dichte von 2 km. Trianguliert wird mit einem Bildflug im Maßstab 1 : 8000 bis 1 : 10000 mit 60 % Längsüberdeckung und 30 % Querüberdeckung. Die Auswertung erfolgt am Stecometer. Die Bündelausgleichung wird mit dem finnischen Programm in Helsinki gerechnet. Für 8 Blöcke mit einer Gesamt-Fläche von 448 km² wurde die erreichte Genauigkeit mit Hilfe von 700 Streckenmessungen geprüft. Der mittlere Fehler eines photogrammetrisch koordinierten Punktes beträgt 5 cm.

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Simultaneous Compensation of Systematic Errors with Block Adjustment by Independent Models

by H. Ebner and W. Schneider, Stuttgart

Preface

In modern aerial triangulation systematic errors are of central importance again. This was so already, years ago when the polynominal methods were introduced into strip and block triangulation. But during the following phase which was characterized by simultaneous least squares adjustment of all bundles or models of a block the interest concentrated on random errors whilst systematic errors were neglected most of the time.

The recent change of thinking was caused by the results of various practical block adjustments which indicate clearly that systematic errors of considerable size are present in photogrammetric data usually [1]. Some of the typical phenomena which can be caused by not compensated systematic deformations are:

- A reduction of control leads to a higher decrease of accuracy than predicted by theory.
- The accuracy decrease with increasing block size is higher than expected from theory.
- Replacing 20 % sideward overlap by 60 % side lap the accuracy is improved only slightly or even not at all.
- Starting from the same data a block adjustment by independent models can give more accurate results than a bundle block adjustment.

(See [2], [3], [4], [5], [6].)

The mathematical model for compensation of systematic errors

Among the possibilities to compensate the inherent systematic errors of photogrammetric data the concept of selfcalibration by additional parameters is the most promising one being available today [7], [8]. In the adjustment we treat these parameters as random variables with appropriate weights [9], [10]. This approach has two essential advantages:

- It is fully general and leads to optimal accuracy results. Random variables (or observations) are the general case of parameters. Free unknowns as well as constants are special cases of observations and can be represented by weight zero and infinite weight respectively.
- Additional parameters put up as free unknowns can cause serious numerical problems. If some of the unknowns are highly correlated with each other the normal equations become ill conditioned. This problem is avoided when the additional parameters are treated as observations with proper weights.

The block adjustment can be formulated in different ways [10]. If the additional parameters generally are common to groups of models (to whole strips for instance) the following formulation is suitable:

$$\begin{aligned} v_1 &= Ax + By - f \\ v_2 &= \qquad\ Iy - s \end{aligned} \qquad G = \begin{bmatrix} G_{ff} & \\ & G_{ss} \end{bmatrix} \tag{1}$$

f = vector of observations
v_1 = vector of residuals belonging to f
s = vector of additional observations
v_2 = vector of residuals belonging to s
x = vector of unknowns
A = coefficient matrix belonging to x
y = vector of additional unknowns
B = coefficient matrix belonging to y
I = unit matrix
G_{ff} = weight coefficient matrix of the observations f
G_{ss} = weight coefficient matrix of the additional observations s

In equations (1) the additional parameters are put up as unknowns and these unknowns are observed. Usually the additional observations s will be zero. But if some of the additional parameters are known from a priori calibrations the corresponding amounts can be introduced into the adjustment.

The formulation presented here fits into the approach of Generalized Least Squares [11]. This approach itself is related to the concept of Bayesian Estimation [12]. Furthermore it can be shown that the present formulation according to equations (1) fits into the mathematical model of Least Squares Collocation if we set $s = 0$ (additional observations of amount zero) [13]. In this case we obtain:

$$Ax - v_1 + Bv_2 = f \qquad G = \begin{bmatrix} G_{ff} & \\ & G_{ss} \end{bmatrix} \tag{2}$$

Ax = trend
$-v_1$ = noise
Bv_2 = signal
G_{ff} = weight coefficient matrix refering to noise
$BG_{ss}B^T$ = weight coefficient matrix refering to signal

Realization in case of independent model block adjustment

As the basic method for block adjustment by independent models we choose the planimetry height iteration used in the PAT–M43 program [14]. Concerning the additional parameters we suppose that the systematic deformations are common to a certain group of models at times but change from group to group. In addition some systematic can be common to all models.

With the formulation of identic deformations for different models a problem appears resulting from the fact that the coordinate origin is arbitrary for each model. The same formulation $\Delta x = axy$, $\Delta y = 0$ for instance leads to different model deformations, depending on the origin of x (see figure 1).

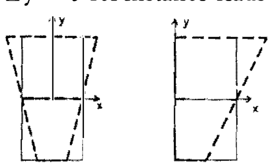
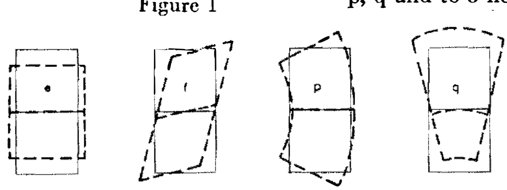


Figure 1

This problem doesn't appear in bundle block adjustment where the origin of each image is well defined by the centre point. To solve the problem also in case of independent models we search for parameters whose effects are not changed by shifts of the coordinate system in x and y direction. This condition leads to 4 planimetric parameters e, f, p, q and to 6 height parameters r, s, t, u, v, w. The effect of these pa-



$\Delta x = ex \quad + fy \quad + p(x^2 - y^2) \quad + 2qxy$

$\Delta y = -ey \quad + fx \quad + 2pxy \quad + q(y^2 - x^2)$

Figure 2

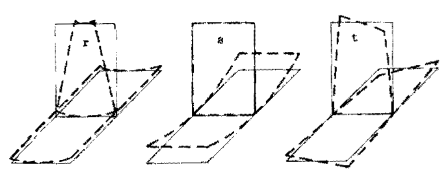


Figure 3

model points	$\Delta z =$	rx^2	$+ sy^2$	$+ txy$
left (right) hand side perspective centres	$\Delta x = -2rxz$			$-tyz + (--)u$
	$\Delta y =$		$-2syz$	$-txz + (-)v$
	$\Delta z =$	rx^2	$+ sy^2$	$+ txy + (-)w$

rameters and their contributions to the observational equations for planimetry and height are shown in figure 2 and figure 3.

The parameters e and f allow for a compensation of affine deformations of the planimetric model coordinates. The parameters p and q are the only one parameters of degree 2 whose effects are independent of coordinate shifts in x and y direction. They also appear in conformal polynomial strip adjustment and are able to compensate the trapezoid shaped model deformations gained in [1].

The effects of the affinity terms e and f are independent of the flight direction. In contrast to that the effects of the parameters p and q change when the flight direction is turned.

The height parameters r and s compensate for second degree z deformations in x and y direction whilst the parameter t corrects for twisted models. The terms u, v and w compensate for systematic errors of perspective centre coordinates.

Test results

To gain practical experience with the suggested concept a preliminary computer program was written by the second author. This program is fully operational and is capable to adjust blocks of medium size with a reasonable computing time. The additional parameters may be common to any group of models or/and to all models of the block. The weight of each of those parameters can be varied separately in a range between zero and infinite. At a later time this program shall be replaced by an extended version of the PAT-M package [14].

The practical tests were performed to get answers to the following questions

- For which groups of models shall be put up identical additional parameters and which weights shall be used for these parameters?
- Which accuracy improvement can be attained by an extended block adjustment with additional parameters?
- Is the accuracy obtained in agreement with the corresponding theoretical accuracy predictions?

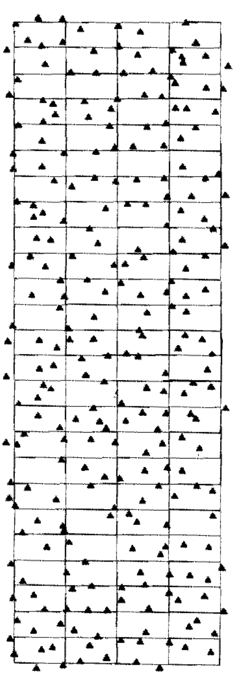


Figure 4

So far as the test material is concerned use could be made of the data of the OEEPE project Oberschwaben. From the compehensive material of this project we selected a subblock consisting of strips 5, 7, 9 and 11 of the block Frankfurt. The test block and the 258 available control points are shown in figure 4.

The total number of models is 100 and the block sizeis 20 km × 62.5 km. All control points and tie points were signalized. The photography was taken with a Zeiss RMK A 15/23 camera at a photo scale of 1 : 28000. The image coordinates were measured with a Zeiss PSK stereo comparator and the independent models were formed computationally.

The test is not yet finished completely. In particular the investigation on height block adjustment with additional parameters is still at work. For that reason only the planimetric results are available up to now. The control distributions investigated here are represented in figure 5.

The results obtained shall be discussed according to the questions raised at the beginning of this chapter.

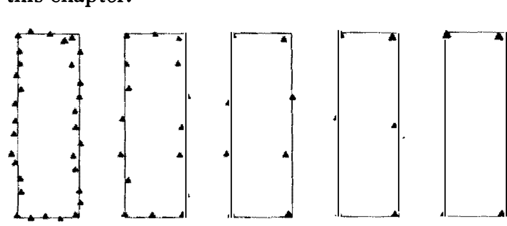


Figure 5

Identical additional parameters and proper weights

At the beginning each strip was given its own set of additional parameters e, f, p and q. Considering the standard deviations σ of those parameters we have learned that the terms p and q are very well determined even if only 4 control points are used. Unfortunately the determination of the affinity terms e and f is much poorer. If only 4 control points are used the standard deviations are in the order of the amounts of the parameters themselves. However, if the affinity terms are common to all models of the block the standard deviations are reduced significantly. Respecting this it can be recommended to put up individual parameters e and f only if there is a real reason to do so. In case of our test block it was found as adequate to put up common affinity terms only.

Concerning proper weights of the additional parameters it was found that the amounts of the terms e, f, p and q, being computed in the block adjustment are only slightly dependent on their weights. This is true also in case of poor control distributions. Therefore it can be recommended to choose the weights of the additional parameters according to their expected amounts or somewhat smaller. With that the accuracy is optimized and problems with respect to the condition of the normal equation matrix are avoided. The amounts of the additional parameters themselves are in agreement with the model deformations obtained in [1].

Accuracy improvement by additional parameters

Using the control distributions represented in figure 5 the test block was adjusted without and with additional parameters. The corresponding results are represented in the following table. The accuracies are related to the photo scale.

Let us start the discussion with σ_0 representing the standard deviation of the planimetric model coordinates. Without additional parameters σ_0 depends significantly on the control distribution used. This is in disagreement with theory. When additional parameters are introduced into the block adjustment σ_0 becomes considerably smaller (at a factor 1.4 to 1.6) and the dependency on control distribution disappears. With 4.2 μm sigma nought is close to the noise limit we can expect from todays photogrammetry at all.

Although the discussion of σ_0 is most illuminating, the real power of the new concept is only shown by the comparison of the absolute accuracies, expressed by μ_{xy}, the RMS value of the coordinate errors at check points. We see that the additional parameters improve the accuracy the more the poorer the control distribution is. The improvement increases up to a factor 3.0 in case of 6 control points used. In figure 6 the corresponding results are represented graphically.

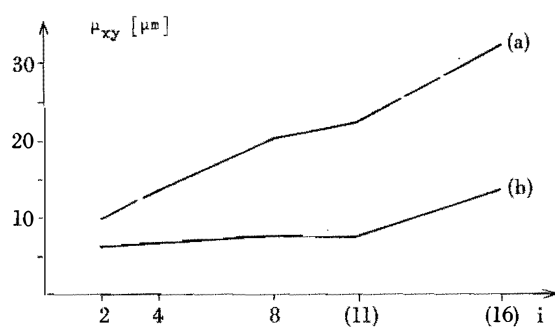


Figure 6 (a) without, (b) with additional parameters

The test shows that absolute accuracies of about 7 μm at the photo scale can be realized today, even when the control spacing along the block perimeter is in the order of 4 to 8 base length. If we put this accuracy of 7 μm $\triangleq$ 20 cm in relation to the length of the block (62.5 km) we obtain a relative accuracy which is better than 1 : 300000.

Comparison with theory

Now a comparison is made between the accuracy obtained by block adjustment with additional parameters and the corresponding theoretical accuracy being based on random errors only [4]. However, to allow for a correct comparison we have to consider that the check points used in the test are not errorfree. Therefore the theoretical accuracy figures obtained from [4] are superposed by the

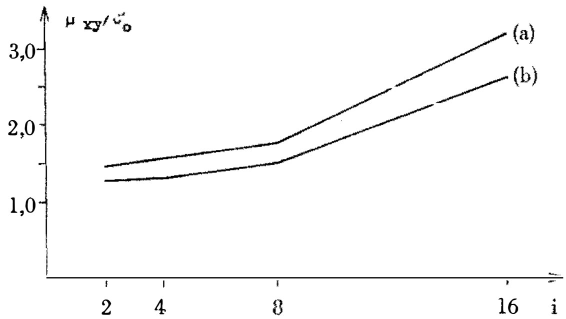


Figure 7 (a) test, (b) theory

random accuracy of check points which we assume with 10 cm in the terrain. This assumption can be considered as realistic. The result of the comparison is given graphically in figure 7.

The plot shows that the accuracy obtained in the test is somewhat poorer than the accuracy as predicted by theory. However, the discrepancies are less than 20 % and can be explained by the facts that our test is only one sample and doesn't meet the premises of the theory rigorously (different block shape for instance).

Considering this we can say that the accuracy results of the test are in agreement with the corresponding theoretical expectations. This agreement is most important because it indicates that the systematic errors of the model coordinates are compensated very well by the additional parameters used and that the remaining errors can be considered as random.

control version	control points	check points	without add. param.		with add. param.		accuracy ratios	
			σ_0 [μm]	μ_{xy} [μm]	σ_0 [μm]	μ_{xy} [μm]	σ_0	μ_{xy}
i = 2	32	226	6.8	9.9	4.3	6.3	1.6	1.6
i = 4	16	242	6.5	13.4	4.2	6.6	1.5	2.0
i = 8	8	250	6.2	20.0	4.2	7.4	1.5	2.7
(i = 11)	6	252	6.1	22.1	4.2	7.3	1.5	3.0
(i = 16)	4	254	5.9	32.4	4.2	13.5	1.4	2.4

Table

Summary

An advanced concept of block adjustment by independent models is presented, allowing for a simultaneous compensation of certain types of systematic errors of model coordinates. To gain practical experience with this concept a corresponding computer program was written. The test results obtained up to now allow for the following conclusions:

- The practical application of the concept causes no problems.
- The accuracy of adjusted block coordinates is improved up to a factor 3.
- The obtained accuracy corresponds very well with the accuracy as predicted by theory.

Zusammenfassung

Es wird ein erweiterter Ansatz für die Blockausgleichung mit unabhängigen Modellen vorgestellt, der eine simultane Kompensation systematischer Fehler der Modellkoordinaten mit Hilfe zusätzlicher Parameter erlaubt. In der Blockausgleichung werden diese Parameter als zufällige Variable mit entsprechenden Gewichten behandelt. Um Erfahrungen mit dem neu vorgestellten Ansatz sammeln zu können, wurde ein praktischer Test mit dem Material des OEEPE-Blocks Oberschwaben durchgeführt. Das dafür geschriebene vorläufige Rechenprogramm soll zu einem späteren Zeitpunkt durch eine erweiterte Version des Programmpakets PAT–M ersetzt werden. Die bisher erzielten Ergebnisse lassen die folgenden Feststellungen zu:

- Die neue Konzeption läßt sich in der Praxis problemlos anwenden.

- Der erweiterte Ansatz führt in der Lage zu einer Genauigkeitssteigerung bis zum Faktor 3. Die dabei erhaltene und zur Blockgröße in Relation gesetzte Genauigkeit ist besser als 1 : 300 000.
- Die erzielten Genauigkeiten stimmen mit den entsprechenden theoretischen Vorhersagen gut überein. Daraus kann geschlossen werden, daß die angesetzten zusätzlichen Parameter die vorhandenen systematischen Fehler weitgehend kompensieren.

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Investigation on the Applicability of Block Adjustment in Austria

by G. OTEPKA, Vienna.

1. Introduction

The performed investigation is limited to two problems of the Austrian Federal Bureau of Standards and Surveying (BAfEuV) and conditions which are present in this organisation and in Austria. The two problems mentioned are:

a) to produce control points for the plotting of Austria in the topographic map "Österreichische Karte (ÖK) 1 : 50000"

b) to intercalate points between the existing Fifth-Order Trigonometric-Network. These points are called "Einschaltpunkte (EP)".

For years the BAfEuV uses photogrammetric methods for both problems. Up to now the advantages of block adjustment never had been used though general computer programs are available and a suitable computer exists.

In the first part of this investigation two typical routine projects of the BAfEuV were used for block adjustment to show the efficiency of block adjustment. In the second part the costs of different methods are compared, methods which are used by the BAfEuV on one hand and block adjustment on the other for solving the two first mentioned problems. The results of these computations and comparisons are reported.