AEROTRIANGULATION WITH INDEPENDENT MODELS ON THE ZEISS PLANIMAT - A STUDY OF THE INSTRUMENT

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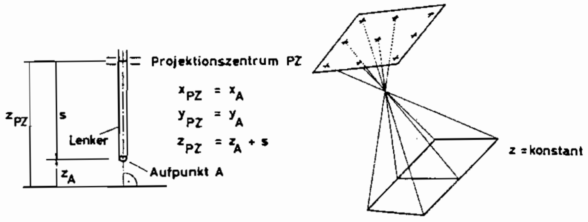
1. Introduction

Owing to its universal applicability, the technique of strip and block triangulation with independent models is today well on its way towards becoming an interesting alternative to aerotriangulation based on bundles. While the latter method is based on image coordinates and thus linked to the use of comparators, the method of independent models uses the spatial coordinates of relatively oriented photo pairs. In other words, the measurements may also be performed in precision stereoplotters. No exchange of base is required. Tie-in of the different models in the direction of the strip is made by computation, using the tie points on the ground and the perspective centers, whose coordinates have to be determined in the different models.

It is the purpose of the present study to determine the accuracy of a modern precision stereoplotter, the ZEISS PLANIMAT, in measurements of independent models, and to ascertain the effect of instrumental errors on aerotriangulation.



Method A (vertical guide rod)

Method B (resection in space)

Figure 1: The two methods for determining the coordinates of the perspective centers.

Projektionszentrum PZ= Perspective center PZ
Lenker=Guide rod
Aufpunkt A=Space point A

z=konstant=z=constant

2. Features of ZEISS PLANIMAT allowing measurement of independent models

In the PLANIMAT, the requirement for automatic recording of coordinates, which is indispensable for practical aerotriangulation, is satisfied by the possibility of connecting an ECOMAT unit. The actual problem in the measurement of independent models consists in determining the coordinates of the perspective centers. Since the position of the perspective centers in space changes in the course of relative orientation in the PLANIMAT (the axes of rotation for φ and ω do not intersect the projection cardan), it is necessary to determine the coordinates separately for every model. In instruments whose perspective centers do not vary their position in space, on the other hand, it will, in principle, suffice to determine the position of the perspective centers just once in relation to the system of coordinates used for measurement, which in this case, however, has to remain unchanged until all the models have been measured. Although determination of the perspective-center coordinates in every single model in the PLANIMAT involves a certain amount of additional work, this is practically offset by the possibility of disconnecting the coordinate counters during relative orientation and using the freehand guiding feature.

In the ZEISS PLANIMAT there are two different approaches for determining the coordinates of the perspective centers:

Method A (guide rods vertical, Fig. 1): Each of the spatial guide rods has annular marks at calibrated distances from the space point. With the aid of magnetic bubble levels, the guide rods are set perpendicular to the horizontal x, y-plane. Then z is varied until the annular mark of the guide rod coincides with an index representing the center of the projection cardan. The subsequent recording directly gives the x and y-coordinates of the perspective center. The z-coordinate is derived from the recorded value by adding the distance s between the annular mark and the space point.

The same principle is used in the Kern PG-2 and PG-3 plotters. In these instruments, however, the bubble levels have been replaced by an autocollimating system [1].

Method B (resection in space, Fig. 1): It is a special feature of the PLANIMAT that each photoholder plate has nine reseau crosses forming 90 mm squares, which are projected into the model space together with the photograph. After relative orientation, the coordinates of four or six projected reseau crosses are measured at an arbitrary z-level. Finally, the coordinates of the projection center are computed from these values with the aid of a resection in space.

There is the following basic difference between these two methods: method A is aimed at determining the position of the mechanical perspective centers. The errors of the perspective centers are thus fully evident and are superimposed by the measuring errors of method A. The resection in space, on the other hand, uses a bundle of rays that is assumed to be free from error. Method B is thus free from the errors of the machanical perspective centers, but dependent on the errors of the projected reseau crosses and the grid measurement in the corresponding z-plane.

When using method A, the perspective centers are determined practically without any additional computation. Method B on the other hand calls for a computer program for the spatial resection, but it offers the advantage that the operator may use almost the same measuring technique as for the model points. In both cases about 3 to 5 minutes are required per photo pair.

Apart from these two methods for determining the perspective centers, mention should be made of a third one which is based on measuring unambiguous image points in at least two z-planes and which does not require any knowledge of interior orientation [2]. This principle is generally also used in adjusting photogrammetric plotters. Since the photoholder plates of the PLANIMAT are provided with reseau crosses, this method would be basically well suited. However, in order to permit measurement at two extreme levels (as is advisable for reasons of accuracy), a variation of base would be required during measurement. On account of the sources of error and the greater number of measurements involved, this method will not be investigated in the present paper.

3. Experimental determination of instrument accuracy

3.1 Approach

Suitability of the PLANIMAT for triangulation with independent models is not only determined by the necessary provisions for the recording of coordinates and determination of the perspective centers, but also by the magnitude of systematic instrumental errors and the interior accuracy with which the perspective centers are referred to the relatively oriented model. To ascertain the magnitude of instrumental errors, the use of grids appears appropriate, since in this case the nominal model is known. The interior accuracy of perspective-center determination can be easily obtained with the aid of double measurements.

In the case of the PLANIMAT, the crosses on the photoholder plates can be directly used for the grid measurements. Since for aerotriangulation the different models are generally tied in with the aid of six points near the Gruber points, no additional points are required. The nominal model (1:1) is supported by the grid constant a = 90 mm and the calibrated focal length c_1 and c_2 (Fig. 2). The measuring process equivalent to practical conditions is the following: A grid model is obtained by relative orientation of the photo carriers. Next, the six points of the grid model are set in the instrument and their coordinates recorded. This is followed by determination of the coordinates of the perspective centers according to the aforementioned methods A and B.

In the subsequent phase of computation, the model thus measured is transformed to the nominal model with the aid of a similarity transformation in space, all six nominal grid points being used as control points. The resulting overdetermination permits an adjustment by the method of least squares which has to be iterated due to the non-linearity of the transformation.

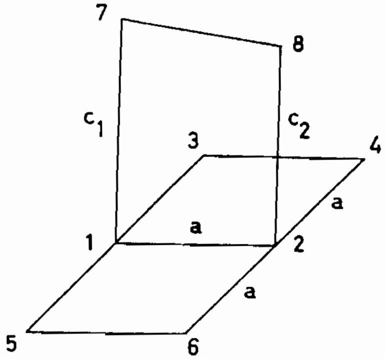


Figure 2: Nominal grid model and designation of points

In order to determine the instrumental errors with sufficient accuracy, the entire process from relative orientation to numerical transformation of the model was done twenty times. Thus the twenty adjusted coordinates obtained for every nominal coordinate can be averaged.

The diviations of these averages from the nominal coordinates are the systematic errors of the instrument, if the grids are assumed to be free from error.

The deviations of the twenty adjusted grid-model coordinates from the corresponding average were used to compute the standard deviation of the mean which was used as a measure of the accuracy obtained in determining the instrumental errors and as a measure of the constancy of systematic instrumental errors, and thus of stability.

Twenty grid models were investigated for the following three variants:
Variant 1: wide angle, c = 153 mm near-horizontal photographs
Variant 2: ultra-wide angle, c = 85mm near-horizontal photographs
Variant 3: ultra-wide angle, c = 85mm photographs of constant tilt $\varphi' \approx \varphi'' \approx +2^g, \omega' \approx \omega'' \approx +2^g$

All measurements were made in two passes with subsequent averaging of the recorded coordinates. Relative orientation was performed by opto-mechanical means. Magnification was 2,5 x in all cases. The measurements were made by Dipl-Ing. W. Wagner at the Oberkochen ZEISS factory, as a part of his thesis. Thanks are due to the ZEISS company for its kind assistance.

Work started with the wide-angle measurements. The ultra-wide-angle measurements followed about four month later on another instrument. A comparison of variants 2 and 3 was made to reveal whether photo tilt had a noticeable effect on accuracy.

3.2 The interior accuracy of perspective center determination

The double measurements permit an estimation of the accuracy with which the coordinates of the perspective centers can be determined in relation to the relatively oriented model according to the two methods, A (guide rods vertical) and B (resection in space). Table I shows the standard deviations in μm (as referred to the photo scale).

Table 1: Mean interior accuracy of determination of perspective-center coordinates for a magnification of V = 1.0.

A	σ_x [μm]	σ_y [μm]	σ_z [μm]
WA	4	5	3
UWA	3	3	3

Method A (guide rods vertical)

B	σ_x [μm]	σ_y [μm]	σ_z [μm]
WA	7	6	3
UWA	3	4	2

Method B (resection in space)

In the case of ultra-wide-angle measurements, roughly the same interior accuracy is obtained for the two methods. In the wide-angle variant, method A is more accurate than method B, but both of them are less accurate than the ultra-wide-angle measurements. Method A leads us to expect identical standard deviations for x and y, proportional to the calibrated focal length (identical random angular errors); σ_z should be constant. Both of these expectations are confirmed by Table 1. The accuracy of the resection in space depends to a large extent on the aperture angle of the bundle of rays and thus on the calibrated focal length. It increases with wider aperture angles. This explains the large difference which method B shows in the standard deviations for wide angle and ultra-wide angle. With respect to the interior accuracy of perspective-center determination, preference should therefore be given to method A for wide-angle measurements. For ultra-wide angle, the two methods would be roughly equivalent.

3.3 Summary of instrumental errors

Table 2 summarizes the instrumental errors for the three variants studied. These are listed in the form of systematic coordinate errors of the Gruber points 1 to 6 and the perspective centers 7 and 8 (for designation of points, see Fig. 2). The notes "A (vertical guide rods)" and "B (resection in space)" at points 7 and 8 indicate the method used to determine the coordinates of the perspective centers. It should be mentioned once more that the ultra-wide-angle measurements were made about four month after the wide-angle ones on a different instrument. For points 1 to 6, the instrumental errors were determined with an average accuracy of 0.5 μm (0.3 to 0.7). For the perspective centers, the wide-angle measurements resulted in a standard deviation of systematic errors of 2 μm (0.5 to 2.5), the ultra-wide-angle measurements of 1 μm (0.5 to 1.7). The systematic instrumental errors are thus sufficiently significant. The small standard deviations also point to high stability of the instrument. The standard deviations of the coordinates measured in the different models exceeds the above mentioned standard deviations by the factor $\sqrt{20} \sim 4.5$.

Table 2: Summary of systematic coordinate errors f_x, f_y, f_z of the Gruber points 1—6 and the perspective centers 7 and 8 for a magnification of V = 1.0.

A (guide rods vertical) and B (resection in space) identify the two methods of determining the perspective centers.

Point	f_x [μm]	f_y [μm]	f_z [μm]
1	—2	+1	0
2	+4	+4	—4
3	—2	—4	+2
4	+2	—4	0
5	—7	+2	—2
6	+5	+1	+4
quadratic mean 1—6	4	3	3
7 A	+34	+28	+8
8 A	+26	+22	—7
Δ 7, 8 A	—8	—6	—15
7 B	—14	+7	+1
8 B	+12	+19	+1
Δ 7, 8 B	+26	+12	0

Variant 1: WA, c = 153 mm,
near-horizontal photographs.

Point	f_x [μm]	f_y [μm]	f_z [μm]
1	—6	+2	+1
2	0	+1	+4
3	—2	—1	—3
4	+2	—6	+1
5	—3	+6	+2
6	+9	—2	—5
quadratic mean 1—6	5	4	3
7 A	+27	—15	+8
8 A	+39	+4	+8
Δ 7, 8 A	+12	+19	0
7 B	—4	+3	—3
8 B	+4	+6	—5
Δ 7, 8 B	+8	+3	—2

Variant 2: UWA, c = 85 mm,
near-horizontal photographs.

Point	f_x [μm]	f_y [μm]	f_z [μm]
1	—2	+1	+4
2	—1	+2	+3
3	—4	0	—5
4	+3	—7	+1
5	—4	+4	0
6	+8	0	—4
quadratic mean 1—6	4	3	3
7 A	+21	—22	+11
8 A	+34	—6	+9
Δ 7, 8 A	+13	+16	—2
7 B	+7	+1	0
8 B	+3	+4	—3
Δ 7, 8 B	—4	—3	—3

Variant 3: UWA, c = 85 mm,
tilted photographs $\varphi \sim \omega \sim 2^g$

3.4 Discussion of the systematic errors of the Gruber points

Table 2 shows that the systematic coordinate errors referred to the photo scale reach a maximum value of 7μm in the wide-angle case and do not exceed 9μm in the ultra-wide-angle measurements. The quadratic means of the Gruber points 1 to 6 lie between 3μm and 5μm, results in the three variants studied being approximately the same. In any case, these errors of the instrument can be considered as sufficiently small. The PLANIMAT thus reaches the accuracy of previous first-order instruments. Mention should also be made here of the fact that the systematic errors obtained refer, in principle, to the combination of PLANIMAT and grid, and the share of the grids may well reach a value of 3μm.

The distribution of instrumental errors in the model is illustrated by diagrammatic sketches. Fig. 3a shows the horizontal errors obtained, Fig. 3b the vertical errors at the six Gruber points.

Although the wide-angle and ultra-wide-angle measurements were performed on two different instruments, the horizontal position in each of the three variants shows a scale affinity (length difference in x and y) of roughly identical magnitude and identical sign. It is quite possible that this is partly due to the grid errors. The two ultra-wide-angle cases reveal only relatively slight differences in both the horizontal and the vertical results (generally by 1μm, but not more than 4μm). The effect of photo tilt on the systematic deformation of the model at the Gruber points is thus rather small.

3.5 The systematic errors of the perspective centers

According to Table 2 the coordinate errors of points 7 and 8 reach values of up to 39μm. However, since for triangulation the perspective centers are only used for connecting the models, their absolute position being of secondary importance, the differences $\Delta 7,8$ are much more interesting than the systematic coordinate errors themselves. They have therefore been included in Table 2.

While the values $\Delta 7,8$ A (guide rods vertical) do not vary greatly for wide angle and ultra-wide angle, method B (resection in space) gives larger differences of error $\Delta 7,8$ for wide angle than for ultra-wide angle, above all in the x-component that is important for the φ connection. This is probably due to the principle of spatial resection used in method B: if the systematic errors of the grid points in a z-plane are approximately identical for wide angle and ultra-wide angle, this means that the errors of the perspective centers are larger in the wide-angle case, due to the smaller angle of the bundle, than in the case of ultra-wide-angle photography. This agrees with the results obtained with respect to the interior accuracy of the two methods (see 3.2). In the triangulation, the error differences $\Delta 7,8$ will act together with the instrumental errors on the Gruber points. For further details see chapter 4.

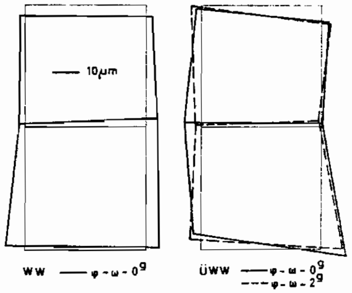


Figure 3a: Systematic horizontal-position errors of the Gruber points for a magnification of V = 1.0, for two different instruments.

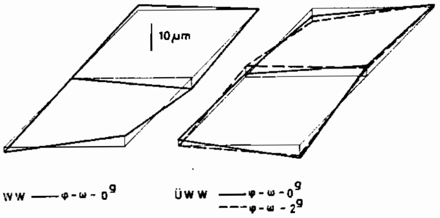


Figure 3b: Systematic vertical errors of the Gruber points for a magnification of V = 1.0, for two different instruments.

Table 2 also shows that the ultra-wide-angle variants 2 and 3 differ by up to 10 μm (method A) and 11 μm (method B) in the coordinates of the perspective centers. In other words, the effect of photo tilt on the systematic errors is greater in the case of the perspective centers than of the Gruber points.

4. The effect of systematic instrument errors on triangulation

After the systematic coordinate errors have been determined for individual models, we shall now investigate their effect on triangulation. Apart from the absolute values of the resulting errors we are here interested in a comparison between wide angle and ultra-wide angle and in the effect of the two different methods of determination of the perspectice centers. Since for triangulation with independent models the perspective centers are only needed for tying-in the models in the direction of the strip, we may well restrict ourselves to a single triangulation strip.

Let us assume twenty models per strip, which is frequently the case in practical work. Repeated connection of nominal models with a calibrated focal length of c = 153 mm (WA) and c = 85 mm (UWA) gives a nominal strip; the marginal points at the two extremes and in the center can be used as control points (total of six control points). The identical models, which are deformed by the amount of the instrumental errors determined (variants 1 and 2, $\varphi \sim \omega \sim 0^9$, Table 2) were subjected to a strip adjustment with the aid of the computer program [3] . As regards the coordinates of the perspective centers, a distinction was made between the methods of determination. A (guide rods vertical) and B (resection in space). The deviations of the adjusted coordinates from the nominal strip coordinates give the systematic errors of triangulation. They are represented in Fig. 4a (horizontal position) and 4b (elevation). The errors refer to the photo scale.

The most obvious systematic effect seen in Fig. 4a is the constriction of the strips between the two series of control points. This is primarily due to the affine scale errors of the model coordinates (see Fig. 3a, wide angle and ultra-wide angle $\varphi \sim \omega \sim 0^g$). The deformed strips clearly show the affine defomation of the different models. The maximum coordinate errors in the x and y-directions are between 10 μm and 20 μm, respectively, and the quadratic means between 6 μm and 11 μm. In the variant A the wide-angle measurements reveal somewhat smaller errors than the ultra-wide-angle measurements; in the variant B ultra-wide angle is more accurate. In the case of wide-angle photography, method A gives slightly smaller systematic errors than method B. In the ultra-wide-angle strip the two methods are equivalent. The distribution of errors is not rigorously symmetrical, since the individual models are likewise affected by asymmetric deformation.

It should be mentioned with regard to the values of the systematic errors that they are small by comparison with the horizontal errors to be expected in the case of a practical strip triangulation over twenty models supported by only six control points.

The most striking systematic effect in Fig. 4b (elevation error) is a deformation of the strip between the series of control points. This is the result of the joint effect of the x-coordinate errors at the Gruber points and the perspective centers, which propagate by double summation.

As in the case of horizontal position, the distribution of errors is not strictly symmetrical here either. However, unlike the results for horizontal position, a clear dependence of vertical errors on the angular field and the method used to determine the perspective centers can, in part, be notices here.

Method A yields systematic vertical errors of comparable magnitude for wide-angle and ultra-wide-angle measurements. (max.37 μm and 30 μm, respectively; quadratic means 21 μm and 19 μm). The different signs are of no importance. Method B, on the other hand, shows a heavy dependence of vertical errors on angular field; the results for wide angle are considerably poorer (max. 51 μm, mean 33 μm) than those for the ultra-wide-angle strip (max. 11 μm, mean 7 μm). We thus have the same tendency as under 3.5.

In the present case, method A would be preferable for wide-angle photography, a fact that was also established in 3.2 with respect to the interior accuracy of perspective-center determination. Method B would be preferable for ultra-wide-angle photography. According to 3.2, the results for the two ultra-wide-angle variants were roughly equivalent.

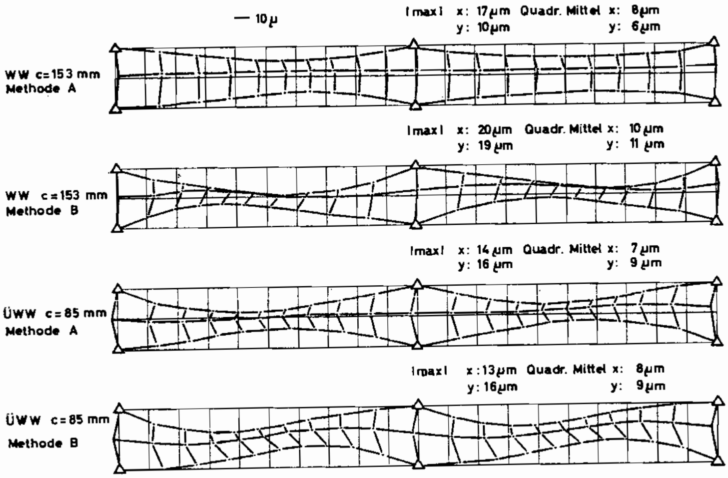


Figure 4a: Systematic horizontal-position errors after strip adjustment, caused by the instrumental errors determined.

Method A: guide rods vertical; method B: resection in space

WW = WA OWW = UWA
Quadr. Mittel = Quadratic mean

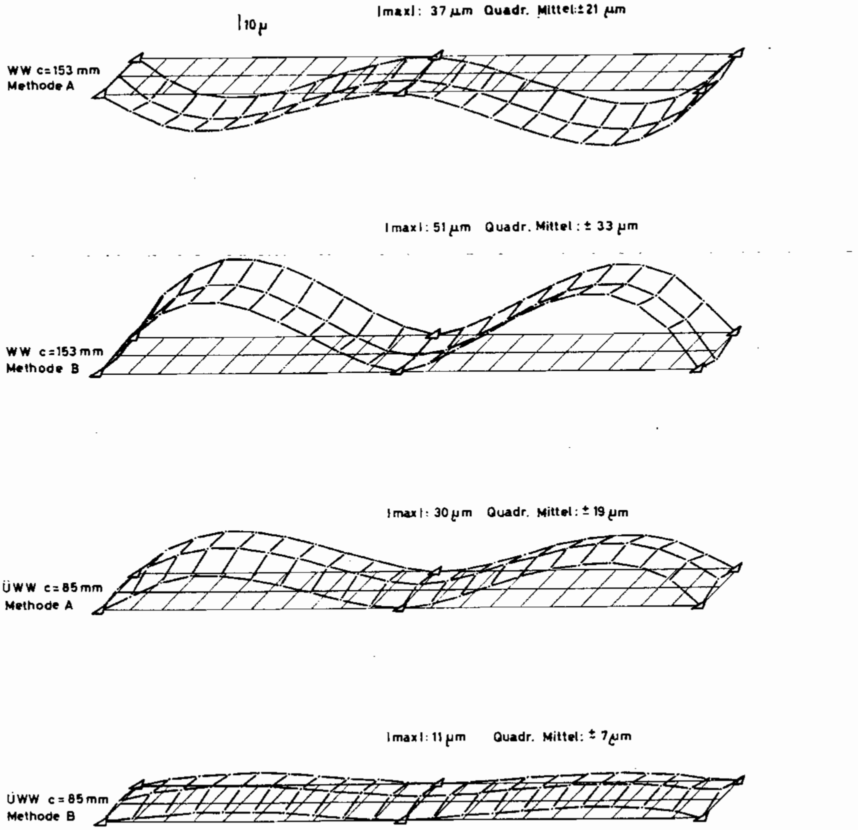


Figure 4b: Systematic vertical errors after strip adjustment, caused by the instrumental errors determined.

Method A: guide rods vertical; method B: resection in space

For practical strip triangulation over twenty models, supported by six control points, we may expect a maximum error in z of about 150 μm to 200 μm at the photo scale. It is against these values that the systematic errors obtained have to be seen. The maximum of 11 μm for ultra-wide angle and method B must then be considered as excellent, the 30 μm or 37 μm for ultra-wide angle and method A or wide angle and method A, respectively, as good, and even the maximum error of 51 μm (wide angle, method B) is still acceptable.

In this connection, mention should also be made of the possibility of determining the instrumental errors before or after a triangulation project and correcting the recorded model coordinates of all independently measured models accordingly by computation. In this manner, the systematic errors of the adjusted strip coordinates could be further reduced quite noticeably.

In conclusion, it may be said that the ZEISS PLANIMAT is well suited for triangulation with independent models. According to the systematic instrument errors found, it ranks among the instruments previously known as of first order. In the case of wide-angle triangulations, the method A (guide rods vertical) for determining the perspective centers should furnish better results than the method B (resection in space). With ultra-wide-angle photography, the accuracy of the two methods will be about the same.

Summary

The ZEISS PLANIMAT is suitable for triangulation with independent models. The coordinates of the perspective centers can be determined by two different methods. The paper investigates the interior accuracy of the two methods by repeated measurement of grid models including the perspective centers. In addition, it determines the systematic instrument errors in the wide-angle and ultra-wide-angle positions for near-horizontal and tilted photographs. Finally,

the authors show the effect of these errors on strip triangulation. The resulting accuracy of the instrument justifies the expectation of good results in practical triangulation work with the PLANIMAT.

References

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