THE THEORETICAL HORIZONTAL ACCURACY OF ADJUSTED BLOCKS OF UP TO 10 000 INDEPENDENT MODELS

by H. Ebner, Stuttgart

Translation by W. Treml, Ottawa

1. Introduction and Synopsis

In recent years, Ackermann has investigated the theoretical horizontal accuracy of the adjustment according to the anblock method of blocks of up to 200 independent models (1), (2), (3). He found that the standard deviations of the adjusted coordinates of model tie points within the block are nearly constant, with only a weak dependency on block size, as long as the periphery of the block is well supported by control points. In a block with 200 models, the maximum standard error is only 1.2 times the standard error of unit weight.

Large, modern computers like CDC 6600, UNIVAC 1108, or the large models of the IBM 360 series already permit the adjustment of blocks of 1000 and more models. It is thus of interest to investigate whether the accuracy properties determined by Ackermann are valid also for blocks of this size.

In the following, the theoretical horizontal accuracy of blocks of up to 10 000 independent models will be investigated for the case of high control density at the periphery of the block (distance between control points two base lengths).

In accordance with the theoretical character of the investigation, overlap, point arrangement etc. are assumed to be schematically ideal: 60 % forward overlap, 20 % lateral, tie points in the model corners, flat terrain. The control points are assumed to be error-free. The functional model (the statistical concepts of the functional and the stochastic model must be distinguished here from the photogrammetric model) of the anblock adjustment presupposes an adequately accurate levelling of the independent models and permits a plane similarity transformation for each. The stochastic model considers the measured coordinates of the independent models as uncorrelated and having equal accuracy. This simple error theory is at present being tested by the author with the help of more general mathematical models. The theoretical foundations for this are published in (4).

In (3) it was determined that for high control point density at the block edges the accuracy is practically independent from the shape of the block. The investigation was therefore restricted to square blocks. For the computation, the hypermatrix code described in (5) was used. As a result, the standard errors of the coordinates of the tie points in units of the standard error of unit weight σ_o was obtained.

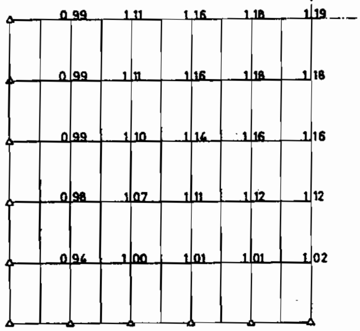
2. Results and Discussion

The investigation comprises seven square blocks of different sizes, in which the number of strips varies between 10 and 70 in steps of ten. The number of models ranges therefore between 200 and 9 800. The control points are arranged uniformly at the periphery of the block with a distance of two base lengths between points. Figure 1 shows the standard error of the coordinates of selected tie points in units of the standard error of unit weight for blocks of 10, 30, 50 and 70 strips.

The equality of σ_x and σ_y is a characteristic of the anblock method in connection with the simple stochastic model on which it is based. Considering the slight accuracy differences within the individual blocks, it appeared sufficient to indicate the standard errors of only 81 tie points in a regular 9 x 9 grid. Since each block has two axes of symmetry, it is also sufficient to show a quarter of the block and thus the standard errors of 25 tie points only.

.10 strips

200 models

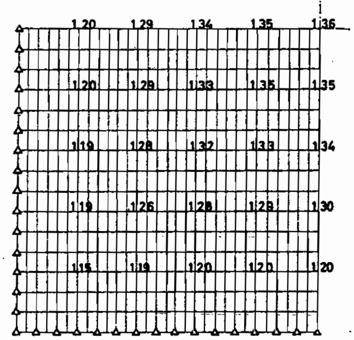


$\sigma_{max} = 1.19\ \sigma_o$

$\sigma_{mean} = 1.08\ \sigma_o$

30 strips

1800 models

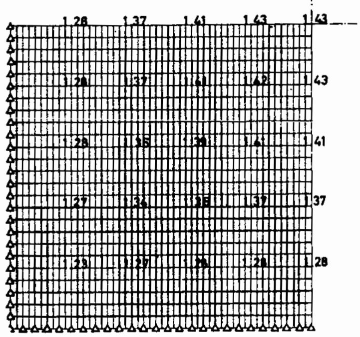


$\sigma_{max} = 1.36\ \sigma_o$

$\sigma_{mean} = 1.27\ \sigma_o$

50 strips

5000 models

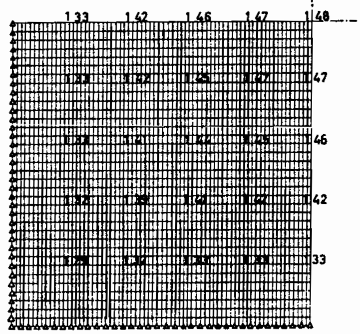


$\sigma_{max} = 1.43\ \sigma_o$

$\sigma_{mean} = 1.34\ \sigma_o$

70 strips

9800 models



$\sigma_{max} = 1.48\ \sigma_o$

$\sigma_{mean} = 1.39\ \sigma_o$

Figure 1. The standard errors $\sigma_x = \sigma_y$ of the coordinates of selected tie points in units of the standard error of unit weight σ_o for square blocks of 10, 30, 50 and 70 strips

For a representative indication of accuracy, the maximum standard error σ_{max} (in the middle of the block) and the mean value σ_{mean} (mean square value of the 81 standard errors) are also shown. For the block of ten strips, the standard errors agree with the corresponding values found by Ackermann. Only $\sigma_{mean} = 1.08\ \sigma_o$ differs slightly from the mean square value $\mu = 1.06\ \sigma_o$ computed from the standard errors of all tie points.

It shows that the accuracy stays very homogeneous even in extraordinarily large blocks, and that the maximum standard error even for 9 800 models only reaches the magnitude $1.48\ \sigma_o$. Figure 1 also shows that the accuracy difference between points arranged symmetrically to the diagonal of the block becomes smaller with increasing number of models; in the block of 70 strips with three-digit indication it even disappears: the shape of the individual model becomes pratically immaterial. (A block of this size at a photo scale of 1 : 40 000 would, for instance, cover the territory of the German Federal Republic, and the theoretical standard errors of all coordinates would be smaller than 1 metre).

To show the interrelationship between accuracy and block size, figure 2 shows the mean and the maximum standard error in units of σ_o over the number of strips.

For blocks of 4, 6, 8 and 10 strips, σ_{max} and μ were taken from (3). At ten strips, the above mentioned jump between $\mu = 1.06$ and $\sigma_{mean} = 1.08$ occurs.

Figure 2 shows strikingly that the characteristic of the curves is better than linear. They can be very well approximated with the following logarithmic functions (maximum residual deviations are 1 %):

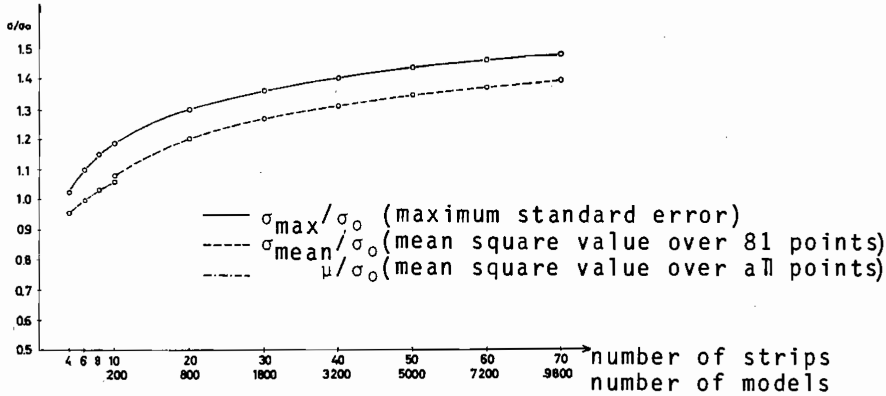


Figure 2. Maximum and mean standard errors of the horizontal coordinates of the tie points in units of the standard error of unit weight σ_o for square blocks with high control point density at the edges

$$\sigma_{mean}/\sigma_o = 0.71 + 0.37 \log n_s \quad (10 \leqq n_s \leqq 70) \qquad (1)$$
$$= 0.65 + 0.185 \log n_M \quad (200 \leqq n_M \leqq 9800)$$

$$\sigma_{max}/\sigma_o = 0.81 + 0.37 \log n_s \quad (4 \leqq n_s \leqq 70) \qquad (2)$$
$$= 0.75 + 0.185 \log n_M \quad (32 \leqq n_M \leqq 9800)$$

n_s = number of strips
n_M = number of models

The similarity of these empirical determined relationships with the logarithmic function for the mean square value of the standard errors in levelling nets derived theoretically in (6) seems very remarkable. It confirms the opinion gained from the comparison of the accuracy structures of geodetic nets (7) and of photogrammetric horizontal blocks, that the error propagation in two-dimensional systems is subject to similar laws in spite of the differences in the individual elements.

Equations (1) and (2) show a constant difference between σ_{mean} and σ_{max}. This means that the quotient decreases with increasing block size. Table 1 shows the values:

n_s	σ_{max}/σ_o	σ_{mean}/σ_o	$\frac{\sigma_{max}}{\sigma_{mean}}$
10	1.19	1.08	1.10
20	1.30	1.20	1.08
30	1.36	1.27	1.07
40	1.40	1.31	1.07
50	1.43	1.34	1.07
60	1.46	1.37	1.06
70	1.48	1.39	1.06

Table 1. The relationship between maximum and mean standard errors for square blocks of 10 to 70 strips with high control point density at the edges

In the planning of aerialtriangulation projects, the given quantities are usually the size of the area on the ground and the desired accuracy. Required is then the optimum flight disposition: type of camera, photo scale, flying height, number of strips or photographs, and the number and distribution of the control points. In our examples, the problem is restricted to the case of square blocks. According to (8) a standard error of unit weight σ_o = 16 μm at photo scale is assumed for wide-angle photographs 9 x 9 in. on film. If the length D of a side in kilometers is chosen as a representative measure of the size of the block, and if the accuracy is denoted by σ_{max} in centimeters, then we obtain for high control point density at the block edges the following relationship between block size, accuracy and number of strips from equation (2):

$$\frac{\sigma_{max}\,|cm|}{D\,|km|} = \frac{1}{n_s}\,(7.2 + 3.3 \log n_s) \qquad (3)$$

Figure 3 gives a graphic illustration of equation (3).

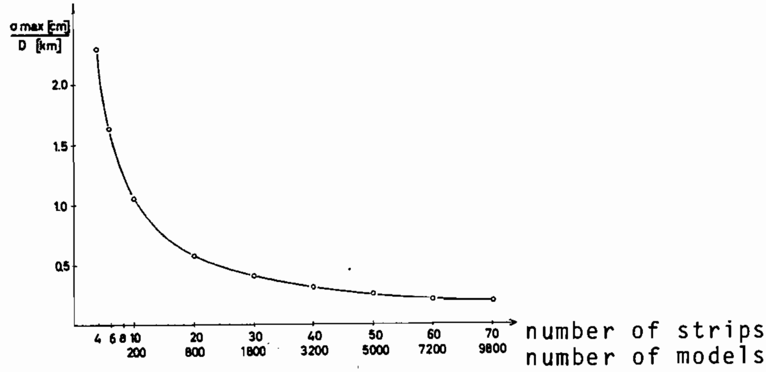


Figure 3. Relationship between block side length D, maximum standard errors and number of strips for square blocks with high density of control at the edges

It is remarkable that for $n_s \rightarrow \infty$, σ_{max}/D converges $\rightarrow 0$. This means that the accuracy can theoretically be increased at will relative to the size of the block. For $n_s = 70$ (9 800 models), it is better than 1 : 500 000. This is an impressive demonstration of the efficiency of block adjustment.

Finally, some remarks concerning the accuracy of the distances between the adjusted tie points. In the inversion of the reduced normal equations, only positive weight coefficients of the adjusted coordinates were obtained for all block sizes. This means that the accuracy of all distances between tie points is better than the accuracy that would result from the standard errors alone (disregarding the correlation). For the distances between directly adjacent points (s_1 = base length, s_2 = two base lengths) the standard errors σ_{s1} and σ_{s2} expressed in units of σ_0 turned out to be practically constant for all block sizes and nearly independent of the position of the points within the blocks:

$0.92 \leqq \sigma_{s_1}/\sigma_o \leqq 0.94$ for all block sizes and all
$1.15 \leqq \sigma_{s_2}/\sigma_o \leqq 1.22$ positions within the blocks

These results prove the high horizontal accuracy of adjusted blocks with high control point density at the edges for practically any block size. For 10 000 models, the maximum standard error of the coordinates of the tie points still remains below 1.5 σ_o. The photogrammetric determination of points through block adjustments is therefore very suitable for cadastral and reallotment purposes as well as for the densification of geodetic networks. It appears therefore to be possible and expedient for mapping large spaces or even whole continents, as for example in the Brazilian or Australian 1 : 100 000 mapping projects, to limit the geodetic surveys to the establishment of a widely spaced traverse framework and to densify each resulting loop with a single photogrammetric block. The individual stations of the traverses would then supply the control point structure for the block edges which is essential for the accuracy.

3. Computer processing details

To keep the numerical effort as small as possible, the reduced normal equations based on the unknown coordinates were established directly. Because of the schematic arrangement of the models and of the control point distribution, the x and y coordinates are not interrelated in this case: The system of normal equations is split up into two equal, independent partial systems. As far as the numerical solution is concerned, this means that the number of unknowns as well as the bandwidth is reduced to one-half, with a consequent reduction of the numerical effort to 1/8. If n_s denotes the number of strips, then the number of unknowns of the adjustment is obtained as $2\,n^2 - n + 1$. The bandwidth was minimized by arranging the consecutive numbering of the tie points transversely to the direction of the strips. In the numerical solution, advantage was taken of the fact that only the standard errors of 81 tie points in a regular 9 x 9 grid were of interest, of which only 25 points located in a quarter of the block are different from one another because of the symmetrical properties of the blocks. It was therefore possible to substitute a solution of the normal equations with 25 righthand sides (the corresponding 25 columns of the unit matrix) for the complete inversion of the reduced normal equation matrix. It was thus possible to use the FORTRAN-ASA programme HYCHOL (5) by Klein for the solution of large equation systems subdivided into sub-matrices with many right-hand sides.

The computation was carried out in the CDC 6600 with core storage for 128 K words of 60 bits each and external disc storage of 11 000 000 words. Central processor time is determined primarily by the number of arithmetic operations, input/output time by the number of disc instructions. For the solution of large systems with HYCHOL on the CDC 6600, we obtain in good approximation:

$$\begin{array}{l} \text{CP in seconds: } 6 \times 10^{-6} \times \text{number of multiplications} \\ \text{IO in seconds: } 4 \times \text{number of submatrices} \end{array} \qquad (6)$$

For the computed blocks, Table 2 shows the photogrammetric data and the data determining the numerical solution, and the computing times on the CDC 6600 (including compilation).

number of strips n_s	10	20	30	40	50	60	70
number of models $2n_s^2$	200	800	1800	3200	5000	7200	9800
Unknowns $2n_s^2 - n_s + 1$	191	781	1771	3161	4951	7141	9731
band-width $2n_s$	20	40	60	80	100	120	140
No. of right-hand sides	25	25	25	25	25	25	25
CP-time	11^s	21^s	50^s	1^m52^s	3^m37^s	6^m31^s	10^m57^s
IO-time	3^m11^s	6^m05^s	9^m32^s	13^m43^s	17^m38^s	21^m01^s	24^m03^s
system time	34^s	1^m05^s	2^m00^s	3^m51^s	6^m24^s	10^m23^s	16^m07^s

Table 2. Data on the numerical solution of the reduced normal equations with 25 right-hand sides. Square blocks of 10 to 70 strips and high control point density at the block edges.

The system time of only 16 minutes 7 seconds for the block of 9 800 models (9 731 unknowns) demonstrates strikingly the efficiency of the CDC 6600 and of the HYCHOL programme.

Bibliography

(1) ACKERMANN, F.: On the theoretical accuracy of planimetric block triangulation. Photogrammetria 21, p. 145 - 170, 1966.

(2) ACKERMANN, F.: Theoretische Beispiele zur Lagegenauigkeit ausgeglichener Blöcke. BuL 35, p.114 - 122, 1967.

(3) ACKERMANN, F.: Gesetzmäßigkeiten der absoluten Lagegenauigkeit von Blöcken. BuL 36, p. 3 - 15, 1968.

(4) EBNER, H.: Zur Überprüfung der unter vereinfachten Annahmen erhaltenen Genauigkeit von Ausgleichungen mit Hilfe übergeordneter Modelle. Z.F.Verm. 94, p. 264 - 268, 1969.

(5) ACKERMANN, F., EBNER,H., KLEIN,H.: Ein Programmpaket für die Aerotriangulation mit unabhängigen Modellen. BuL 38, Seite 218 - 224, 1970 (A programme package for block adjustment with independent models).

(6) MEISSL, P.: Über die Fehlerfortpflanzung in gewissen regel mäßigen, flächig ausgebreiteten Nivellementsnetzen. Z.f.Verm. 95, p. 103 - 109, 1970.

(7) GOTTHARDT, E.: Genauigkeitsuntersuchungen an schematischen trigonometrischen Netzen. Festschrift zum 70. Geburtstag von Prof. Walter Grossmann, Verlag Konrad Witwer, p. 123 - 131, Stuttgart 1967.

(8) ECKHART, D.: The Anblock Method: Problems of Programming, Practical Results, lecture at the International Symposium on Aerialtriangulation, Urbana/Illinois, 1966.