by means of the relationship $m_s = \Delta s/(3\cdot\sqrt{2})$. For the picture scale of 1:6,000 and with the use of the new error tolerance of the Baden-Württemberg Automation Directive [9]

$$\Delta s = (0.5\sqrt{s} + 0.04s + 8)\ [\text{cm}],\ s \text{ in } [\text{m}] \qquad (2)$$

the values of Figure 2 are obtained.

In the second group of observations - the straight-line arrays - discrepancies are less suited for interrogation [7]. Therefore, an error tolerance will be derived for a more descriptive quantity. For this, the distance e of the middle point j from the straight line through the two end points i and k is suitable (Figure 3). The coordinates of the foot point (j) and of the points i and k satisfy the proportion:

$$\frac{x_{(j)} - x_i}{x_k - x_i} = \frac{y_{(j)} - y_i}{y_k - y_i} \qquad (3)$$

Relationship between the coordinates of the points (j) an j:

$$x_{(j)} = x_j - e \sin t$$
$$y_{(j)} = y_j + e \cos t \qquad (4)$$

If equations (4) are inserted in (3), we obtain after some rearrangement:

$$e = \frac{(Y_k - Y_i)(x_j - x_i) - (x_k - x_i)(y_j - y_i)}{\sqrt{(x_k - x_i)^2 + (y_k - y_i)^2}} \qquad (5)^*$$

The standard error of the quantity e and thus also the error limit M_g depend only on the standard error m_k of the coordinates and on the ratio ϑ of the distances of the centre point from the two end points:

$$M_g = 3\, m_k \sqrt{2 - \frac{2\vartheta}{(\vartheta + 1)^2}} \qquad (6)$$

In Figure 4, $m_k = \pm 3.6$ cm

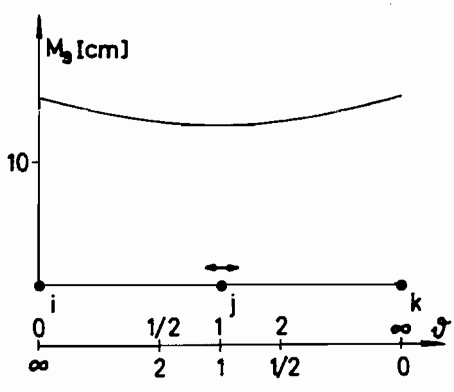

Figure 4: Error limit for the straight-line conditions

*The numerator represents the discrepancy of the previous straight-line equation [7] . In extending the program to include the possibilities mentioned in Section 5, we will make use of this more general relationship (5) also for the straight-line condition equations.

3. Program description

The program has been written in ALGOL and is suitable for EDP equipment providing 20,000 words core storage and tape input-output. A computer of this size must be available if a practically feasible program is to be used with reasonable computation times. The program must be capable of recognizing nearly all possibilities of error that can occur in practice, it must print out the erroneous data with suitable error messages, and it must produce homogeneous results after elimination of such data. The project manager must decide whether these results can be considered as final by examining the printout to test the applicability of the simple selective rules of the program.

This programming comfort is obtained at the expense of storage capacity and computing time. At present, with the use of the Telefunken TR 4 computer, only subsystems can be formed, i.e. interconnected inter-distances and straight-line arrays with a maximum of 70 conditions. Considering that distances longer than 60 metres will augment the accuracy of the coordinates only insignificantly and that in the future such long distances will no longer be measured, the subsystems will generally become smaller automatically. To ensure adequate packing of the data even under these conditions, the subsystems are stored continuously on tape and then adjusted subsystem by subsystem.

The program consists of two parts

A) the preparatory program, which stores computable subsystems on tape, and
B) the main program, which calls the prepared subsystem into core, executes the adjustment, lists the results, and punches them on cards.

Individually, the preparatory program has the following functions:

a) Storage of a list of coordinates, arranged in rising numerical sequence, obtained as output after block adjustment [1].
b) Locating the coordinates of both ends of the distances belonging to all inter-distances of a subsystem; similarly for all straight lines of the subsystem.
c) Numbering errors, if any, must be listed and the relevant data eliminated.
d) All data needed to establish the condition equations of a subsystem are stored on tape at the end of the operations.

The diagram, Figure 5, shows the entire data input.

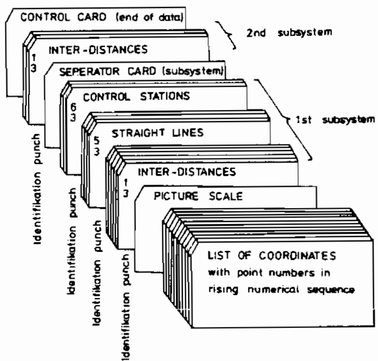

Figure 5: Data input

For the keypunching of the inter-distances, straight-line arrays, and points to be held-to - separated as to subsystems - the new survey plats with the encircled subsystems are available. The delineation of the subsystems is carried out - independent from the subdivision shown on the plats - exclusively according to the exigencies of the theory of adjustment. Thus, for example, the interconnected short inter-distances of a road intersection must be adjusted within one and the same subsystem.

The main program executes the following operations:

a) After transfer of a subsystem into core storage, the error detection operation continues. Each inter-distance is computed from the photogrammetric coordinates and the difference is interrogated twice:

   aa) If the difference falls outside the error limit (1) of the procedure, an error message is printed out and this specific interdistance eliminated.

   bb) For each inter-distance, it is stated whether the difference falls inside or outside the official error limit (2).

   The layout of the printout (Table 1) is patterned after the Baden-Württemberg Automation Directive.

NF	Plat	From point to point	Coordinates Y	X	Interdistance measured	computed	Differences ins.	tol.	outs.
22	02	2007301	48496.42	87242.58	5.73	5.77	0.04	0.09	
22	02	2007302	48500.24	87246.91					
22	02	2007301	48496.42	87242.58	12.03	12.04	0.01	0.10	
22	02	2007304	48508.43	87243.42					
22	02	2007303	48504.36	87247.16	6.52	5.53		0.10	-0.99
22	02	2007304	48508.43	87243.42					
Last interdistance not used									
22	02	2007302	48500.24	87246.91	4.13	4.13	-0.00	0.09	
22	02	2007303	48504.36	87247.16					

Table 1: Printout, before adjustment

Each straight-line aggregate is processed accordingly, using the quantity e (5) and the error limit (6), except that no comparison with an official error limit is made.

b) Adjustment of a subsystem by conditions. At the end, the corrected interdistances and coordinates are printed out (see Table 2) and the final coordinates of the property boundary points are punched.

List of old and new coordinates and differences

Point No.	Y (old)	X (old)	Y (new)	X (new)	Y difference	X difference
	m	m	m	m	m	m
2007301	48496.42	87242.58	48496.43	87242.60	0.01	0.02
2007302	48500.24	87246.91	48500.23	87246.89	-0.01	-0.02
2007303	48504.36	87247.16	48504.35	87247.16	-0.01	-0.00
2007304	48508.43	87243.42	48508.43	87243.42	0.00	0.00
2007305	48535.30	87245.58	48535.27	87245.58	-0.03	0.00
2007306	48535.89	87241.47	48535.89	87241.46	-0.00	-0.01
2007401	48572.09	87249.57	48572.12	87249.58	0.03	0.01

List of distances and differences

NF	Plat	Point No.	Point No.	Distance measured	Distance computed	Distance difference
		m	m	m	m	m
22	02	2007301	2007302	5.73	5.74	0.01
22	02	2007301	2007304	12.03	12.03	-0.00
22	02	2007302	2007303	4.13	4.13	0.00
22	02	2007304	2007305	26.93	26.93	-0.00
22	02	2007305	2007306	4.17	4.17	-0.00
22	02	2007305	2007401	37.08	37.06	-0.02

Table 2: Printouts, after adjustment

4. Experience from the Hermuthausen/Steinbach reallotment

The basic photogrammetric and terrestrial material has been described by ACKERMANN [1] in connection with the horizontal block adjustment (photoscale 1:6,000, 32 models, 1,090 hectares). This material was routinely processed in Ludwigsburg by the Landesamt für Flurbereinigung und Siedlung Baden-Württemberg (Office of Reallotment and Settlement of the State of Baden-Württemberg). At the time the field operations were carried out, the possibility of combining distances and straight-line aggregates as a means of checking and increasing the accuracy of the coordinates, as indicated in Fig. 1, was still unknown. Inter-distances were used exclusively - for each corner point at least two in different directions.

Of the 5,900 inter-distances, divided into 147 subsystems, the computer used 5,648 for the adjustment of 4,540 points. For 252 distances (4.9%), the differences between photogrammetric and terrestrial determination fell outside of the error limit (1) (Fig. 2) before adjustment. We know from experience (see [4]) that these gross errors are caused primarily by keypunching errors and distance measuring errors, not so much by photogrammetric errors. Some 40 differences exceeded the error limit only by a few centimetres.* The decision whether such inter-distances can nevertheless be included in the adjustment will be made by the Project Manager after detailed investigation - for example uncertain setting of points, considerable terrain slope, etc. Had the official error limit (2) been used as the criterion for the elimination of gross errors, then a total of 8.3% of the interdistances would not have been used in the adjustment. Thus, 4.0% of all

*Moreover, if we assume normal distribution for the differences, 0.27% of the inter-distances - in our case already 16 - fall theoretically outside an error limit based on three times the standard error.

measured interdistances would have been discarded although the differences between terrestrial and photogrammetric measurement are clearly within the range of accidental errors.

The standard error of unit weight, i.e. the standard error of the photogrammetric coordinates before adjustment, was 3.6 cm; at a picture scale of 1:6,000, this corresponds to 6 microns in the photograph. This value agrees well with the accuracy data of the horizontal block adjustment [1] and once again demonstrates the high accuracy photogrammetry can achieve in practice. After adjustment, the accuracy of the coordinates - provided the minimum requirement of two interdistances in two different directions has been adhered to - is 2.5 cm (= $3.6/\sqrt{2}$). Interconnected systems of interdistances, as they occur to a large extent in reallotment operations, result in even better accuracies [6] . The combined photogrammetric-terrestrial cadastral survey thus achieves the accuracy of the conventional terrestrial survey procedures. In addition, the final coordinates, together with the corrected distances, are forming a perfectly discrepancy-free property boundary record; after adjustment, moreover, all differences against the measured interdistances lie within the official error limit (2).

The automatic elimination of grossly erroneous input data, as described in Section 3, brought rewards in the form of short computing times. The single computation of the entire reallotment procedure in the TR 4 took 3.3 hours. Such a single computation yields homogeneous results. The respective subsystem will be recomputed only if data subsequently screened to eliminate blunders are not in agreement with these results within the official error limit. Assuming that one hour of computing time in the TR 4 costs DM 1,200.-, then the cost of calculating the final coordinates of one point was 0.87 DM (= 1,200 x 3.3/4540).

5. Prospects

Although the programs's efficacy has thus been proved, it should only be considered as one step on the path of a development aiming at a universal adjustment program for hybrid measurements. We intend to rewrite the program in the near future to go from the medium-sized TR 4 computer to the large CDC 6600 computer. This will enable us to refine the automatic treatment of data errors even further and to compute far larger subsystems that will be defined automatically by the computer. In addition, the program is to be expanded in four points:

a) If the photogrammetric coordinates of individual corner points are missing, the available terrestrial and photogrammetric numerical material is in many cases still adequate to calculate the final coordinates of such points. In the condition equations, these coordinates appear as unknowns. The problem would therefore have to be solved with the adjustment algorithm for "conditions with unknowns". But if we choose a small weight for these quantities, the present algorithm for adjustment by conditions can be retained ([12], page 100). With this program expansion, the proportion of uncoordinated points is reduced and consequently also the effort required for terrestrial remeasurements.

b) In the field, ordinates can also be measured in addition to the parameters now used (Fig. 3). The relationship (5) can be used as a condition equation between the ordinate e requiring correction and the coordinates, also to be corrected, of the three points involved. In this case, the required linearization will be achieved more rapidly by forming numerical differences than by using differential quotients. The weight of the ordinate depends on the length of the ordinate and on the ratio ϑ between the distances of the foot point from the two other points.

This program expansion eliminates the necessity to target auxiliary points. In case (c) of Figure 1, the auxiliary point will in future no longer be required; instead, only the distance of the last point from the straight line through the other two points is measured.

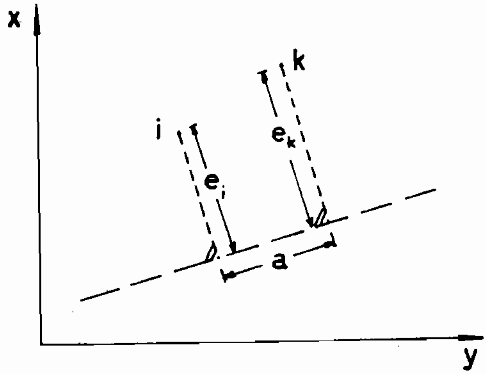

Figure 6: Expansion of the straight-line condition to orthogonal (off-set) surveying (see equation (7))

c) These considerations would suggest the extension of the system to the abscissae. The condition equation reads (Fig.6):

$$a+v_a = \sqrt{(x_k+v^x{}_k-x_i-v^x{}_i)^2+(y_k+v^y{}_k-y_i-v^y{}_i)^2-(e_k+v^e{}_k-e_i-v^e{}_i)^2} \qquad (7)$$

From this, we obtain with the discrepancy

$$w_a = \sqrt{(x_k - x_i)^2 + (y_k - y_i)^2 - (e_k - e_i)^2} - a = a^\circ - a \qquad (8)$$

the linear form:

$$- v_a - \frac{e_k - e_i}{a^\circ} v^e{}_k + \frac{e_k - e_i}{a^\circ} v^e{}_i + \frac{x_k - x_i}{a^\circ} v^x{}_k + \frac{y_k - y_i}{a^\circ} v^y{}_k - \frac{x_k - x_i}{a^\circ} v^x{}_i - \frac{y_k - y_i}{a^\circ} v^y{}_i + w_a = 0. \qquad (9)$$

The weight of the abscissa depends on the length of the two ordinates and of the abscissa itself. This program extension, in connection with the two preceding ones, facilitates above all the application of photogrammetry in the form of the so-called simplified cadastral resurvey [10] , [8] . In this form, locally separated individual measurements - mostly orthogonally measured lines - are transformed into the uniform state co-ordinate system.

d) For the control of conventional orthogonal and polar surveys, too, interdistances are measured, or the conformity to the straight-line condition of the respective point aggregates is tested. Eminent authors ([5] , [8]) have also advocated the inclusion of these control data in the automated calculation of coordinates. Our program can solve this task, too, if we adapt the weights to the accuracy of orthogonal and polar surveys.

Summary

In general, photogrammetry cannot quite meet the high relative accuracy officially specified for cadastral surveys in the Federal Republic of Germany. Satisfactory accuracy will be achieved, however, if short distances and other data measured in the terrain are included in the calculation of the final coordinates. Besides augmenting accuracy, these ground data also have the important function of providing a thorough test for all coordinates. In this paper, the ALGOL program is described and the practical results of a large combined photogrammetric-terrestrial cadastral survey are reported.

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