AEROTRIANGULATION WITH INDEPENDET MODELS

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1. A Progress Report from the Institute for Photogrammetry of the University of Stuttgart

This issue of "Bildmessung und Luftbildwesen" is devoted to the theme of aerialtriangulation with independent models. On this theme, a selection of papers from the Institute for Photogrammetry of the University of Stuttgart is presented here. Since its foundation in the summer of 1966, or rather intensively since about two years, aerialtriangulation has been the most important field of work of the Institute. Several efficient computer programmes were developed and it is now possible to report on results and applications.

In the period of time delimited approximately by the International Photogrammetry Congresses in London (1960) and Lausanne (1968), the methods of block triangulation have been essentially developed and the possibilities of mastering the numerical problems have been demonstrated and discussed (compare the reviews (1), (2)). In this period, a number of organizations have developed suitable computer programmes and have since applied block triangulation routinely in practice, especially since various investigations have also resulted in the clarification of important accuracy questions. External appearance notwithstanding, it is however not possible to speak of a conclusion of the development of numerical aerialtriangulation. Not only are important comparative investigations on the theory of errors and on the accuracy of the methods still lacking, but the numerical problems of treatment of very voluminous adjustments or of the general case of aerialtriangulation can not yet be considered to be satisfactorily solved. It must be noted that the encouraging findings and results obtained by a number of agencies have so far met reluctant acceptance as far as their general practical application is concerned.

The true potential of aerial triangulation has not yet been utilized on a broad basis or been made generally accessible to the practitioner.

There are several reasons for this: until now, the computer programmes have often been tied too closely to specific computers and procedures, i. e. they were greatly limited and not freely applicable. Especially in the field of public surveys in Germany, computers of inadequate capacity are still largely used which do not permit to make use of all possibilities of analytical treatment of blocks of models and of large quantities of data. Since the development of efficient computer programmes is very costly, it is often found that either a narrow limit is placed on the scope of the programme to be developed, or the development is stopped as soon as the immediate requirements of the developing organization are satisfied.

Based on this assessment of the situation, it was clearly recognised that in our programme-developments the hitherto existing restrictions would have to be overcome, i. e. we recognised the requirement of very generally applicable, highly efficient and practical procedures with a minimum of restrictive coding conventions. In particular, the programme systems were to be independent of make and type of computer and freely interchangeable; they were therefore to be written in ALGOL 60 and/or FORTRAN IV (Asa norm).

For practical applications as well as for theoretical investigations there is a considerable need to be able to handle large adjustments in which the number of unknowns to be solved may be as great as 10^4. The most important programming target was therefore considered to be the adjustment of systems of large size, i. e. of blocks of at least 1000 models. This requirement originated in the conviction that an exhaustive, comprehensive processing of large quantities of data through efficient numerical methods is not only technically feasible but that it represents at present the least costly and therefore most economical way to achieve an effective increase in efficiency. The conditions for this are given, since in coming years large computers will be increasingly available and accessible through terminals.

For the development and application of the computer programmes for aerialtriangulation, the Institute for Photogrammetry has so far used two computers belonging to the University of Stuttgart: the Telefunken TR 4 of the computer centre of the University, and since early 1969 the CDC 6600, manufactured by Control Data Corporation, installed at the Regional Computer Centre at the Institute for Statics and Dynamics of Aeronautical and Astronautical Constructions. Briefly, here are some technical data: TR 4: 32 K words core memory of 36 bits each; programming languages ALGOL and FORTRAN; floating-point multiplication of 11-place decimal numbers takes 31 micro-seconds in assembler. CDC 6600: 128 K words core memory of 60 bits each; 12 peripheral computer-units, each with 4 K words memory of 12 bits each; 10 functional units (adding, dividing, multiplying, shift moduls); 12 input and output channels of 12 bits each; parallel processing permits the simultaneous processing of different programmes; 1000 lines-per-minute high-speed printer; card reader for 1200 cards per minute; external magnetic tape storage and disc storage for $11 \cdot 10^6$ words per disc; programming languages FORTRAN and ALGOL; assembler floating-point multiplication of 16 place decimal numbers takes 1 micro-second.

The fortunate circumstance that the large CDC 6600 computer has been available since early 1969 has enabled the Institute for Photogrammetry to complete a first part of the planned programmes for the adjustment of large blocks sooner than expected. The experiences made so far are very favourable and confirm that the heuristic principle of processing the given information exhaustively and as rigorously as possible in the form of comprehensive adjustments in large computers has great practical and economical significance.

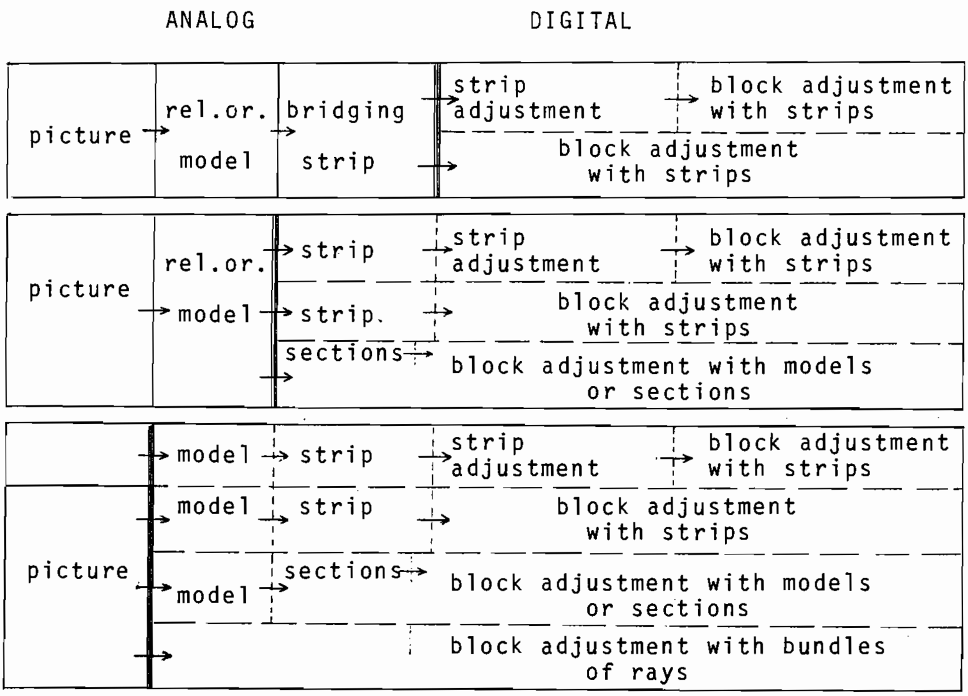

Figure 1: Schematic Diagram of the Methods of Aerialtriangulation

2. The Method of Independent Models

According to the stated intention to programme practical and, as far as possible, generally applicable procedures for the calculation and adjustment of aerialtriangulations, we have decided to concentrate first on the method of independent models, also known as semi-analytical aerial triangulation.

The method of independent models is characterised (see Figure 1) by the connection of independently formed and measured individual models by means of transformations (usually similarity transformations) into a strip or block, simultaneously fitting them to the given control points. They form thus the elementary computational units for strip or block adjustment. A significant point here is that the projection centres are also carried along as model points and are used in the connection of neighbouring models in this strip (see Figure 2). This consequently eliminates the requirement of classical aerialtriangulation to form strips in the triangulation instrument, or computationally, through continuing connection of photographs.

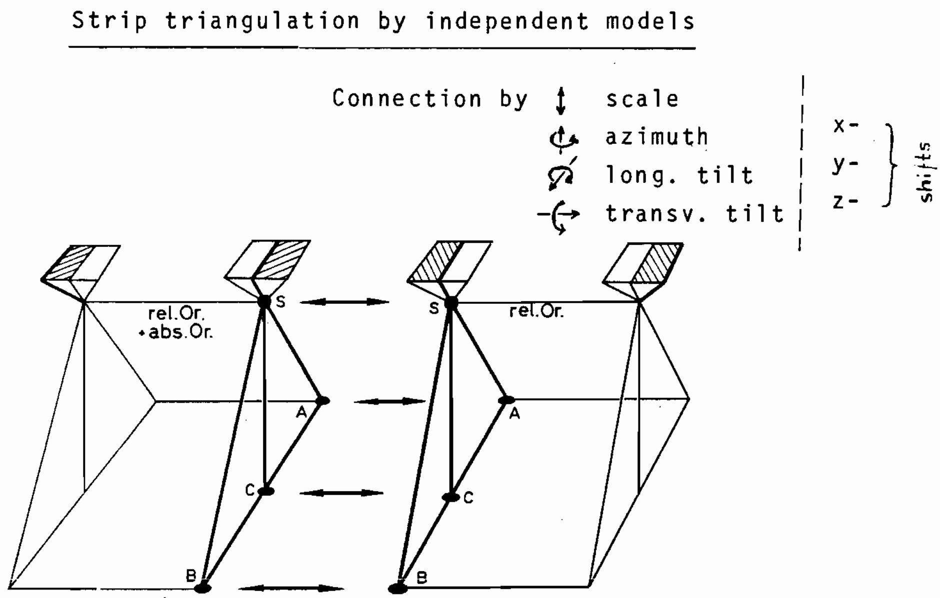

Figure 2: Schematic Diagram of the Connection of Independent Models

The decision to use the method of independent models is based on the favourable combination of their properties from the point of view of practical applicability: the method can be considered to be sufficiently rigorous. The principle of processing a large block of pictures or points in small independent units leads to a favourable numerical structure in the adjustment, it permits the use of simple transformations and of simple error premises, and it is also relatively insensitive to approximations in the premises. The most important property for the present utilization of aerialtriangulation, however, is the general applicability of the method in connection with its theoretical effectiveness. One aspect of general applicability is the suitability of the method to process arbitrarily composed blocks of models (e.g. 60 % lateral overlap, cross-strips, high-altitude flights). Furthermore, the measuring data of aerialtriangulation are so far obtained essentially as picture, model, or strip coordinates (see Figure 1). In each of the three cases, the adjustment can be carried out according to the method of independent models. In this regard, the following remarks may be made.

a) The most rigorous form of aerialtriangulation consists of the measuring of picture coordinates and of the direct use of these data for the computation of the block ("bundle-block"). Doubtlessly, this yields the best accuracy results, relatively as well as absolutely. It has not yet been determined, however, how great the difference in accuracy over the computation of blocks from independent models really is. It could well be 30 % or more (see (3)), and justifies therefore to aim for the aerialtriangulation with bundles of rays as the ultimate method.

   Since the bundle method is dependent on the measured data from comparators, the conditions for its practical application are at this time fulfilled only to a very small extent. The introduction of comparators or, to express it more generally, of analytical procedures into photogrammetric practice has so far been accepted only very reluctantly. Apart from universities and research institutes, only one stereocomparator is presently being used for practical work by a government agency in the German Federal Republic. Photogrammetric companies in this country are not working analytically at all. A programme system for aerialtriangulation based on comparator data could therefore find only few possibilities of application today.

Conversely, however, image coordinates can always be used to compute models which can subsequently be used directly for aerialtriangulation according to the method of independent models. The effect of the approximations that must be accepted in this procedure will have to be determined by further investigations.

b) In practice, a large portion of aerialtriangulations is measured in stereo projection instruments, with the universal triangulation instruments (also known as first-order instruments) still bearing a portion of the load. In these instruments, the independent models can be measured and used immediately as input in the simultaneous adjustment according to the method of independent models. In this case it is of special practical significance that this method no longer requires an inside-outside change of the base. As a consequence, the so-called precision plotters without base change (for example Planimat, A 8, A 10, PG 3, Stereometrograph, Presa 225; Stereocartograph III) also advance to the ranks of full-fledged triangulation instruments. The more recent designs are as accurate today as the present first-order instruments (see for example (4)) and fulfill - if equipped with registration devices - all requirements for aerialtriangulation. It is a significant aim of a computer programme with independent models to open the field of aerialtriangulation fully for these instruments.

If use is also made of the possibility to determine systematic instrument errors by means of grid measurements and to correct them computationally, it will be possible to attain a standard of accuracy for aerialtriangulation with projection instruments closely approaching that of purely analytical methods. This will enable even smaller companies and government agencies, where a comparator could not be fully utilized, to carry out fully acceptable aerialtriangulations with the precision plotters.

Some of the listed instruments (Planimat, PG 2, PG 3) are equipped with special devices for the convenient and accurate determination of the projection centres (see (5)).

It is to be expected that in all cases where comparators are not purchased, the precision plotters will become the preferred triangulation instruments, thus gradually eliminating the universal first-order instruments as we know them (with base change). At least there is no likelihood that there will be newly-designed first-order universal instruments.

c) At present, a large portion of all aerialtriangulations is measured in triangulation instruments (first-order machines) with base change. Although independent models can also be measured immediately in these instruments, it is still common - and will largely remain so for some time to come - that the material supplied for adjustment will be in the form of strips formed by the aeropolygon method in which the projection centres have not been measured.

In this case it is still possible to adjust according to the method of independent models, as long as only the first projection centre at the beginning of the strip has been determined. The remaining projection centres can then be determined from the base components. Even if no projection centre at all had been measured, such Aeropolygon strips can be adjusted according to the method of independent models by introducing fictitious projection centres that have been determined only approximately. The decisive criterion is only that identical (strip-)coordinates are assigned to each projection centre in the two models involved. A further condition is that points measured twice in the area of common overlap of neighbouring models must not be meaned.

3. Other Details

The measurement of independent models for aerialtriangulations is very simple and therefore of immediate economic significance. It is composed of relative orientation, measurement of coordinates, and determination of the projection centres. For the determination of the projection centres three methods exist:

a) Analog to the z_0 determination in the adjustment of an instrument, the machine coordinates of a projection centre can be computed by measuring at least two grid or image points (at arbitrary projector inclination) at two different heights. For this standard procedure, which can be used in principle with all stereoprojection instruments, accuracy information is given in (6). According to this source, the accuracy is not sufficient if only two points are used, especially if the z range of the instrument is small.

According to so far unpublished investigations (7) of Commission AB of the OEEPE, it seems not to be necessary to determine the projection centres of an instrument for every pair of photographs, but it is obviously sufficient to determine them once and for all at the beginning and/or at the end of the triangulation measurements. It is then necessary, however, to keep the base constant and above all to leave the machine coordinate system unaltered. This means that one must either give up the convenience of freehand movement during relative orientation or reconnect the coordinate system every time.

b) Some of the newer instruments (PG 2, Planimat, PG 3) are equipped with devices (levels, autocollimation, engraved distance markings) with which the projection centres can be determined immediately and, except for a constant shift in z, recorded directly. The procedures are fast and accurate and do not constitute a significant increase in the amount of work required for the measurements, even if the projection centres are determined in every model, which is essential in the Planimat because of the change of the projection centres during relative orientation (see (5)).

c) If calibrated crosses are etched on the picture carrier, the respective projection centre can be computed by means of a resection in space. This procedure is very accurate especially with superwide-angle photographs (see (5)).

d) In paragraph 2c) reference has already been made to the possibility to do away with the determination of the projection centres in the instrument when a strip triangulation according to the aeropolygon procedure with continuous connection of pictures (bridging) is carried out. It should be noted that in this case, too, we are dealing with a true adjustment of independent models, i.e., the connection of the pictures with its attendant unfavourable accuracy (see (3)) will be improved correspondingly during adjustment.

e) For the sake of completeness it should be noted that for analytically computed models from comparator measurements the projection centres are also immediately computed and thus always available.

In contrast to the adjustment methods practised so far, the method of independent models also requires a shortening of the base lengths when correcting for earth curvature, to eliminate the convergence of the camera axes.

The methodic conditions for programming and application of aerialtriangulation with independent models are thus fulfilled. Although this paper, as well as the other papers published in this issue, emphasizes the operational points of view of the method of independent models - its efficiency is based on them - this should not create the impression that all details have been fully clarified. Above all, error-theoretical problems still require solution. It remains to be investigated, for example, which weights and correlations must be assigned to the model points, and especially to the projection centres. In the computer programmes, very simple assumptions for the weights have at first been made. The analysis of the empirical results obtained with them is expected to yield clues to improvements.

Little is known also about the effect of systematic image or instrument errors on a strip or on a block, and as a whole the accuracy of the method of independent models compared to polynomial adjustments on the one hand and bundle triangulations on the other must yet be determined theoretically and empirically. These problems are now the subject of various investigations (for example (8)) to which the Institute hopes to be able to contribute further through the computer programmes developed there.

4. Numerical Problems

In the programming of large adjustments for photogrammetric block triangulation, the main problem does not consist of the mathematical formulation, the linearization or development of normal equations or of partially reduced normal equations, although here, too, considerable differences may result. The real difficulty lies in the handling and control of the numerical process. As an additional incidental condition, a large computer should be utilized for more than one hour only in exceptional cases for an adjustment, not so much because of the cost involved than for operational reasons.

As far as the numerical process is concerned, photogrammetric block triangulations are in many respects distinguished by difficult conditions which are considerably less favourable than for example in the case of geodetic net adjustments:

- the number of unknowns (even of the reduced systems) ranges up to an order of magnitude of 10^4, it lies thus 1 to 2 powers of 10 higher than for geodetic adjustments.
- only weak approximate values for the unknowns are known at the start.
- with an unfavourable arrangement of control points, the system may be poorly conditioned.
- even in the conventional block formations, but especially in those with a 60 % lateral overlap or with cross-strips, large band widths are encountered in the reduced normal equations.
- little is known about the problems of the necessary computational acuity for large systems in connection with poor conditions.
- little is known also on the effectiveness of various termination criteria for iterations.
- the convergence behaviour of iterative solutions is difficult to predict.

Photogrammetric literature contains little information on these problems. Experiences and investigations made in the field of geodesy are usually not farreaching enough to be really conclusive. The best information can be found in the papers published by D. Brown's group (for example (9)).

In the development of programmes for aerialtriangulation with independent models, experiments and considerations have led to a clear preference for the direct solution of the normal equation instead of the iterative solution. The development of an especially efficient programme for the solution was an essential precondition, though. With few linearizations and a direct solution of the respective normal equations, the overall solution is obtained in an efficient process even with very large systems.

With the decision for direct methods of solution, a number of the numerical problems mentioned earlier is avoided. The questions connected with conditioning, computational acuity and approximate values can be answered only by investigations and further experience with the developed programmes. The experiences made so far with the computer programmes confirm however, that at least for practically feasible block and strip triangulations no difficulties are to be expected in the numerical process.

5. The Computer Programmes

The programme developments at the Institute for Photogrammetry of the University of Stuttgart have so far led to four practical programmes in the field of aerialtriangulation; the other papers contained in this issue report on these programmes or their applications. These are the programmes:[1)]

- Spatial strip triangulation with independent models, in ALGOL and FORTRAN, for TR 4 and CDC 6600
- Spatial block triangulation with independent models, in FORTRAN, for CDC 6600,
- Horizontal adjustment with independent models (Anblock), in FORTRAN, for CDC 6600 (an earlier version in ALGOL for TR 4 is no longer used),
- Terrestrial distances and straight-line adjustment; in connection with the Anblock programme: combined photogrammetric-terrestrial block adjustment; in ALGOL, for both TR 4 and CDC 6600.

All programme systems - with exception of the last, which is to be considered as an extension - are based on the method of independent models. All are designed for large quantities of data or for a large number of models in the blocks or strips.

The present status of the programmes indicates that it has been possible to change from the TR 4 to the CDC 6600 in the course of their development. It also reflects that their development took place in two different working groups. In particular, the Anblock programme was given priority to permit practical adjustments in the field of cadastral photogrammetry with a working version as early as possible.

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1) The ALGOL programmes have been discarded in the meantime.

In the continuing development, emphasis is placed on the spatial block programme which is conceived as a programme package. It is not only expected to fulfill all practical requirements, but should also offer all possibilities for scientific investigations, with regard to numerical as well as error-theoretical questions. The condition of suitability for large adjustments which was imposed from the very beginning has been satisfied. Not only is there no programme-imposed limitation of block sizes but it has also been possible to achieve such short computer times that the target of adjustment of blocks of at least 1000 models can be reached on the CDC 6600 in approximately two seconds per model without operational difficulties.

6. Applications

The block triangulation with independent models is a general method of aerialtriangulation or photogrammetric point determination. It is therefore suited for all fields of application of aerialtriangulation.

The programme system presented here is characterised by the elimination of many restrictions which have so far stood in the way of a broad development of aerialtriangulation. Significant progress is achieved thereby in two fields of application:

- in surveys for small-scale mapping, blocks of 1000 models or larger must be handled, as demonstrated by the examples Australia (10), Brazil, Canada (11). The programme for these fields of application must yet be extended to auxiliary data, however.
- For large-scale precision surveys, for example point measurements for cadastral purposes, the optimum treatment of voluminous masses of data through simultaneous adjustment is advantageous in many regards (accuracy, homogeneity, saving of control points, independence from picture scale). Here, the concept of aerialtriangulation loses its significance or fades into the background in favour of the feature of integrated data processing.

According to the experiences made so far and according to the predetermined target projections, the author is tempted to enter here a passionate plea for the exhaustive treatment of all computational possibilities in numerical photogrammetry. The full efficiency potential of photogrammetry will be utilized only with general computational procedures and exploitation of large computers. As initially stated, the consequent introduction of the most rigorous and general computational procedures with large computers not only results in a significant increase of efficiency,but it also represents the simplest and most economical method of increase of efficiency which becomes effective according to the criteria of automation. From this point of view the increase in accuracy or the elimination of various restrictions in the context of the fields and methods of work applied so far is not even the decisive argument, but very generally conceived and automated procedures will make accessible ever new fields and new achievements.

In the six following papers published in this issue, the programme developments and some applications and investigations are treated in detail. On this occasion I wish to express my cordial thanks for their work to all collaborators of the Institute and to all who have contributed to the success of the developments presented in this paper.

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