A PROGRAMME PACKAGE FOR BLOCK ADJUSTMENT WITH INDEPENDENT MODELS

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1. System Concept

The Institute for Photogrammetry of the University of Stuttgart centres its programme developments on a programme system for spatial block triangulation with independent models; the target concept is far-ranging, directed towards very large blocks; it was therefore never aimed at the most direct programming of a certain procedure, but at the systematic contruction of a comprehensive system, which we have named the programme package PAT-M (= Programme Package Aerial-Triangulation with Models). It consists of self-contained units (subprogrammes) which required some basic research. Very useful experiences and results from the preceding programming of strip triangulation with independent models were incorporated.

The programme system for block triangulation was to be as universal as possible, capable of future expansion, even if it was not possible to develop all its variations immediately. In particular it was to fulfill the following requirements:

- although intended for large computers (CDC 6600), the programme should not be restricted to them.
- as far as the computer programme is concerned, the block size should in principle be unlimited. Total capacity of the computer determines the limits.
- This important requirement means two things: on the one hand it should be possible to adjust even extremely large blocks in large computers, if necessary with computing times of several hours. On the other hand, computers with small central processing units

(for example 32 K) would be capable of treating larger blocks, if adequate external storage is available and if long computing times are possible.

- The programmes are written in FORTRAN-ASA, a completely machine-independent language available everywhere.
- All frequently required operations (matrix multiplications, disc operations) are provided as independent subroutines in the programme, which can be replaced by subroutines in the assembler as required. Further optimizing is thus possible for any computer without requiring rewriting the programme.
- The programme should not impose any restrictive conditions to disposition and execution of the block triangulation: any kind of model aggregate should be accomodated, for instance with cross-strips, any lateral overlap, multiple overlap, high and low-altitude photography, any number of tie points, model points, control points; arbitrary numbering of the terrain points, without coding for point type; ordering and establishing relationships of the measurements must be done by the programme.
- The programme system must provide for several variants of the functional model for the block triangulation including the use of auxiliary data.

  A first variant is the programme

  PAT-M 43: spatial similarity transformation of models, determined separately horizontally and vertically for the sake of a reduced numerical effort (4 + 3 parameters per model).

  A further variant

  PAT-M 7 provides for the spatial similarity transformation with simultaneous determination of all 7 transformation parameters per model. At greater numerical effort, it is the theoretically more rigorous solution.
- The stochastic model should also be variable and provide the possibility to assign different weights for the x, y, z coordinates of the model points and of the projection centres. It

should also be possible to treat the terrestrial control point coordinates as stochastic quantities of arbitrary weight. Apart from error-theoretically more rigorous results, this procedure promises easier detection of control point errors (see (1)).

- A central subprogramme for the solution of the normal equations should be developed which can be used with all variants. The solution of the equation systems forms the major share of the numerical effort. It was therefore essential to optimize this subprogramme to a high degree.

2. The subprogramme HYCHOL for the solution of equation systems of any size

The programme package was to provide for the simultaneous adjustment of blocks of at least 1 000 or more models. The large number of unknowns required an especially effective solution. As in strip adjustment, Gauss methods (direct solution) and the iterative methods of conjugated gradients were considered. Among the Gauss methods, the Cholesky method as applied to the submatrices of a hypermatrix (hyper-Cholesky) was chosen for considerations of numerical effort and computational acuity. As for strips, theoretical comparisons of computer-time requirements have been made also for the block. While these considerations indicate a smaller difference between direct and iterative methods of solution than in strip adjustment, they did not indicate a clear advantage in required effort for the method of conjugated gradients. Because of the unfavourable dependency of computing time, when using the gradient method, on the always inadequate quality of the approximate values of the unknowns in the case of aerialtriangulations, and especially on the number and distribution of control points in a block, we decided to use the Gaussian solution in the form of a hyper-Cholesky method. The programme was therefore named HYCHOL. The decision to use a non-iterative solution is in agreement with (3) where a Gaussian solution is preferred, too.

For a number of theoretical investigations, HYCHOL offers the advantage that further column vectors can be processed at the same time without much additional effort as right-hand members of the system of equations, thereby permitting the simultaneous computation of several solutions. In this way, a substitute for inversions is available by introducing selected columns of the unit matrix as right-hand members, from which the corresponding columns of the inverted coefficient matrix are obtained as solutions (see (4)).

The subprogramme HYCHOL is written in FORTRAN-ASA. It handles most effectively band structures; zero submatrices within the band are skipped, i. e. neither stored nor computed (figure 1).

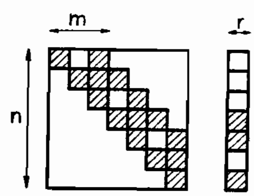


Figure 1. Structural diagramme of a system of equations with n unknowns, bandwidth m and r right-hand sides; with 0-submatrices within the band and at right.

The system of equations to be solved is stored externally (disc or drum), transferred in the course of computation submatrix by submatrix to core storage, reduced, and again stored externally. After the subsequent resubstitution, the solution is again stored externally.

Computing time is essentially determined by the number of multiplications and the number of submatrices (external storage). The number n_{Mult} of the multiplications required for a solution can be estimated for HYCHOL, in good approximation, according to the following relationship:

$$n_{Mult} \approx \left(\frac{s^2}{2} - \frac{s}{2} + \frac{1}{3} \right) ut^3 + (2s - 1)ut^2 r \qquad (1)$$

where s = mean number of submatrices per hyperline

u = number of hyperlines

t = mean number of columns or rows of a submatrix

r = number of right-hand vectors (for pure solution: r = 1)

for Figure 1, for example, s = 3, u = 7.

A large number of computed runs in the CDC 6600 (128 K words of 60 bits each of core storage, magnetic disc with $11 \cdot 10^6$ words) of the Regional Computer Centre of the University of Stuttgart permits to make reliable statements on the computing time required for the solution of large systems

$$CP[sec] \approx 1,5 \cdot t_{Mult} \cdot n_{Mult} \qquad (2)$$

$$IO[sec] \approx 4\ (s + 1)\ u \qquad (3)$$

where CP = computing time in central processor

IO = input-output time (disc traffic)

t_{Mult} = CP-time for one individual matrix multiplication/number of the individual multiplications contained therein.

For the CDC 6600,

$t_{Mult} = 4 \cdot 10^{-6}$ seconds in FORTRAN, or

$t_{Mult} = 2 \cdot 10^{-6}$ seconds in the assembler.

The system time is computed from CP and IO time:

$$\text{system time} = CP + k \cdot IO \qquad (4)$$

where k = proportion between core storage used and total core storage (for submatrices 50 x 50, k is approximately 0.2).

Computer costs are determined by system time. Depending on priority, one second of system time costs DM 1.25 to DM 1.80.

In the development of the HYCHOL programme, great efforts were made to optimize the programme as far as possible. The quantities expressed in relations (1) to (3) show that the programme is indeed extraordinarily fast.

3. The programme PAT-M 43 with horizontal and vertical iteration

3.1 Error equations

The first version of the programme package solves the problem of the general spatial block adjustment with independent models in the form of several iterations of separate horizontal and vertical adjustment. This version requires a relatively low numerical effort, since the seven transformation parameters per model are subdivided into groups of four and three parameters. Accordingly, the error equations are also subdivided into two groups:

1. group: Error equations for horizontal adjustment (Anblock), linear:

$$\begin{aligned} v_x &= -xa + yb - \Delta x + x_G \\ v_y &= -ya - xb - \Delta y + y_G \\ v_{Px} &= x_G - x_P \\ v_{Py} &= y_G - y_P \end{aligned} \qquad (5)$$

where x, y = measured model coordinates

x_G, y_G = unknown ground coordinates

x_P, y_P = terrestrial control point coordinates

$a, b, \Delta x, \Delta y$ = parameters of the plane similarity transformation

v_x, v_y, v_{Py}, v_{Py} = corrections

The error equations (5) are formulated for all model points with the exception of the projection centres. The latter do not participate in the horizontal adjustment; they would disturb the results of the initial horizontal adjustment.

Formulation (5) treats the terrestrial control point coordinates as stochastic quantities, i. e. as observations requiring correction. With arbitrarily chosen weights for the control points, depending on net accuracy and photo scale, this formulation is error-theoretically more rigorous than the forced connection to the control points usually employed so far, but adds practically nothing to the computational effort if used according to formulation (5). Advantages can be expected here particularly in the detection of control point errors which manifest themselves in the form of large corrections to the terrestrial coordinates. If the terrestrial coordinates of the control points are to be held to, they need only be assigned correspondingly high weight numbers.

2. group: linearized error equations for vertical adjustment

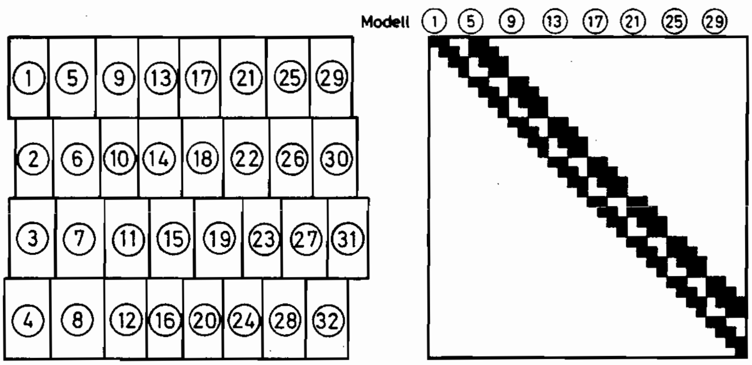
- for the projection centres:

$$
\begin{array}{lllll}
v_x = -zc & & + x_G & & - x \\
v_y = & + zd & & + y_G & - y \\
v_z = xc - yd - \Delta z & & & + z_G & - z
\end{array}
\qquad (6)
$$

- for the other model points

$$
\begin{array}{ll}
v_z = xc - yd - \Delta z & + z_G - z \\
v_{Pz} = & z_G - z_P
\end{array}
\qquad (7)
$$

where x, y, z = measured model coordinates
x_G, y_G, z_G = unknown ground coordinates
z_P = terrestrial control point elevations
$c, d, \Delta z$ = transformation parameters of the (linearized) elevation transformation
v_x, v_y, v_z, v_{Pz} = corrections



■ 4x4 matrix of horizontal block
3x3 matrix of vertical block

Figure 2. Numbering of models transverse to strip direction, and corresponding structural diagram of the reduced normal equation matrix for the transformation parameters

According to (6), the projection centres are used only for the determination of elevation and tilt of the models. They are not used for the horizontal connection of the models - an acceptable approximation. Regarding the correction and the weight of the vertical ground control points, the same comments apply as for the horizontal coordinates.

3.2 Direct formation of the reduced normal equations

To ensure that the programme fulfills the requirements regarding generality with respect to any number of tie points, it is appropriate in the horizontal as well as in the vertical adjustment to reduce the normal equations to the transformation parameters, i.e. to eliminate the unknown coordinates of the points to be determined. To keep the numerical effort as small as possible, the non-disappearing 4x4 or 3x3 submatrices of the reduced system of normal equations pertaining to the parameters of the individual models are formulated without the detour via error equations and complete normal equations. The internal computational dimension is kilometers, in order to obtain coefficients of comparable magnitude. The system of normal equations reduced to the transformation parameters is stored externally as submatrices (size approximately 50 x 50), in the CDC 6600 on magnetic disc. Computer time for the solution of a system of equations of a given size is very largely dependent on the bandwidth (see equations (1), (2), (3)). The bandwidth in turn is determined by the sequence in which the model parameters are arranged in the system of equations. It is therefore necessary to find first an optimum model sequence by means of a sorting programme. In most cases, sorting the models in transverse direction to the strip results in the smallest bandwidth (figure 2). This sorting process, for which there are still other favourable solutions, can be automated.

3.3 The iteration process

The block adjustments begin with a horizontal adjustment by establishing reduced normal equations from the x,y model coordinates, then solving them with HYCHOL. This is followed by a rigorous spatial similarity transformation of all models with the computed horizontal parameters (a, b, Δx, Δy). The subsequent elevation iteration passes through the same steps: establishing reduced normal equations, solution with HYCHOL, and rigorous spatial similarity transformation of each model with the three determined transformation parameters (c, d, Δz). In contrast to the horizontal adjustment, the elevation transformation is not linear. An iteration process would therefore be necessary for this reason alone, not only because of the mutual influence between horizontal position and elevation. For large model tilts it may therefore be useful to carry out two elevation iterations after the first horizontal adjustment. But in the standard case, the simple cycle horizontal-vertical-horizontal-vertical will be sufficient. For reasons of a better convergence of the iteration process and of the computational acuity, the models are transferred onto centre-of-gravity coordinates preceding each horizontal and vertical adjustment, where the centres of gravity are computed omitting the projection centres.

The sequence and number of horizontal and vertical iteration steps can be chosen arbitrary. After each single iteration the maximum coordinate-differences to the preceding step are printed for x, y and z, to document the convergence of the process.

The adjusted ground coordinates are computed by forming the mean from the model coordinates transformed after the last iteration step. This procedure is theoretically rigorous if the weights of the participating coordinates are taken into consideration. For the determination of the final control point coordinates, the weighted mean must therefore be computed.

3.4 Data input and printout of results

The data input is subject to only very few formal conditions. Point numbering is arbitrary, but it must be referred to the terrain points (including the ground control points). Data must be arranged by models (as they are immediately obtained in the measuring of independent models). Each model number is followed by the listing, in arbitrary sequence, of the points measured in that model. Every point must be accompanied by: number of the associated terrain point, model coordinates x, y, z in hundredths of millimeters. Only the projection centres are in a sense excepted, as they must always appear as the first two points in the list of points of a model. The model end is signalized by the separation code -99. The control points with their ground coordinates (in metres) are grouped separatedly according to horizontal and vertical control points in two so-called zero models, both of which have the model number 0. The end symbol -999 signifies the end of the block of data.

In the printout, the input data are shown first. Next come the results of the last transformation of the model coordinates, model by model, with all corrections, followed by the standard error of unit weight and the mean square values of various groups of corrections, for the last horizontal and vertical iteration step. Finally, the listing of the adjusted ground coordinates, i.e. the final result of the adjustment, is printed.

3.5 Detection and elimination of blunders

From the adjustment of practical blocks with up to 300 models we learned that blunders stand out better in blocks than in strips. Therefore the direct strategy, omitting the intermediate step of strip formation, can be retained.

3.6 Theoretical and empirical computing times

Assuming that three iterations each are needed for horizontal and vertical adjustment, computer time for CDC 6600 with programme PAT-M 43 for a complete block adjustment can be estimated as follows:

n_S = number of strips
n_M = number of models
s, u, t, r see equation (1)

Let 4 n_s or 3 n_s transformation parameters be combined in each submatrix. From this follows:

$s = 2$ submatrices per hyperline
$u = n_M/n_s$ number of hyperlines
$t = 4\ n_s$ for the horizontal block
$t = 3\ n_s$ for the vertical block
$r = 1 \ll t$ number of the righthand side; to be disregarded.

Because of the required additional operations, equations (2) and (3) are replaced by:

$$CP[sec] = 10^{-5}\ n_{Mult} \qquad (8)$$

$$IO[sec] = 5\ (s + 1)u \qquad (9)$$

$$k = 0.2 \text{ (see equation (4))}$$

Thus according to equation (1):

$$CP[sec] = \frac{4n_M \cdot n^3{}_s\ (3^3 + 4^3)3}{3n_s}\ 10^{-5} \approx 4 \cdot 10^{-3} n_M \cdot n^2{}_s \qquad (10)$$

$$IO[sec] = \frac{5 \cdot 3 \cdot n_M \cdot 2 \cdot 3}{n_s} = \frac{90\ n_M}{n_s} \qquad (11)$$

System time = CP + 0.2 x IO

Example 1: n_M = 300 models, n_s = 10 strips
CP = 120 seconds
IO = 2 700 seconds
System time = 660 seconds = 2.2 seconds per model

Example 2: n_M = 1 050 models, n_s = 15 strips
CP = 945 seconds
IO = 6 300 seconds
System time = 2 205 seconds = 2.1 seconds per model

Practical spatial blockadjustments with the programme PAT-M 43 have confirmed that normally not more than 3 horizontal and 3 vertical iteration steps are necessary. Our empirical computer times are even shorter than expected theoretically above. Up to now they are in the order of 1.5 sec. per model. Those times demonstrate the efficiency of both, the CDC 6600 computer and the PAT-M 43 programme. Assuming an average cost of DM 1.50 per second of system time, this means that the computation of block triangulations is economically favourable even if it may be necessary to repeat the adjustment several times because of blunders in the data.

4. Other programmes

As repeatedly stressed, the programme PAT-M 43 is only the first working version of the projected programme package for aerial-triangulation. Another version will be PAT-M 7 with a simultanous determination of all 7 model-parameters within an iteration step. Among the programme developments yet to come, the inclusion of auxiliary data, especially APR, is considered to have priority.

Parallel we are working on the programme PAT-B for a rigorous block adjustment with bundles (full analytical method), based on the same general concept as the PAT-M system. First results obtained with this programme are also very promising.

Summary

A program-package for block triangulation (x, y, z) with independent models is presented. First the general requirements for adjustment of very large systems are listed, then the subprogramme HYCHOL for the solution of systems of equations of any size is discussed. Finally the adjustment program PAT-M 43 is reviewed, which operates with planimetry-height iterations. The computing times given demonstrate the high performance of the system.

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