HORIZONTAL BLOCK ADJUSTMENT WITH LARGE NUMBERS OF POINTS

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1. Advantages of Block Adjustment

In geodesy, the principle has long been observed that the adjustment of a system is carried out not only with a selection, but with all relevant measuring data. For a long time, this principle has rarely been used, if at all, in aerialtriangulation. The rationale appears to be that it did not seem technically feasible to handle the extensive adjustments under consideration of all data and relationships. Graphical or other approximate procedures were therefore regarded as adequate for practical purposes, a historically ingrained concept from which only a gradual detachment seems possible even with the use of modern computers.

With the extremely efficient and fast computers now available practically all restrictions are removed which until now have impeded the full utilization of all measured data in adjustments of greatest possible rigorousity. The decisive moment in this connection is that the optimum exploitation of large quantities of data is not only technically feasible, but also considerably more economical if high speed-computers are used rather than simple methods in small computers. For example, the efficiency ratio (computing speed) between the Zuse Z 25, a computer widely used in Germany for survey applications, and the CDC 6600 is about 1 : 30 000, while the cost-ratio per hour of computer time is only about 1:30. Even if various marginal conditions diminish this difference and rigorous adjustments require much higher computational efforts, there still remains a significant economical advantage if large data quantities must be processed in large computers. Based on the extensive data from a reallotment operation, the photogrammetric determination of horizontal points from individual models will in the following be compared to a block adjustment. For this case of determination of a very large number of targeted boundary points (several thousand) for cadastral purposes, several favourable properties of the block computation compared to individual model restitution can be indicated ab initio:

- a horizontal block aggregate which considers all ties is more homogeneous internally than individually oriented models; the neighbouring accuracy is therefore more favourable.
- due to the better homogeneity, blunders in the block are more easily detected and eliminated.
- for equal accuracy requirements, a horizontal block requires far fewer control points than the restitution of individual models.
- with an existing trigonometric third-order net, no additional control points are required at all, or only a few.
- with a block adjustment, there is no requirement to position control points in specific locations in the terrain. In particular, control points don't have to be located in the corners of individual models.
- with this independence from a rigid control point placement, the block becomes practically independent of picture scale. In particular, this means freedom to chose larger picture scales with their desirable advantages of improved accuracy, confident point identification, and the possibility to fly in poor weather.
- in the block, the entire organization and arrangement of data is done in the computer, a factor of considerable significance in the processing of thousands of points. The block adjustment includes of course the transformation of all individual points, thus eliminating separate computational steps for this work.

The number of technical and practical advantages is obviously so great that the block adjustment, i. e. the optimum exploitation of the measurements in large computers, should always be used for photogrammetric cadastral surveys if large amounts of data must be processed.

2. The Anblock Programme

The block adjustments discussed in the following have been carried out according to the "anblock" method (interlinked Helmert transformations, see [1]). The programme was written by the mathematician K. Ballein.

A first version of the programme was originally written in ALGOL for the Telefunken TR 4 computer. In this version, an iteration method was used for the solution of the reduced normal equations, but the experience with it was not satisfactory.

Since in reallotment operations in Baden-Württemberg and elsewhere, up to ten thousand new points are established in one operation, with an average of at least 50 tie points per model, the programme was rewritten for the CDC 6600 and expanded especially in view of the large quantity of data. Attempts to work with the ALGOL compiler were soon discarded and an entirely new programme was written in FORTRAN.

It is not possible to give here a programme description, but it should be mentioned that it is not the solution of the normal equations from the transformation parameters, but the fully automatic organisation and arrangement of the large volume of data that presents the greatest difficulties.

In future, the adjustment of reallotment for cadastre blocks will no longer be carried out with the anblock programme, but with the programme for spatial block adjustment with independent models which makes it possible to eliminate the levelling of the individual models in the instrument and to get along with few vertical points that need not even be known with very great accuracy.

3. The reallotment operation Hermuthausen-Steinbach

Through the courtesy of the Baden-Württemberg government office for reallotment and settlement (Landesamt für Flurbereinigung und Siedlung) in Ludwigsburg, the data of the completed photogrammetric measurement of the reallotment operation Hermuthausen-Steinbach was available for block adjustments. Briefly the most important information:

- area 1090 hectar, regarded as a medium-sized operation
- photoflight spring 1968, by the firm K. HÄUSSERMANN, with WILD RC 8, 6-inch focal length, 9 x 9 in. picture format, picture scale 1:6 000
- 4 strips in north-south direction with a total of 32 models.

The photogrammetric restitution was done in 1968 at the reallotment office in Ludwigsburg, according to the customary procedure of restitution of individual models. This included the measurement of all targeted points in the C8 stereoplanigraph in two passes after levelling and approximate scaling of each model. The individual models were transformed into the ground coordinate system with on the average 5 to 6 control points by means of a Helmert transformation (in the IBM 1401); the coordinates of common tie points were meaned. This represented the final results of the photogrammetric determination of the coordinates.

27 trigonometric points existed in the area, and an additional 64 control points had to be established in the terrain for the restitution of the individual models.

The total number of new points was 4 826, 860 of which were determined more than once. On the average, each model contains 123 individual points, 62 new points measured in more than 1 model (in the block, these points become tie points) and 5.6 control points. The corresponding maximum numbers per model are 419, 135, and 9 respectively.

For the block adjustment, the original machine coordinates measured in the C8-Stereoplanigraph (mean of 2 passes) - a total of about 6 000 measurements - and the correct coordinates of the 91 control points were available on punched cards. Blunders had already been eliminated or corrected.

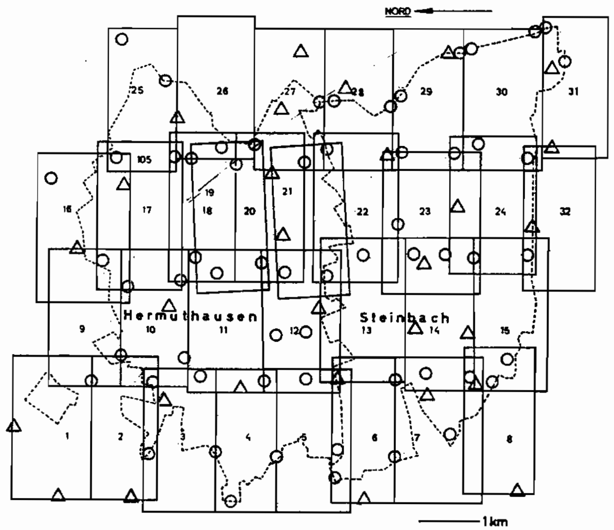


Figure 1. Control point distribution of adjustment I

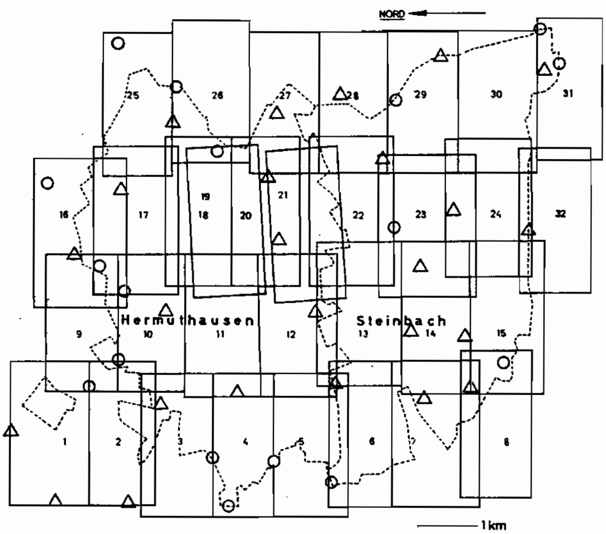


Figure 2. Control point distribution of adjustment II

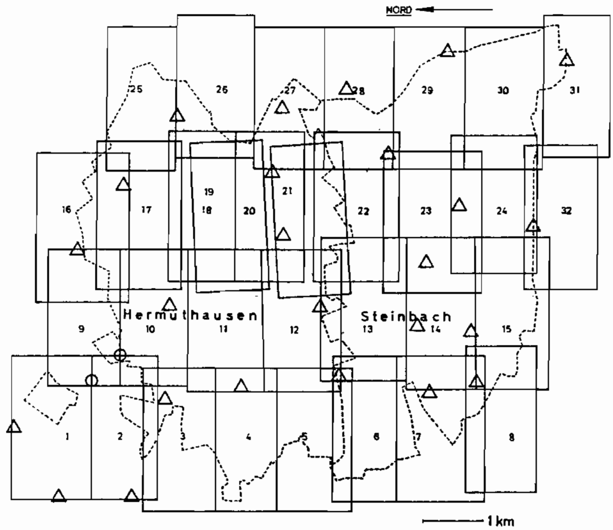


Figure 3. Control point distribution of adjustment III

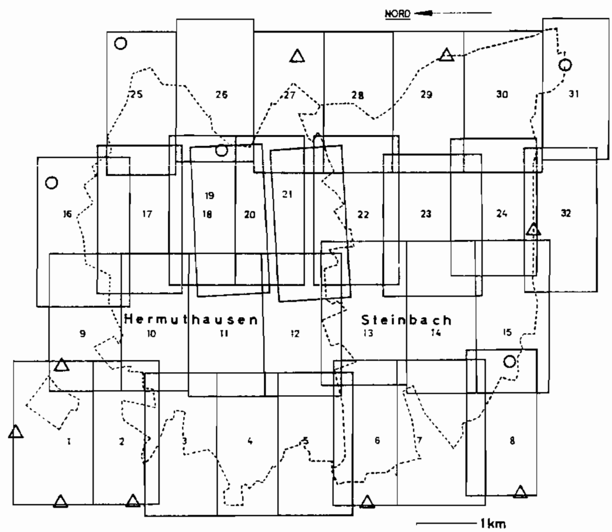


Figure 4. Control point distribution of adjustment IV

4. Comparative Horizontal Block Adjustments

With the basic material described, 4 different horizontal block adjustments were carried out on the CDC 6600 of the University of Stuttgart.

I Complete adjustment of the given material, with all control and tie points. This adjustment is considered as the optimum utilization of the given data and serves as a standard for the comparison with the individual model restitution.

II Adjustment with the 27 given trigonometric points and a few supplementary points at the edges of the block and with all tie points. This is to become standard procedure in the future.

III Adjustment only with the given trigonometric points and with all tie points.

0 Corresponding to I to IV, the designation 0 is used in addition to denote the results of the individual model restitution of the reallotment office.

First, the effectiveness of the block adjustment in comparison with the individual model restitution (0) was to be demonstrated with adjustment I; in this form, it had not yet been demonstrated up to now. With the other versions it is shown to what extent the control point arrangements can be varied without significant adverse effect on the results. Even without absolute comparison measurements, they were to supply an indirect confirmation of the relationship known only theoretically so far.

5. Results

In view of the large amount of data, the results can be represented here only in condensed form.

Version	n_M	n_{PP}	n_{VP}	n'_{PP}	n'_{VP}	r	σ_o	$\bar{v}_{PP}$	$\bar{v}_{VP}$	v_{PP}^{max}	v_{VP}^{max}
0	32	91	860	182	1720	(236)	(7,0cm) (11,6µm)	5,6cm 9,3µm	6,9cm 11,5µm	21cm 35µm	18cm 30µm
I	32	90	855	179	1810	2140	4,8cm 8,1µm	7,3cm 12,1µm	2,9cm 4,9µm	30cm 50µm	18cm 30µm
II	32	42	892	66	1907	2134	4,4cm 7,3µm	9,2cm 15,3µm	2,8cm 4,7µm	28cm 47µm	18cm 30µm
III	32	27	898	45	1918	2002	4,1cm 6,8µm	8,2cm 13,6µm	2,7cm 4,4µm	20cm 33µm	19cm 32µm
IV	32	14	906	19	1941	1980	4,2cm 7,1µm	11,2cm 18,7µm	2,8cm 4,6µm	27cm 45µm	19cm 32µm

Table 1. Results of Block Adjustments.
0 - single model restitution / I - all control points / II -trig.points plus perimeter control points / III -trig. points / IV - perimeter control points

Table 1 contains the important data necessary for the assessment of the various block adjustments or of the individual model restitution. The following quantities are given:

n_M = number of models in the block

n_{PP} = number of control points in the block

n_{VP} = number of unknown tie points

n'_{PP} = number of control point measurements inthe various models

v'_{VP} = number of measurements of tie points

r = redundancy of the adjustment = $2(n'_{PP} + n'_{VP})-4n_M-2n_{VP}$

σ_o = $\sqrt{[vv]/r}$ = standard error of unit weight = accuracy of the coordinate measurements (before adjustment)

$\bar{v}_{PP}$ = $\sqrt{[vv]_{PP}/ 2n'_{PP}}$ = mean square value of all corrections to control points

$\bar{v}_{VP}$ = $\sqrt{[vv]_{VP}/ 2n'_{VP}}$ = mean square value of all corrections to tie points

v_{PP}^{max}, v_{VP}^{max}: maximum values of corrections to control and tie points

Note: The standard error of unit weight of the single model restitution (0) has been computed only from the Helmert transformations, i.e. only from the residual errors of the control points. It should also be remembered that on the average each model contains about 120 individual points that do not contribute to the adjustment and that are therefore not listed in table 1.

The availability of check points with the adjustments II, III, IV, allowed estimates of the absolute accuracy of the adjusted block. For instance, adjustment II had 48 check points, the ground coordinates of which were known, although not used in the adjustment. The root mean square value of the planimetric coordinate errors of the 48 check points amounted to 8,0 cm (or 13,7 μm, thus demonstrating the high absolute accuracy capability of planimetric block adjustment by independent models.

In the comparative assessment of the various adjustments, the changes of the adjusted coordinates of the new points in each case are of interest. The mean square values and the maximum values of the coordinate changes between two adjustments are therefore shown in table 2 with the following designations:

$$\bar{d}_x = \sqrt{|x_{II} - x_I|^2/n}, \qquad \bar{d}_y = \sqrt{|y_{II} - y_I|^2/n}$$

$$d_x^{max} = |(x_{II} - x_I)|^{max}, \qquad d_y^{max} = |(y_{II} - y_I)|^{max}$$

n_{VP} = number of comparison points (tie points).

II-I : version II compared with version I

In the comparison, only the tie points are considered. The inclusion of the large number of individual points was deemed to be too involved. They would alter the result of table 2, which holds for the overlap regions of the models, only in a favourable sense, i. e. reduce the differences.

comparison between	n_{VP}	$\bar{d}_x$	$\bar{d}_y$	d_x^{max}	d_y^{max}
I - 0	853	4,6cm 7,7μm	3,6cm 6,0μm	14 cm 23 μm	12 cm 20 μm
II - I	855	2,7cm 4,5μm	3,1cm 5,2μm	7 cm 12 μm	9 cm 15 μm
III - I	855	9,0cm 15,0μm	4,4cm 7,3μm	28 cm 47 μm	18 cm 30 μm
IV - I	853	9,1cm 15,2μm	4,5cm 7,5μm	21 cm 35 μm	14 cm 23 μm

Table 2. Differences between the adjusted coordinates of the tie points between various adjustments

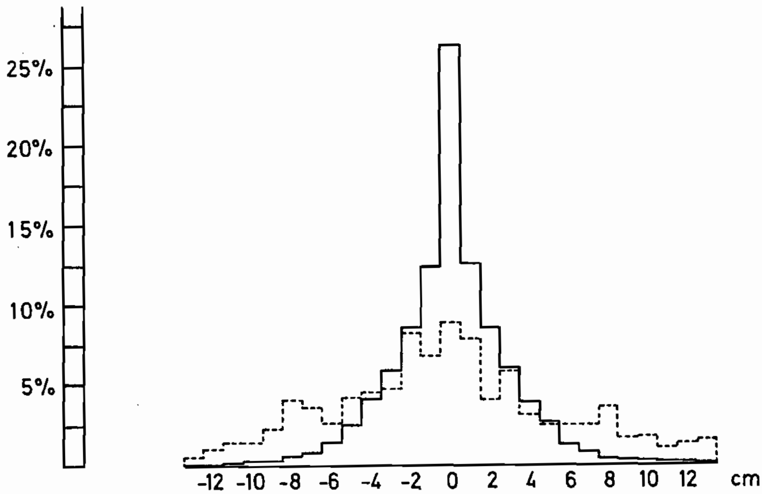
Table 1 shows that the residual errors of the tie points are remarkably small, while conversely the residual errors of the control points are relatively large. To show the relationships in more detail, the frequency distributions of the residual coordinate errors v of the tie and control points are shown in figure 5.

The diagram distinctly shows the entirely different distributions of the two groups. In the distribution of the residual errors of the tie points it is remarkable that the quantities -1 cm, 0, +1 cm account for more than 50 % of all corrections and that only 1 % of all corrections is larger than 9 cm.

6. Brief discussion of the results

6.1 Comparisons between individual model restitution and block adjustment

The individual model restitution (0) and version I of the block adjustment have been carried out with practically the same data.



tie points n_{VP}= 1810, $|v|$>13 cm: 0,2 %, $|v|_{max}$ = 18 cm
control points n_{PP}= 179, $|v|$>13 cm: 5,8 %, $|v|_{max}$ = 30 cm

Figure 5. Frequency distribution of the coordinate corrections v of adjustment I

The comparison of the results confirms the expectation that the block adjustment is more homogeneous within itself. In the block, the mean value of the gaps at the model transitions is very effectively reduced by more than half (58 %): The mean square residual errors $\overline{v}_{VP}$ of the tie points are reduced from 6.9 cm $\hat{=}$ 11.5 microns to 2.9 cm $\hat{=}$ 4.9 microns, a remarkably good result significantly confirmed by more than four thousand values.

Conversely it is shown that the fit of the models to the tie points has become worse by 30 % in the block. In view of the large weight by which the many tie points outnumber the control points, this result is understandable. It is corroborated by the mean coordinate change (I - 0) of 4.1 cm $\hat{=}$ 6.9 microns. It should still be further investigated how the residual errors of the control points are to be assessed. In this connection, it is not unlikely that net tensions are also involved here. If a flawless trigonometric net serves as a basis, the possibility exists to force a closer fit to the net by assigning higher weights to the control points. This procedure is not, however, considered proper.

6.2 Comparisons between the various block adjustments

First, it must be noted that the standard errors of unit weight σ_0 are remarkably small with 4 to 5 cm (7 to 8 microns at picture scale). The overall quality of the data is therefore excellent.

Of greatest practical interest is version II. In it, all available trigonometric points and a few additional control points are used to support the edges of the block. It is suggested that this version be used in reallotment and cadastral practice, since for reasons of relative accuracy between adjacent points, all points must be included in the adjustment anyway. Table 1 shows that version II is practically equivalent to the optimal version I. The differences II - I (table 2) also support this favourable impression with average values of 2.9 cm ≙ 4.8 microns.

The remaining adjustments III and IV again displayed the good internal homogeneity of the block adjustments with practically unchanged, small values for σ_0 and for the gaps at the model transitions. Also preserved is the phenomenon that the tensions at the control points are considerably greater than those at the tie points.

Of particular interest is the finding that through the various combinations of control points, the field of points changes only by an average of 7 cm ≙ 12 microns in its coordinate values. This result is remarkable since the accuracy of the net can be assumed to be about 5 cm. Indirectly, it confirms the theory, [2], according to which blocks that are supported by control only at the edges are only very slightly inferior to blocks that are densely populated with control points, or to the individual model restitution.

The figures of Table 2 indicate, confirmed by an analysis of the corrections v_x and v_y, that the residual tensions in x direction (transverse to the direction of flight) are always greater than in y direction (in the direction of flight). In spite of the extraordinarily small magnitudes of the corrections v, significant systematic errors thus still exist. Through the application of affine transformations, which would be permissible with such large numbers of points, the results could presumably still be noticeably improved.

Summing up, the intriguingly high accuracy of the block adjustments must once more be emphasized. The results prove that block triangulation can replace individual model restitution. Version II (existing trigonometric points + edge control points) can be particularly recommended for application to reallotment operations or for cadastral surveys of similar character.

7. Supplement

In the course of reallotment operations in Baden-Württemberg, a very large number (in this case, approximately 6 000) of distances are measured in the terrain in order to satisfy the requirement of independent checking. Hitherto, the distances computed from photogrammetrically determined coordinates have merely been compared with these directly measured distances and tested for compliance with the official error tolerances.
If these data are used not only for checking purposes, but if they are also - according to the principle of utilization of all data - considered as determining elements for the coordinates, yet another increase of accuracy can be achieved, especially in the relative accuracy involving short distances. The simultaneous adjustment of the photogrammetric and terrestrial measurements of the Hermuthausen-Steinbach operation has been carried out on the basis of block adjustment I. A report on the very satisfactory results and on the other properties of this simultaneous adjustment of photogrammetric and terrestrial data can be found in [3] .

Bibliography

[1] VAN DEN HOUT, C.M.A.: The Anblock Method of Planimetric Block Adjustment. Photogrammetria 21, p. 171-178, 1969.

[2] ACKERMANN, F.: Gesetzmäßigkeiten der absoluten Lagegenauigkeit von Blöcken. BuL 36, p. 3 - 15, 1968.

[3] KRAUS, K. and BETTIN, R.: Kombinierte photogrammetrisch-terrestrische Katastervermessung. BuL 38, p. 241-248, 1970. (Combined Photogrammetric-terrestrial Cadastral Surveying).