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SOME PRACTICAL EXAMPLES OF SYSTEMATIC ERRORS OF STEREO-MODELS

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1. Introduction

The detection and correction of systematic errors presently plays a very important role in aerial triangulation. It is hoped that the development of suitable techniques for the correction of systematic errors will result in a further improvement of the accuracy of block adjustment. A particularly efficient method for bundles and models is that of block adjustment with simultaneous self-calibration |1|.

In order to develop efficient and exhaustive correction parameters, information about the magnitude and the characteristics of the systematic errors actually encountered in practical photography is of the utmost importance. The Institute of Photogrammetry of Stuttgart University recently studied several practical blocks, analyzing the residual errors at the tie points for systematic errors. These empirical studies essentially had a two-fold aim:

- They were to provide information on the magnitude and type of systematic errors.
- An analysis of different practical examples was to provide above all information regarding a dependence of systematic errors on such parameters as camera, direction of flight, overlap, timing of flights, plotter used and the number of points. Also, the constancy of systematic errors within a block was of special interest.

This paper gives and discusses several examples of systematic model deformations and outlines a few consequences for a fairly extensive correction of systematic errors.

2. Method of empirical analysis of systematic errors

The following examples of systematic model deformations refer almost exclusively to controlled block adjustments for which additional check points were frequently available for accuracy tests.

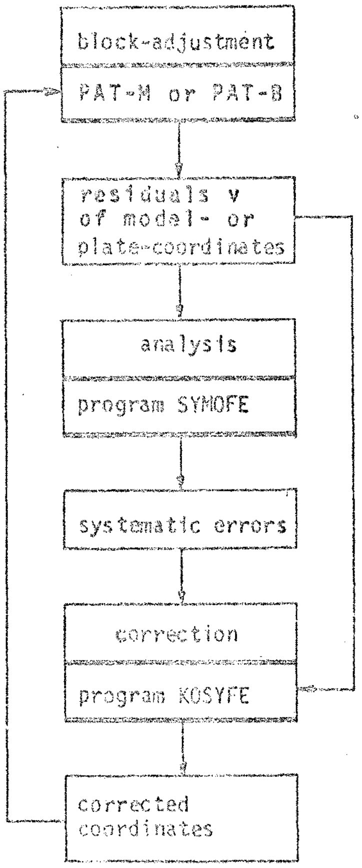
The principle of the empirical method used for determining the systematic model errors is relatively simple:

After block adjustment of independent models without previous correction, systematic model errors are derived by a superposition of models by analyzing the residuals v at the tie points. Theoretically, this method is based on the assumption that systematic model errors can be represented as a function of the location of a point in the model and are constant for a given location in the model over a larger group of models.

The suggestion to determine systematic errors in this manner was originally made by Masson d'Autume |2|. This method, which can also be applied to photographs, may, in principle, be counted among the self-calibrating techniques, since the information used refers directly and exclusively to the block adjustment.

At our Institute, this method has primarily been applied to a study of the results of practical block adjustment and has been suitably modified. Fig. 1a is a flow chart of empirical analysis the way it is presently being used with the aid of suitable computer programs:

After block adjustment with independent models, the ground-referenced and transformed model coordinates with their corrections are first converted into a uniform model-coordinate system. This is done by rotating the coordinate system for the corresponding flying direction and transforming the models to a uniform scale with the aid of the average base separation. The resulting average or reference model is subdivided into six or 15 areas, independently of the number of points, and the residual errors of the tie points - possibly also those of the control points - are referenced to the centers of the corresponding areas (see Fig. 1b).



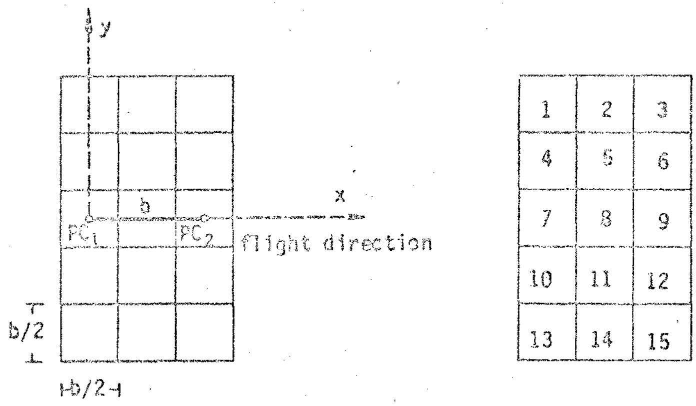


Fig. 1a: Empirical analysis of systematic errors after block adjustment.

Fig. 1b: Reference model for representation of systematic model deformations.

This is followed by computation of average residual errors, first separately for the different models (if there are two or more points in one area) and then for the different strips. This allows areas of constant systematic behavior to be detected and analyzed. If the average systematic model errors of the different strips do not show any significant differences, the average values are determined for strips of identical flight direction, strips of identical flight date or for all models of a block in common.[1]

In the majority of cases, the model deformations are only computed from the residual errors of the tie points, but there is a possibility of including the residual errors of control points, especially in blocks with a large number of control points. The results of empirical analysis are not completely identical with those of simultaneous self-calibration in which all observations affect the computation. In spite of this, the results obtained are highly significant, above all in the case of well-controlled blocks.

Although the primary aim of this paper is the detection of systematic model errors and their properties, possibilities of correcting them should likewise be pointed out because this will best answer the question for a possible increase in accuracy, which is of decisive importance for practical work:

Up to now, the empirically determined model deformations are being corrected iteratively with the aid of correction polynomials.[2] The polynomials used can be adapted to the number of points and the groups of constant systematic errors, in other words, they are applicable either to strips, partial areas of the block or the entire block. Corrections for all points of a model are then computed from the results of the analysis. The corrected model coordinates are used to repeat the block adjustment, and the result is again analysed by the method outlined in Fig. 1a. In the majority of cases, the correction procedure can be broken off after a single iteration.

A study of various practical examples has shown that an empirical analysis of residual errors is a very effective means of correcting systematic errors. Nevertheless, it might be possible to use this method in special cases exclusively as part of a more rigorous correction technique. Thus it is possible, for example, to use the results from the analysis to facilitate the selection of suitable groups of parameters for simultaneous self-calibration. If necessary, the results may also be introduced into the self-calibrating block adjustment as correction values known a priori. Or the opposite method may be useful, in which the

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[1] The systematic errors at the perspective centers, which are not discussed in greater detail in this paper, can be determined in a similar manner.

[2] In principle, more rigorous interpolation methods could also be used, above all, if information is available for 15 model areas.

results of a self-calibrating block adjustment (especially if only an insufficient number of additional parameters was used) are additionally treated by this empirical method of analysis and correction.

3. Projects studied

Table 1 lists the practical blocks studied together with the most important object parameters. In the following, only the purpose of the different projects and points of special interest for an analysis of the systematic errors will be briefly described.

- Oberschwaben

  The OEEPE (Organisation Européenne d'Etudes Photogrammétriques Expérimentales) made the Oberschwaben test for the purpose of an extensive empirical study of accuracy for aerotriangulation. The test field was photographed with 60 % side lap both with a ZEISS RMK 15/23 camera (wide-angle) and a ZEISS RMK 8.5/23 (super-wide-angle). In addition to 540 trigonometric points available as control and check points, the tie points in the six standard points of the model had also been marked by double signals.

  The Oberschwaben test block and the results of normal block adjustment with independent models have been described in detail by Ackermann |3|, |4| (see also Wiser/Ackermann |5| and Haug |6|).

- Appenweier

  The large-scale Appenweier test was performed by the Baden-Württemberg Topographic Survey in cooperation with the Institute of Photogrammetry of Stuttgart University for the purpose of photogrammetric control extension. 24 known control points and 85 check points had been signalized in the test field, partly with 2 - 4 additional auxiliary points. Moreover, the tie points had also been signalized in the six standard points of the model by groups of three. The flight was made with a ZEISS RMK A 15/23 wide-angle camera in four different directions (WE/EW/NS/SN), see Fig. 3a. The test setup and the results of block adjustment were published by Ackermann in |7|.

- Block S1

  The S1 block served for the photogrammetric extension of grid-form vertical control. A ZEISS RMK A 15/23 wide-angle camera was used for the flight. The photography was plotted by three different organizations using ZEISS PSK and PSK-2 stereocomparators as well as a ZEISS PLANIMAT. In addition, plotting included measurement of an average of 150 - 200 natural points per model, which in view of 60 % side lap resulted in a very close connection between adjacent models.

Table 1:
Project specifications

project	camera	photo-scale	overlap [%] 1)	flight-direction	no. of strips	no. of models	flight-mission	control- and check points tie-points	measuring instrument
Oberschwaben OEEPE-testblock	wide-angle ZEISS RMK A 15/23 no. 111 680	28 000	p = 60 q = 60	NS SN	7 8	175 200 375	1969 8.4., 9.4. 12.5.	signalized points: 548 trigonometric points (control- and check points) all standard tie-points (2 x 6 per model) pin-point flying	stereocomparator-measurements in 4 different restitution centres: ZEISS PSK WA Frankfurt SWA The Hague WILD StK1 WA Vienna SWA Delft
	super-wide-angle ZEISS RMK A 8.5/23 no. 111 164	28 000	p = 60 q = 60	NS SN	8 7	200 175 375	1969 10.4. 26.4.		
Appenweier photogrammetric densification of trigonometric networks	wide-angle ZEISS RMK A 15/23 no. 111 678	7 800	p = 64 q = 28	WE EW NS SN	7 7 8 8	112 112 112 112 448	24.4.1973 24.4.1973 12.5.1973 12.5.1973	signalized points: 24 control- and 85 check points (35 of them by excentric subsidiary points) all standard tie-points (3 x 6 per model)	stereocomparator ZEISS PSK2
Block S1 dense grid of height points	wide-angle ZEISS RMK A 15/23 no. 111 678	8 000	p = 60 q = 60	WE EW	14 14	158 165 323	23.3.1975	signalized points: plan: 43 control points height: 105 control points natural points: 88 check points for height control grid and tie-points (150-200 per model)	3 different restitution centres: ZEISS PSK2 139 models ZEISS PSK 51 models ZEISS PLANIMAT 133 models model-scale: 3 500
Burkheim cadastral block	wide-angle WILD RC 8 15/23	7 500	p = 60 q = 20	WE EW	5 3	41 21 62	1974	signalized points: plan: 93 control points 480 check points natural points: 86 control points for height (natural details) standard tie-points (6 per model)	3 ZEISS PLANIGRAPH C 8 no. I, II, III model-scale: 3000
Block S2 large scale plotting project	wide-angle WILD RC 5 15/23	3 500	p = 60 q = 20	WE EW	2 2	34 33 67	5.5.1974	signalized points: plan: about 50 pre-marked check points height: about 90 control- and check points natural points: 57 other check points (natural details) standard tie-points (6 per model)	WILD A 8 model-scale: 1500
Rheidt testfield individual models	wide-angle ZEISS RMK A 15/23 no. 111 680	11 000	p = 60	-	-	47	5.4.1969	signalized points: 123 control- and check points	stereocomparator ZEISS PSK
	super-wide-angle ZEISS RMK A 8.5/23 no. 120 281	11 000	p = 60	-	-	8	5.6.1974		stereocomparator ZEISS PSK2

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1) p = end-lap
q = side-lap

- Burkheim

  Burkheim was a practical cadastral project in which horizontal accuracy was of primary importance. The test material was made available by courtesy of the Reallotment Service of Bamberg in Bavaria. Almost 600 horizontal control points had been signalized for this project, while only isolated topographic points were available for vertical control. A WILD RC-8 wide-angle camera was used to photograph the reallotment area. The models were plotted in three different ZEISS C-8 Stereoplanigraphs. In view of the large data volume, the residual errors of the control points were likewise used for computing systematic model errors.

- Block S2

  The S2 block was a large-scale project flown with a WILD RC-5 wide-angle camera and plotted in the analog mode in a WILD A-8.

- Rheidt

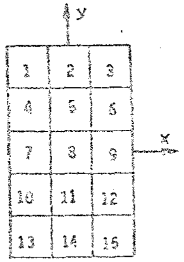
  The Rheidt test field was created by Bonn University for special studies of photogrammetric pictures and models. The examples of systematic model deformations submitted are the result of studies of single models with regard to optimum angular field; see Stark |9|. Unlike the other examples, model deformations here were not computed from tie points but exclusively from residual errors of control points after absolute orientation of single models.

4. Results

Figs. 2 - 6 give a graphical outline of the model deformations found in the different examples. Table 2 contains the corresponding numerical values for the six standard points of the model. The values given are corrections resulting directly from the average residual errors after block adjustment (for Table 2 and Figs. 2 - 6, see Annex).

The results of the different projects can be directly compared, since they refer uniformly to the system of model coordinates in which the positive direction of the x-axis coincides with the line of flight. When interpreting the final results it should be noted, however, that Table 2 only gives the average values for certain groups of constant object parameters of the corresponding block. The systematic model errors of the different strips may vary slightly. This applies above all to those blocks in which strip-specific properties of model deformations were found, as in Appenweier, Block S1, Block S2 and Burkheim. For the sake of simplicity, the systematic errors of these blocks had to be combined in groups of identical object parameters. However, the graphical representation of the different group results are highly significant for the model deformations of the different blocks, since the results for individual strips generally differ from the data in Table 2 only by slight amounts and not in their general form.

Table 2. Systematic model deformations
Results of different projects in units of μm



block		Oberschwaben q = 60 %		Appenweier results of 4-fold-block (WE/EW/NS/SN)				Block S1[1] q = 60 %		Burkheim q = 20 %		Block S2 q = 20 %		Rheidt[1]	
camera		WA	SWA	WA				WA		WA		WA		WA	SWA
measuring-instrument		ZEISS PSK WILD StK1		ZEISS PSK2				ZEISS PSK	ZEISS PLANIMAT	C8 I+III	C8 II	WILD A8		ZEISS PSK	ZEISS PSK2
number of models		375	375	112 (WE)	112 (EW)	112 (NS)	112 (SN)	190	133	41 (WE)	21 (EW)	34 (WE)	33 (EW)	47	8
x	standard position														
	1	+1.9	-7.6	-1.0	-2.2	-2.4	-1.2	-1.9	-1.1	+1.2	+2.8	+4.6	-0.6	+5.4	-1.2
	3	-3.9	+5.4	+1.6	+2.8	+4.3	+4.1	+1.8	+0.5	+1.4	+1.8	-5.1	+0.3	-2.0	-0.6
	7	-1.5	-2.3	-0.6	-0.7	-1.1	-0.3	+0.2	+0.4	-0.2	-3.8	-4.9	-3.7	+0.9	+1.3
	9	+1.9	+2.9	-1.6	-1.4	-0.3	-1.5	0	0	+1.9	+3.0	+4.9	+2.9	-2.4	-2.1
	13	-5.5	+2.4	+3.9	+4.1	+2.8	+3.9	+2.7	+1.4	-3.8	-2.2	-2.0	+3.1	-1.7	+4.8
	15	+7.6	+0.4	-2.0	-2.8	-2.0	-4.3	-2.5	-1.1	+3.4	+0.5	+4.9	-0.9	+3.4	-1.2
y	1	-1.3	-2.7	+2.7	+2.5	+0.4	+1.2	-1.1	+0.3	-4.7	-7.8	+8.0	+0.9	-4.2	+5.5
	3	-3.2	-1.5	-0.5	-0.4	+0.3	+1.8	+0.1	-2.1	+1.6	+3.6	-7.7	-0.6	-4.7	+0.4
	7	+2.2	+1.5	-0.8	-1.1	-2.2	-3.0	+0.2	-0.8	+1.8	+1.8	0	+2.6	+0.7	+1.5
	9	-0.3	-1.6	-1.8	-2.1	-1.0	-0.1	+0.1	+0.5	-0.1	-1.1	0	-2.3	+0.7	-3.2
	13	+2.3	+6.5	-0.5	-0.4	+0.7	-1.4	+1.6	+2.1	+1.6	-0.3	-6.0	+0.9	+0.1	+4.1
	15	-1.3	-2.1	+1.4	+1.5	+1.7	+2.4	-0.2	0	-1.7	+1.1	+4.6	-3.4	+1.3	-2.2
z	1	+1.5	+2.1	+1.7	+1.6	+0.6	-0.2	+0.4	-3.0	+3.0	+5.7	-1.4	-3.1	+9.1	+2.0
	3	+3.9	+3.9	+1.1	+0.1	+1.1	+0.8	+2.5	+2.9	-2.0	-8.2	+1.4	+3.4	+11.9	+2.4
	7	-3.3	-3.1	-1.9	-1.2	-1.1	-1.6	-3.0	-1.2	-0.7	-0.6	+5.7	-3.7	-1.4	-5.2
	9	-7.6	-9.2	-2.0	-0.3	-1.8	-0.9	-3.7	-2.6	+0.6	-0.1	-5.7	+3.4	-2.2	-5.2
	13	+2.6	+3.0	+1.1	-0.6	+0.7	+0.4	+1.2	+1.4	-2.6	-0.6	-0.3	-2.9	+1.2	+6.9
	15	+2.9	+3.6	+2.6	+1.2	+0.2	+1.0	+2.5	-0.9	+0.6	+2.8	-0.9	+2.0	+0.5	+1.7

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1) systematic errors were computed for 15 points

5. Discussion of results

The results of the extensive empirical study make it possible to answer a number of important questions regarding the detection and correction of systematic errors. In the following discussion, special reference is made to the magnitude and type of systematic model deformations as well as their dependence on different factors.

5.1 The magnitude of systematic model deformations

Table 3 uses the average values of the strips to summarize the maximum systematic coordinate errors and the general mean values of all model errors of a block. On the basis of this summary, the general statement may be made that all the examples tested clearly show systematic model deformations in plan and height. Some maximum values go up to over 10 µm, while mean values in plan and height are around 5 µm.

Table 3: Maximum and mean values of systematic model deformations

Project	maximum values (analyzed per strip)			mean values		
	x [µm]	y [µm]	z [µm]	x [µm]	y [µm]	z [µm]
Oberschwaben						
w.a.	11.4	7.2	11.4	4.5	2	4.5
s.w.a.	10.5	10.8	11.8	4.5	3	5
Appenweier	6.9	5.9	6.1	3.5	2.5	2.5
Block S1	8.1	7.8	9.1	4	4	5
Burkheim	5.4	8.8	9.0	4	5	5
Block S2	6.5	13.4	7.4	5	6.5	5
Rheidt [1]						
w.a.	5.4	4.7	11.9	2.5	2	5
s.w.a.	4.8	6.2	6.9	2.5	3	4.5

[1] single models

If we look at individual values in Table 3, there are surprisingly slight differences between the mean values of the different blocks. Only the Appenweier test revealed clearly smaller figures for the systematic model errors. It is also interesting to note that the figures for the systematic errors of comparator and analog plotting do not lie very much apart.

5.2 Type and shape of systematic model deformations

The type and shape of the model deformations is of importance for the selection of effective correction parameters. In the examples studied, a uniform type can be found neither in planimetry nor in height, since the model deformations are partly affected quite considerably by the following parameters:

- Camera (photographic system).
- Overlap and flight design.
- Plotting machine.
- Number of points per model.

In order to arrive at conclusions of general validity, the results of the different examples are classified and discussed in groups of identical object parameters.

5.2.1 Comparator measurements. 6 points per model.
(Oberschwaben, Appenweier)

With parallel and antiparallel orientation of the lines of flight for three different cameras (2 ZEISS RMK 15/23 wide-angle, 1 ZEISS RMK 8.5/23 super-wide-angle), a marked trapezoidal shape is obtained as a deformation type for planimetric position. It should be noted, however, that the shape of the model deformations is different for each of the three cameras (see Figs. 2, 3b).

In the case of the crossed and crossed plus antiparallel location of flight strips in the double blocks and the fourfold block of the Appenweier test, an additional deformation of the models in the x-coordinate is clearly visible (see Figs. 3b, 4).

In height, detection of the model deformations depends to an even higher degree on overlap. Above all, existing systematic deformations of the models can be detected only with 60 % side lap or crossed flight strips (see Figs. 2, 3b, 4). In the case of all three cameras, the systematic heighting errors are marked by heavy bending of the models. In spite of this, the models themselves changed only to a surprisingly small extent.

In the Oberschwaben test, there is a transverse tilt of the model center in the x-coordinate in addition to the considerable deformation (11 - 12 µm) in the y-coordinate. As is evident from Figs. 2 - 4, in height there is reasonably good agreement between the different cameras not only as regards the type but also with respect to the shape and magnitude of model deformations. This applies above all to the wide-angle and super-wide-angle cameras used for the Oberschwaben project.

5.2.2 Analog equipment
(Block S1 - PLANIMAT, Burkheim - C-8, Block S2 - A-8)

In the examples based on analog plotting, model deformations are strongly dependent on the equipment used, as is evident from Figs. 5a, and 6b. In the following, the types of deformation are therefore studied separately for the different projects.

In the S1 block, the effect of systematic machine errors on analog plotting can be clearly recognized. While the PLANIMAT plot shows a pronounced vertical bending between the edges and the center of the model, the models of the PSK measurement exhibit the previously mentioned deformation in the y-coordinate. In horizontal position, the differences between model deformations resulting from PSK and PLANIMAT measurement are less, but still clearly noticeable (see Fig. 5a).

In the Burkheim block, two instruments (C-8 I + III) show a pronounced trapezoidal shape in horizontal position, while the deformation in the third instrument has quite a different characteristic. In height, there is a pronounced twisting of the models in all cases, which in the C-8 III reaches particularly high values (see Fig. 6a).

Block S2 shows different deformations for opposite directions of flight for measurement in a plotting machine (WILD A-8) and otherwise constant object parameters (see Fig. 6b). We here have a superposition of image and instrument errors, whose result varies as a function of the reversal of the direction of one of the two components. The direction of triangulation has remained the same. The deformation figures do not fit a simple pattern in planimetry; in height, a pronounced transverse tilt of the model center can be noted for WE direction of flight and a pure transverse tilt of the model for EW direction of flight. The latter can be explained in conjunction with the perspective centers. The systematic heighting errors of the perspective centers differ in the two directions of flight by their opposite signs and, moreover, are of different magnitude (WE 7 µm, EW 21 µm).

5.2.3 15 points per model (Block S1, Rheidt)

If in Figs. 5a and 5b we only look at the model deformations at the six standard points of the model, the comparator measurements reveal the well-known types of deformation: in planimetry, it is essentially a trapezoidal shape and in height, a bending in the center of the model. The systematic errors at the intermediate points, however, generally differ noticeably from the values that would result from linear interpolation between the corresponding six standard points. In the case of the Rheidt test field, for example, heighting differences go up to 5 µm (super-wide-angle). In the mean values of the strips of block S1, they hardly exceed 5 µm for the comparator measurements and even 10 µm for the PLANIMAT plotting. That means that in these cases the model deformations are only insufficiently characterized by the six standard points of the model.

5.3 Dependence of systematic errors on project parameters

5.3.1 Photoflight parameters

Photographic system (camera, film magazine, film transport, pressure plate, ect.)

The systematic model errors are largely a function of the photographic system and particularly the camera, that is, the type of camera. This important fact results from a comparison of model deformations for different cameras from one and the same manufacturer. According to Table 1, four projects (Oberschwaben, Appenweier, S1, Rheidt) of six different flights were flown with two different RMK 15/23 cameras and two projects (Oberschwaben, Rheidt) with two different RMK 8.5/23 cameras. If we now look at the six standard points of the models and the comparator measurements, we obtain the same characteristic model deformations in each of the six cases with trapezoidal planimetric deformation and a bending in height. The fact that the model deformations for the different cameras vary slightly and have different orientation is of secondary importance here and may be due to various causes, such as installation in the camera in the different photographic aircraft, film magazine, film, ect. (see also Meier |9|). Although it is not the purpose of this paper to detect causal error components for the different results, it should be said that in the future greater attention will have to be paid to the technical data of the photographic system (aircraft, filter, film magazine, film, development, prints).

Unfortunately, a similar comparison was not possible for the two WILD cameras, since the corresponding projects (Burkheim and Block S2) were plotted in the analog mode so that the resulting model deformations are highly machine-specific.

Overlap and flight design

Although overlap only affects the detectability and not the generation of systematic model deformations, it does have an essential influence. Thus, model deformations in height can be determined more or less completely only with a side lap of at least 60 % and crossed lines of flight. In planimetry, the most essential sources of error can be detected and eliminated already at normal overlap, only model deformations in the x-coordinate remaining unaccounted for, which likewise become evident only with crossed flight lines.

Direction of flight

The most essential causes of systematic model errors can be found in the photographic system. They therefore refer to the photograph, irrespective of the direction of flight. With reference to the ground or the higher block coordinate system we therefore obtain a dependence on the line of flight, which can be taken into account by simple rotation.

Date of flight

In the Oberschwaben test block, no significant effect of the flight date on systematic model errors could be shown, although the photography was partly flown at intervals of a month. In the Appenweier test, the WE and EW strips were flown on April 24, 1973, and the NS and SN strips only three weeks later, on May 12, 1973. This may explain the differences between planimetric EW/WE and NS/SN deformations that are small in magnitude but of very typical shape (see Fig. 4). This would mean that changes in the photographic conditions (physical atmospheric conditions, different vegetation, different film, development) may have a certain effect on model deformations.

5.3.2 Plotting machine

In studying the effect of the plotting machine on systematic model errors we have to distinguish between comparators and analog instruments.

With a measurement accuracy of between 1 and 2 µm, comparators are the most accurate aerotriangulation instruments so that only slight systematic errors may have to be expected. In the Oberschwaben test, the photography was plotted in different ZEISS PSK and WILD StK1 comparators by four different organizations. A comparison of results revealed the following maximum differences between the different instruments: for wide-angle photography, 2 µm in horizontal position and 0.8 µm in height, for super-wide-angle photography 1.4 µm and 0.8 µm, respectively. The results indicate that in the case of comparators the effect of systematic errors is actually very small.

When analog instruments are used for measurement, model deformations are produced by the joint effect of different sources of error. In addition to systematic errors in the photography, these are primarily systematic instrument errors and residual errors, such as distortion, earth curvature and refraction.1)

Due to a mixture of several errors, the model deformations resulting from analog plotting are heavily related to specific instruments. The results obtained in studying the different projects justify the statement that the machine errors of analog instruments have a predominant effect on model deformations.

In addition, the plotting direction in analog machines may have an effect on model deformations if the models are triangulated in a uniform direction but plotted in or against the direction of flight within a block.

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1) As far as we know, no corrections were applied in any of the projects involving analog plotting.

5.3.3 Number of points per model

The results of the study clearly show that model deformations can be described by the six standard points only insufficiently if the points are distributed over the entire model. A considerable amount of additional deformations act on the model. It follows from this that in these cases the sets of parameters of simultaneous self-calibration used up to now and conceived for the standard case of aerotriangulation using six tie points do not suffice to completely compensate the systematic model error. However, it is not yet clear to what extent this effect may be carried into block triangulation.

5.4 Constancy of systematic errors

The constancy of systematic errors within a block is decisive for the groups of parameters to be used for simultaneous self-calibration. In assessing stability, we have to distinguish between the behavior of systematic errors with modified project parameters and the behavior with constant project parameters within a block.

To check on constancy, the following statements are based on the separate results of this study as well as the differences between strip and block analysis given in Table 4a and the accuracy figures after correction with different formulations.

In blocks with modified object parameters (Block S1, Burkheim, Block S2) marked differences between the model deformations of the different groups of parameters can be recognized. This is clearly proved by the relatively large differences between strip and block analysis in Table 4a and a comparison of the diagrams that have already been split up into groups of constant parameters.

As regards the behavior of systematic model errors with constant object parameters, no uniform statement can be made. In the Oberschwaben test block, the slight differences between strip and block analysis remain without effect on the absolute accuracy of the reference points after correction. Identical treatment of the systematic errors for the different blocks is thus admissible, as is confirmed by the results of simultaneous self-calibration (see |1|). The Appenweier test shows the opposite picture. The slight differences between strip and block-invariant determination of systematic errors have a decisive effect on the absolute accuracy after correction. This can be clearly seen from the results of the different correction terms in Table 4b. While strip correction results in considerably higher horizontal accuracy, a block-invariant formulation only improves σ_{0P}, while absolute accuracy remains practically unchanged. In other words, in spite of the elimination of systematic errors, block-invariant correction does not improve absolute accuracy as compared with normal block adjustment in which systematic errors are partly eliminated by averaging of multiple measurements.

Table 4a: R.M.S. Values of the differences of systematic model deformations as analyzed per strip or per block

Project	differences (r.m.s. values)		
	x [μm]	y [μm]	z [μm]
Oberschwaben			
w.a. q = 60 %	1.2	1.2	1.6
s.w.a. q = 60 %	1.4	1.4	1.6
Appenweier	1.3	1.4	1.8
Block S1	2.1	2.8	2.9
Burkheim	2.2	2.5	2.8
Block S2	2.7	4.0	3.2

Table 4b: Appenweier, block of 4 flights comparison of several procedures for correction of systematic errors

Version	σ_{oP} [μm]	σ_{oH} [μm]	absolute accuracy		
			μ_x [μm]	μ_y [μm]	μ_z [μm]
standard block adjustment	4.9	8.2	4.4	4.6	9.5
corrections per strip	4.1	7.6	3.1	3.3	9.0
corrections per block	4.2	8.0	4.2	4.6	9.2

μ_x, μ_y, μ_z = r.m.s. values of residual errors at check points

5.5 Effectiveness of systematic-error correction

Table 5 compares the accuracy of the different blocks before and after correction of systematic model deformations. The effect of correction on the absolute accuracy of the adjusted blocks is a separate subject that will not be discussed here. It should only be noted that in practically all the cases studied, the correction of systematic model errors resulted in a marked increase of horizontal and vertical accuracy.

Table 5:

Effectivness of the empirical correction of systematic errors

project/block	control	after block adjustment				after empirical correction			
		σ_{oP} [µm]	σ_{oH} [µm]	$\mu_{x,y}$ [µm]	μ_z [µm]	σ_{oP} [µm]	σ_{oH} [µm]	$\mu_{x,y}$ [µm]	μ_z [µm]
Oberschwaben	for all blocks								
WA, q = 20 % 375 models	plan perimeter i=2b	6.6	8.0	11.8	13.1	4.6	7.6	8.2	12.9
SWA, q = 20 % 375 models	height 13 chains i=2b	8.6	7.2	14.5	14.8	6.8	6.7	10.3	14.7
WA, q = 60 % 375 models		6.4	9.1	9.7	11.5	4.6	7.4	6.7	10.0
SWA, q = 60 % 375 models		8.1	9.2	12.1	13.1	6.7	6.7	7.8	11.4
ppenweier	for all blocks								
single blocks 112 models	plan								
WE	all given control points	4.9	7.7	7.7	14.7	4.2	7.4	6.0	14.9
EW	(perimeter + point inside)	4.5	7.3	6.0	12.4	3.5	6.8	5.2	12.2
NS	height	4.4	7.6	5.3	11.4	3.6	7.3	4.4	11.5
SN	grid i=4b	4.9	7.7	7.8	14.7	3.6	7.3	5.3	15.3
mean		4.7	7.6	6.8	13.4	3.7	7.2	5.3	13.6
double blocks 224 models									
WE / NS		4.9	8.3	5.1	11.6	4.4	7.9	4.1	10.9
EW / SN		4.7	7.9	6.0	11.7	3.8	7.3	4.3	11.3
mean		4.8	8.1	5.5	11.7	4.1	7.6	4.2	11.1
4-fold block (WE/EW/NS/SN) 448 models		4.9	8.2	4.5	9.5	4.1	7.6	3.2	9.0
Block S1 WA, q = 60 %	for all blocks plan								
SK-subblock 139 models	43 control points	8.2	12.0	-	16.0	8.0	11.3	-	10.7
Block S1 323 models	height grid i=2b	11.6	13.9	-	15.4	11.2	12.6	-	11.0
Burkheim WA, q = 20 % 62 models	plan dense perimeter height all given control points	8.4	17.6	10.4	-	7.3	16.9	9.1	-
Block S2 WA, q = 20 % 67 models	plan dense perimeter height: i=4b	13.7	18.0	18.3 1)	22.0	12.3	15.1	15.1	20.9
1) pre-marked check points	plan: 6 points height: i=9b (15 points)	13.4	17.7	25.7 1)	49.7	11.7	14.3	24.0	37.7

$\mu_{x,y}$, μ_z = r.m.s. values of residual errors at check points

$(\mu_{x,y} = \sqrt{(\mu_x^2 + \mu_y^2)/2})$

6. Summary and conclusions

A study of various controlled blocks has produced a number of interesting and valuable hints regarding important characteristics of systematic model errors. The most essential result of empirical analysis is proof that the magnitude of the systematic model errors in plan and height is considerable, with only slight variance. As a rule, correction therefore results in considerably increased accuracy so that a correction of systematic errors can be recommended as a standard technique.

Other results of importance for practical work are the following:

- If the points are distributed over the entire model, systematic model deformations may be encountered that cannot be fully covered with the correction terms presently used and adapted to the standard case of aerotriangulation with six points.
- The model deformations in analog plotting reveal a predominant effect of systematic machine errors.
- In the case of comparator measurements, systematic image and model errors are primarily a function of the photographic system.

As regards the dependence of systematic model errors on various object parameters, the following has been found:

- Of the photoflight parameters, the flight date also has a slight effect on systematic errors, in addition to the photographic system. Overlap and flight design have a considerable effect on the detectability of systematic errors, especially with regard to height. The dependence of systematic model errors on the direction of flight is due to reference to the ground system and can be taken into account by simple rotation.
- In comparator measurements, systematic machine errors have only a slight effect on model deformations, while they result in a considerable dependence of model deformations on specific machines in the case of analog plotting.
- In analog plotting, systematic model errors may additionally be a function of the direction of triangulation.
- The constancy of systematic errors varies as the different object parameters are changed within a block. But even if the parameters are constant, no basically block-invariant systematic errors may be expected. In some cases, very slight differences of systematic errors may have a decisive effect on the result of the correction.

Summarizing it may be said that the results of the extensive tests very nicely supplement previous knowledge of the characteristics of systematic errors and are of great importance, above all, for practical aerotriangulation. The partly very clear hints regarding the behavior of systematic errors provide information that will be very useful for developing efficient and operational computer programs for block adjustment with simultaneous self-calibration. As suitable program systems become available for practical use of the technique, even better utilization of the accuracy potential of aerotriangulation may be expected for the future. In this connection, the fact that this increase in accuracy involves only slightly higher computer costs will be of particular interest for practical use.

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