Photogrammetric densification of trigonometric networks – the project Appenweier

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Photogrammetric point determination has reached a level of accuracy, which makes practical application to geodetic problems feasible. In particular photogrammetric determination of low order trigonometric nets or of traverse nets seems possible, meeting accuracy and economy requirements.

1. The project „Appenweier"

1.1 The state survey authorities of the state of Baden-Württemberg have decided in 1973 to try the applicability of the photogrammetric method for the densification of an existing trigonometric net of 3^{rd} order into 4^{th} order. The photogrammetric part of the project was handled by the Photogrammetric Institute of Stuttgart University. The test area Appenweier (in Baden) covers $9.1 \times 10.4\ km^2$. In it 24 targetted trigonometric points of 3^{rd} and 2^{nd} order are given (some of them targetted excentrically by subsidiary points). 85 rather evenly distributed points were to be determined. They are check points for the test, their known terrestrial coordinates being withheld. The accuracy of the terrestrial coordinates is supposed to be about 1 cm.

The planning of the project aimed at a resultant planimetric coordinate accuracy of 3 cm in the terrain, which implies a photo-scale of about 1 : 3000 to 1 : 4000, equivalent to a required accuracy of 10 μm to 7.5 μm at photo-scale.

Because of expected difficulties with point transfer in very large photo-scales, the photo-scale of 1 : 7800 was applied, instead, with 4-fold flying, in 4 directions. 4-fold photo-overlap, with simultaneous block-adjustment, is expected to improve accuracy by at least a factor of 2, compared with single coverage at the same scale. The 4-fold overlap of 1 : 7800 scale gave altogether 448 models, which is about the same number of models as the single coverage of photo-scale 1 : 4000 would have implied.

All 24 control- and 85 check-points were targetted, by 20 cm $\times$ 20 cm size signals, 35 of them by excentric subsidiary points. In addition, for special test purposes, all standard tie-points (6 per model) were targetted with triple signals each.

Aerial photography was taken in two missions on April 24^{th} and May 12^{th}, 1973, with the Zeiss RMK A 15/23 wide-angle camera, by Rheinische Braunkohle AG. The first flight mission was discontinued due to turbulence. Due to bad weather conditions the targets in the field had to be maintained for about 2 months, which caused some difficulties.

1.2 The film-diapositives of the aerial photographs were measured with the Zeiss stereocomparator PSK. Some photographs showed noticeably blurred signals due to image movement. The standard measuring accuracy, of the mean of the double plate-coordinate-measurements, was established from all recordings to be about 1.5 μm.

The machine coordinates were transformed into the image coordinates via the fiducial marks by applying similarity transformation. After correction of image coordinates for lens distortion, earth curvature, and refraction independent models were computed by analytical relative orientation. The computed model-coordinates formed the input for the block-adjustment by the method of independent models.

2. Block-Adjustments

2.1 The data allowed several investigations with regard to different points of view. Several series of adjustments were performed. Here only some of the preliminary results concerning planimetry are reported. They refer to

- Control: Version 1, utilizing all given control points (perimeter points + 8 points inside the block)
- Combined adjustment of the 4-fold block; adjustment of the 2 double blocks of crossing flight directions; separate adjustments of the 4 single blocks, labeled according to their flight directions (EW – WE – NS – SN).
- Post treatment of the adjusted blocks by the method of least squares interpolation.

2.2 Presentation of results

The test results of the various block-adjustments are summarized in table 1. The accuracy figures are given in cm and in μm.

3. Evaluation of the results; comments

3.1 Four-fold overlap

The first comment should refer to the actual case on test, the 4-fold block, with the given control, table 1. The resulting planimetric coordinate accuracy figures are

$$\mu_{xy} = 3{,}5 \text{ cm} = 4{,}5\ \mu\text{m after blockadjustment}$$

$$\mu_{xy} = 2{,}7 \text{ cm} = 3{,}4\ \mu\text{m after least squares interpolation}$$

Such results are, generally speaking, highly satisfactory and meet the planning specifications of 3 cm.

The vector diagrams of residual errors at the check points are not presented in this paper. They show the familiar trend effects within local areas. 4 such areas of high correlation of residuals can be distinguished, each extending over about 1/4 of the block-area. By the least-squares interpolation the local systematic trends have been considerably reduced. They are, however, still discernible. Thus, there are still some systematic error effects left uncompensated.

The magnitudes of the maximum residual coordinate errors of 10.6 cm and 9.4 cm, respectively, are in general agreement with expectation. The large errors have some relationship with the trendfields of the vectors. Nevertheless, they occur in first instance independently, sometimes rather close to control points. Most likely they are caused by disturbed signals. It remains to be seen whether the additional adjustment of the auxiliary signals by straight-line and distance-conditions will be able to reduce the magnitudes of the maximum residual errors.

Table 1 Appenweier, Planimetric results of block-adjustment with independent models (wide angle, photoscale 1 :7800)
All given control points used (perimeter + points inside)

Block	after block-adjustment										after least squares interpolation							
	σ_o	μ_x	μ_y	μ_{xy}	ε_{max}	σ_o	μ_x	μ_y	μ_{xy}	ε_{max}	μ_x	μ_y	μ_{xy}	ε_{max}	μ_x	μ_y	μ_{xy}	ε_{max}
	– cm –					– μm –					– cm –				– μm –			
Single blocks (112 models, 27 control points, 77 check points)																		
EW	3.5	4.0	5.3	4.7	12.8	4.5	5.1	6.8	6.0	16.4	3.2	4.7	4.0	13.3	4.1	6.0	5.1	17.1
WE	3.8	5.4	6.5	6.0	19.1	4.9	6.9	8.3	7.6	24.5	4.5	5.3	4.9	16.3	5.8	6.8	6.3	20.9
NS	3.4	4.7	3.5	4.1	12.8	4.4	6.0	4.5	5.3	16.4	4.0	3.1	3.6	9.9	5.1	4.0	4.6	12.7
SN	3.8	6.0	6.1	6.1	20.3	4.9	7.7	7.8	7.8	26.0	5.1	4.2	4.7	19.7	6.5	5.4	6.0	25.3
mean	3.6	5.1	5.5	5.3	20.3	4.7	6.5	7.1	6.8	26.0	4.3	4.4	4.4	19.7	5.5	5.6	5.6	25.3
Double blocks (224 models, 30 control points, 82 check points)																		
WE/NS	3.8	3.8	4.1	4.0	12.6	4.9	4.9	5.3	5.1	16.2	3.1	3.3	3.2	13.1	4.0	4.2	4.1	16.8
EW/SN	3.7	4.2	5.1	4.7	14.3	4.7	5.4	6.5	6.0	18.3	3.6	3.7	3.7	12.2	4.6	4.7	4.7	15.6
mean	3.8	4.0	4.6	4.3	14.3	4.9	5.1	5.9	5.5	18.3	3.4	3.5	3.5	13.1	4.3	4.5	4.4	16.8
4-fold block (WE/EW/NS/SN; 448 models, 30 control points, 83 check points)																		
	3.8	3.4	3.6	3.5	10.6	4.9	4.4	4.6	4.5	13.6	2.7	2.6	2.7	9.4	3.5	3.3	3.4	12.1
[1]	3.8	3.3	3.2	3.3	10.6	4.9	4.2	4.1	4.2	13.6	2.6	2.4	2.5	9.2	3.3	3.1	3.2	11.8

μ_x, μ_y = r. m. s. values of residual errors at check points; $\mu_{xy} = \sqrt{(\mu_x^2 + \mu_y^2)/2}$; ε_{max} = max. residual coordinate error of check points

[1]) 8 additional perimeter control points, 75 check points

3.2 Comparison of 1-, 2- and 4-fold blocks

The 4 single blocks represent conventional blocks of 20 % lateral overlap, photo scale 1 : 7800. Their summarized accuracy figures are:

$\sigma_0 = 3.6$ cm $= 4.7$ μm (range 4,4 – 4.9 μm)
$\mu_{xy} = 5.3$ cm $= 6.8$ μm (range 6.0 – 7.8 μm) after block-adjustment
$\mu_{xy} = 4.4$ cm $= 5.6$ μm (range 4.6 – 6.3 μm) after l. sq. interpol.
$\mu_{xy}/\sigma_0 = 1.5$ and 1.2, respectively

Such figures qualify the block-results of the independent model method as most satisfactory. The confirm the high level of accuracy which the photogrammetric system is capable of.

Compared with the single blocks the improvement of accuracy by double and 4-fold overlap does not reach expectation. The ratios of the r. m. s. values μ_{xy} of single, double, and 4-fold blocks are:

1 : 0.81 : 0.66 (1 : 1/1,23 : 1/1.52) after block-adjustment
1 : 0.80 : 0.61 (1 : 1/1.25 : 1/1.63) after interpolation

The theoretical ratio, on the basis of random errors only, is expected to be about a factor 1.5 ($> \sqrt{2}$) per step. Thus, only little more than half the expected improvement has been realized.

The general explication of such results must refer to the presence of uncompensated systematic image errors which have been known to disturb unfavourably theoretical expectations which are based on random errors only. Nevertheless such explanation still leaves some questions open. It had been anticipated, originally, that the 4-fold block would compensate systematic errors very well. Evidently such effects did not come about. Strange to say, the least squares interpolation, supposed to remove remaining systematic error effects, has been more effective with the 4-fold block than with the single or double blocks (1 : 1/1.30 against 1 : 1/1.22 and 1 : 1/1.23).

3.3 Least squares interpolation

The conclusion just drawn about the presence of systematic image errors is confirmed by the general effectiveness of the least squares interpolation after the block-adjustments. The accuracy improvement for single, double, and 4-fold blocks amount to 18 %, 19 %, and 23 %, respectively (1 : 0.82; 1 : 0.81; 1 : 0.77). It means that treatment of systematic errors one way or another is essential.

The maximum residual errors have been hardly reduced by least squares interpolation, which confirms their independent, local causes as stated in section 3.1.

4. Summary and conclusion

First of all the test has confirmed the high accuracy level of photogrammetric point determination, indicated by the mean planimetric coordinate accuracy of single blocks of 4.4 cm or 5.6 μm, from the photo-scale 1 : 7800.

With double and 4-fold overlap the accuracies improved to r. m. s. values of planimetric coordinate errors of 3.5 cm (= 4.4 μm) and 2.7 cm (= 3.4 μm). Such results are highly satisfactory in absolute terms. Nevertheless the double and 4-fold coverage has not been as effective as expected.

The results indicate the presence of systematic image errors. Research into their more effective compensation is continued and further improvement of accuracy is expected.

Zusammenfassung

Es wird über die vorläufigen Ergebnisse der kontrollierten photogrammetrischen Netzverdichtung „Appenweier" berichtet, die vom Landesvermessungsamt Baden-Württemberg in Zusammenarbeit mit der Universität Stuttgart durchgeführt wurde.

Um die geforderte Genauigkeit von 3 cm zu erreichen, wurde das 95 km² große Gebiet im Bildmaßstab 1 : 7800 mit 448 Modellen 4 fach beflogen. Nach simnltaner Blockausgleichung der unabhängigen analytischen Modelle und kleinste-Quadrate-Interpolation ergab sich aus 83 Vergleichspunkten eine Lagekoordinaten-Genauigkeit von 2,7 cm = 3,4 μm und ein Maximalfehler von 9,4 cm. Trotz des sehr zufriedenstellenden Ergebnisses hat sich die 4fach-Überdeckung als nicht so effektiv

gezeigt wie erwartet. Die getrennte Ausgleichung der 4 Einzelblöcke erreichte eine Genauigkeit der Lagekoordinaten von 4,4 cm = 5,6 μm.

Die Ergebnisse weisen auf die Existenz systematischer Fehler hin. Mit verfeinerten Ausgleichungsmethoden zu ihrer wirksameren Kompensation wird eine weitere Genauigkeitssteigerung erwartet.

Densification of Trigonometric Nets Practical Experiences with Bundle Adjustment

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Background

Photogrammetry has already for some years been used for cadastral purposes in Denmark. To a certain degree photogrammetric plans have formed the basis of new cadastral maps. Perhaps more interesting in this connection is the fact that photogrammetrically coordinated fixed-points are being used for cadastral measurements.

The Geodetic Institute of Denmark is responsible for the triangulation of heigher order. The Institute establishes points down to a density of approx. 2 km. Further densification is left to the Cadastral Service and the private surveyors.

According to official regulations the surveyors have to tie new cadastral measurements to fixed-points when these are located within a distance of 200 m (for measuring of roads the limit is 500 m). This means in practice that unless you have a net of points with a density of approx. 400 m problems may arise especially in the developing areas of larger towns.

Also the local authorities have a professional interest in the developing areas of the towns. One of the main jobs for the photogrammetric firms is to produce technical plans in the scale 1 : 1000 to the municipalities. The need for ground control for the compiling of these plans corresponds very well with the earlier mentioned 400 m net. The net is furthermore useful for the local technical administration for map revision and setting out purposes.

The jobs that will be presented in this paper are all the result of a cooperation between a state authority, namely the Cadastral Service, a private photogrammetric firm, the local municipality, and the local private surveyor. Such a cooperation implies of course both advantages and difficulties.

The photogrammetrie triangulation

Up to 1970 the only photogrammetric triangulation method used for the described kind of jobs was the Anblock with models measured in analogue instruments such as A8, A7 and Simplex III. This method is still in use for smaller blocks and gives sufficiently good results with a photo scale of 1 : 4000 or 1 : 5000.

In 1970 the first test block in Denmark using 'bundle-adjustment' was made. The measuring and the calculations were done in Finland at the Technical University of Helsinki. The results were promising, and since then some 40 jobs covering approx. 1400 km^2 have been performed mainly in Finland but lately also in Denmark.

All the jobs mentioned in this report were carried out in Finland. The pictures were measured on Zeiss PSK comparator using glas-diapositives and the calculations were made with the Finnish triangulation programme.

Prior to the adjustment the pictures were corrected for

1) film distortion (using affin transformation on the fiducial marks),
2) lens distortion (radial),
3) refraction, and
4) earth curvature.

Criteria had to be established for the practical execution. Up till now only very little has been published for the practical execution. Up till now only very little has been published about bundle-