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SELF CALIBRATING BLOCK ADJUSTMENT

H. EBNER, STUTTGART

INSTITUTE FOR PHOTOGRAMMETRY, STUTTGART UNIVERSITY
KEPLERSTRASSE 11, 7000 STUTTGART-1

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1. INTRODUCTION

Detection and elimination of the systematic errors of photogrammetric image or model coordinates is one of the main objectives of recent research in aerial triangulation. This is understandable and consistent because not compensated systematic errors change the image or model accuracy to the worse and can propagate very unfavourably during block adjustment (see |1|). A striking disagreement between empirically obtained accuracy figures and the corresponding theoretical expectations can be the consequence (see |2| and |3|).

The most direct way to reach the goal consists in the immediate determination of the systematic deformations by comprehensive system calibrations and the subsequent correction of the image or model coordinates. An alternative, but more indirect concept replaces the determination and elimination of the systematic errors by a compensation of their effects. This can be attained by proper flight dispositions in combination with more fold photo coverage or, in a more economic way, by appropriate post treatment of the adjusted block coordinates. A general interpolation method, used for this purpose is linear least squares interpolation (see |4|). This method is very efficient in case of rather dense control. With a smaller number of control points however, the more general methods of system calibration yield significantly better results. In modern aerial triangulation therefore, priority is given to these methods.

A real calibration of the photogrammetric system can be gained by test field calibration or by self calibrating block adjustment. Test field calibration allows for a detailed and accurate determination of the systematic deformations of the photogrammetric data (see |5|). However, additional flight and measuring effort is required and the pre condition exists, that the calibrating data are representative for the practical project, actually treated. In contrast to that, self calibrating block adjustment only uses the project data themselves. Here, the actual photogrammetric system is calibrated with regard to the actual terrestrial one. Consequently, only those systematic errors can be compensated, which show up on the basis of the available tie points and control points.

The following representation of self calibrating block adjustment is restricted to the simultaneous method, where the systematic errors of image or model coordinates are compensated by additional parameters of the adjustment. An alternative solution, suggested by Masson d'Autume and picked up by Schilcher, shall only be mentioned here. This method determines and eliminates the systematic deformations iteratively, by repeated analyses of the residuals of the block adjustment (see |6| and |7|).

2. STATE OF THE ART AND PRESENT PROBLEMS

A first comprehensive report on photogrammetric block adjustment with additional correction parameters was presented at the Ottawa Congress 1972. The authors Bauer and Müller were able to show results, by which the efficiency of simultaneous self calibration was demonstrated impressively |8|. Two years later, at the Commission III Symposium in Stuttgart several papers on this topic were presented. Bauer, Brown |9|, Schut |10| , Salmenperä, Anderson and Savolainen |11| reported on self calibrating bundle adjustment and Ebner and Schneider presented a first application to independent model block adjustment |12|.

Although simultaneous self calibration today is recongnized as the most efficient concept for compensation of systematic errors in aerial triangulation there are essential problems which still have to be solved. Three main problems can be distinguished here.

The first one concerns the proper choice of the additional parameters. As a study of the above mentioned literature shows, the individual authors are still experimenting with the number and type of correction terms. Furthermore it is noticed, that the additional parameters are treated as block invariant terms. This means, that identic systematic deformations are supposed for all images or models of the block. This supposition however, is only correct in case of really homogeneous projects (one camera, film, measuring instrument and so on). In all other cases a variation of the systematic errors within the block must be expected. Summarizing it can be stated, that a general concept for the choice of correction terms is still missed. Such a concept would require several different groups of additional parameters and a sufficient number of effective parameters per group.

The second problem follows from the fact that highly correlated or insignificant additional parameters lessen the block stability and change the accuracy of the adjusted block coordinates to the worse. Therefore the algebraic correlations between the individual correction terms and the correlation with the orientation parameters should be as small as possible. The significance of the computed correction terms can be checked by proper statistical tests. If some of the additional parameters are found insignificant the block adjustment should be repeated without them. Although these requirements are known in principle, frequently not enough attention is payed to them in practice.

The third problem is a purely operational one and concerns the comfort of self calibrating block adjustment programs. In the opinion of the author the user of such an extended program shouldn't be burdened with the selection of the additional parameters and the critical valuation of their amounts, as computed by the adjustment. Consequently this task should be automized as far as possible.

The following chapter contains recommendations for a solution of the problems quoted above. For that purpose a strategy is suggested, consisting of a sufficiently general functional and stochastic model and of proper significance tests. In that way self calibrating block adjustment shall be standardized to a certain extent and the practical application shall be simplified.

Finally, in chapter 4 of the paper practical results of block adjustments by bundles and by independent models are presented, being based on the self calibration strategy, as suggested here.

3. THE SUGGESTED STRATEGY

3.1 The Functional Model

The implementation of simultaneous self calibration in bundle block adjustment or independent model block adjustment consists in an extension of the observational equations by the effect of suitably chosen additional parameters. The task of these parameters is compensation of the systematic errors of image or model coordinates, recognizable from the actually given control points and tie points.

In contrast to the above mentioned programs, which are restricted to block invariant correction terms, the functional model, presented here, is using several different groups of additional parameters if the systematic deformations vary within the block. The operational aspects of this measure will be discussed in chapter 3.3.

With the selection of the additional parameters of bundle adjustment the objective is pursued to compensate all the systematic deformations, appearing in 9 image points. Figure 1 shows the schematized distribution of the 9 points and the polynomial terms, determinable here.

- Figure 1 -

For x and y together 18 terms are obtained, but 6 of them are compensated by the orientation parameters of the individual images. The remaining 12 correction terms, called b_1 to b_{12} are formulated as orthogonal to each other and with respect to the 6 orientation parameters. Figure 2 shows their contributions to the observational equations and their effects on the image points. The orthogonality of the additional parameters leads to well conditioned normal equations and allows for separate statistical checking of the individual correction terms, computed by the adjustment (see chapter 3.3).

- Figure 2 -

The formulation of the additional parameters according to figure 2 exceeds the corresponding suggestions of Schut |10| and Gotthardt |13|, although the most dangerous systematic errors are also fully compensated by them.

The functional model of independent model block adjustment is extended correspondingly. Figure 3 shows the schematized photogrammetric model, consisting of 6 points and the polynomial terms determinable in this case.

- Figure 3 -

The $3 \cdot 6 = 18$ terms, obtained for x, y and z are complemented by 3 more terms, by which the systematic errors of the perspective center coordinates x, y and z are compensated. From the 21 terms 7 are considered by the spatial similarity transformation, put up for every model. Therefore only 14 additional parameters remain.

If the block adjustment is based on the well proved planimetry-height-iteration, as used in the computer program PAT-M43 |14|, 8 correction terms refer to the planimetric block adjustment and 6 are used with height block adjustment. The corresponding parameters, called p_1 to p_8 and h_1 to h_6 again are formulated as orthogonal to each other and with respect to the transformation parameters of planimetry and height. The formulation of the correction terms and their effect on the model points is shown in figure 4.

- Figure 4 -

By the additional model parameters of figure 4 the correction terms suggested in |12| are surpassed.
With ordinary applications of aerial triangulation the functional models as presented here guarantee a fully adequate compensation of the data inherent systematic errors and can serve as standard models. In case of essentially more points per image or model (e.g. cadastre) however, the use of further correction terms can be suitable.

3.2 The Stochastic Model

First of all it seems to be obvious to treat the additional parameters as free unknowns, as done in |8| and |11|. This would lead to the following formulation:

$$v_1 = Ax - By - f \qquad (1a)$$

In (1a) f is the observation vector, containing the measured image or model coordinates, x denotes the vector of unknown terrain coordinates and transformation parameters and y is the vector of the unknown additional parameters.

Two facts however, are ignored with the formulation (1a): the relatively small size of the systematic errors and the fact that they vary from project to project with regard to sign and size (the theoretical mean value is zero). Therefore it is more suitable to treat the additional parameters as observations of amount zero with appropriate weights. This can be done by keeping (1a) and adding the following set of observation equations:

$$v_2 = \qquad y - 0 \qquad (1b)$$

The weights of the additional parameters can simply be chosen according to the expected amounts of the correction terms or somewhat smaller.

If some of the additional parameters can be derived from calibrating data, which are representative for the actual practical project, the obtained amounts can directly be introduced into the corresponding lines of the observation equations (1b), replacing the amounts zero. The weights of these parameters can then be determined from the accuracy of the calibration.

The formulation (1a), (1b), which is also used by Brown |9| and others, leads to banded bordered normal equations, with the additional parameters forming the border. In that way favourable computing times are guaranteed.

The suggested stochastic model shows several advantages. First of all, the treatment of the additional parameters as observations is completely general. Free unknowns and constants are special cases of observations and can easily be implemented by the special weights 0 and ∞ (10^{20}). Furtheron the appropriate weights of the additional parameters guarantee optimal accuracy. The most important advantage however, is the avoidance of unreliable results.

Such results have to be feared if the additional parameters are treated as free unknowns and the available control points and tie points don't allow for an accurate determination of all the correction terms, put up. The normal equation matrix then becomes ill conditioned and in extreme cases even singular. In the latter case the minimum equation of the adjustment is fulfilled by every arbitrary value of the corresponding parameters.

This danger is avoided when the additional parameters are treated as observations. The residuals v_2 then directly influence the minimum condition, which leads to a definite solution for all correction terms used. The geometrically poorly determined parameters in this case show up in form of insignificant amounts of the corresponding correction terms.

Last not least it shall be mentioned that self calibrating block adjustment with additional parameters can also be treated as a collocation problem. The appendix contains the derivation of the corresponding equation system and shows it's equivalence with the observation equations (1a), (1b).

3.3 Operational Points of View

Besides an efficient mathematical model for simultaneous self calibration the secure and comfortable execution of the extended block adjustment is the most important point. Therefore, the additional burden of the program user should be limited to an additional input, determining, which group of additional parameters has to be assigned to the occasional image or model.

With the establishment of the parameter groups it is assumed, that the systematic deformations only change if a corresponding change of project parameters occurs (different cameras or camera installations, different films or film processing, different measuring instruments and so on). A dependency of the systematic deformations on the flight direction is avoided if the image or model coordinate system itself relates to flight direction.

Consequently, a new group of additional parameters is only required if at least one of the project parameters has changed. That means that the number of parameter groups usually will be small.

When the self calibrating block adjustment is executed, the computed correction terms have to be checked critically by statistical means. For that the covariance matrix of the additional unknowns y is needed, which can be obtained from a partial inversion of the normal equation matrix. The numerical effort of this operation is relatively small.

The statistical checking succeeds in two subsequent steps. At first it is investigated, whether the corresponding correction terms of the different parameter groups differ significantly from each other. If this is not the case the concerned parameters are combined to one parameter. Thereby the individual correction terms (b_1, b_2, ...) can be treated separately. This is possible because the additional parameters are nearly orthogonal and influence each other only slightly. With the correspondingly combined parameters the block adjustment then is repeated.

In the second step the significance of the remained correction terms is checked and weight ∞ (10^{20}) is given to the insignificant parameters. A last repetition run then gives the final results.

It easily can be realized, that both steps of this procedure can be performed automatically by the program, so that the program user has not to be burdened by the valuation of the obtained correction terms and the according consequences.

The sophisticated two step procedure, suggested here, guarantees, that only the well determined additional parameters are finally used. In that way optimum reliability is attained.

Of course, the computing time of block adjustment increases when simultaneous self calibration is implemented. In practice however, the additional amount will not be very high, because the correction terms usually will only be put up with the very last runs, whilst the first runs by which the gross errors of the data are detected and eliminated, will be performed as up to now.

4. TEST RESULTS

For a practical test of the suggested strategy a part of the test block Oberschwaben was used. The author appreciates that the OEEPE has made this valuable material available. The chosen sub block is built by the wide angle strips 5, 7, 9 and 11 and consists of 100 models (terrain area = 20 · 62.5 km^2). All control points and tie points were targeted. The flight was performed with a Zeiss RMK A 15/23 camera at a photo scale of 1:28 000. A Zeiss PSK stereocomparator was used for image measurement.

Starting from the same image coordinates the block adjustments were performed by bundles and by independent models. In the second case the models were formed computationally.

The bundle block adjustments were computed at the Technical University of Munich, using the self calibrating program, developed by Dr. Grün |15|. The flexibility of this program with respect to the functional model allowed to use exactly the 12 additional image parameters, suggested in 3.1. The block adjustments by independent models were performed at Stuttgart University, using the self calibration program of Mr. Schneider. This program was developed to test the concept of simultaneous self calibration before an integration into the program package PAT-M |14|. For the given support the author is grateful to Dr. Grün and to Mr. Schneider.

Figure 5 shows the test block and the individual control distributions, used in planimetry and height. The terrestrial points, which didn't serve as control points, were used as check points. (180 to 250 occasionally).

- Figure 5 -

For all 4 strips of the test block the same project parameters can be assumed. From there it follows that only one group of additional parameters has to be put up for the whole block. The justification of this measure was confirmed by test adjustments with strip invariant correction terms, which didn't show significant variations of the parameter amounts from strip to strip.

Because all correction terms are treated as block invariant parameters the present test is not able to demonstrate the full efficiency of the suggested strategy (see 3.3). The separation of the block invariant correction terms into significant and insignificant terms however, can be shown here.

The significance tests were performed on the 99 % level. From the beginning it shall be mentioned that the obtained correction terms proved as only slightly dependent on the control distribution. With the poorest density the additional bundle parameters b_1, b_2, b_6, b_7, b_8 and b_{11} were found as significant. Their common effect on the image points represents the systematic image deformations and is shown in figure 6 (b = 92 mm). The maximum values amount to 9 µm in x and in y.

- Figure 6 -

The further results of the bundle block adjustments are summarized in table 1. Because all image coordinates were treated as observations of weight 1 the standard deviation of unit weight σ_0 here directly represents the mean accuracy of the image coordinates. Without self calibration σ_0 is highly dependent on the control distribution used. This clearly indicates the presence of systematic errors, which influence the residuals and σ_0 the less the poorer the control density is. Simultaneous self calibration reduces σ_0 by a factor 1.3 to 1.6 and eliminates the dependency on the control distribution almost completely. With σ_0 = 3.2 µm the accuracy of the image coordinates is close to the accuracy limit, attainable today at all.

- Table 1 -

In table 1 the accuracy of the adjusted block coordinates x, y and z is represented by the RMS values $\mu_{x,y}$ and μ_z, which were determined from the coordinate errors at the planimetric check points and at the check heights. Both, $\mu_{x,y}$ and μ_z are reduced to the photo scale. Simultaneous self calibration improves the accuracy the more, the less favourable the control distribution is. The factor of improvement increases up to 3.3 in planimetry and up to 2.3 in height.

The accuracy figures σ_0, $\mu_{x,y}$ and μ_z, obtained with self calibration, are slightly smaller than the corresponding results, which Bauer and Müller had published in |8|. These relate to a Oberschwaben subblock, sonsisting of the wide angle strips 1, 3, 5, 7, 9 and were obtained with 3·4 correction terms (3 successive adjustments with 4 additional parameters each).

With the block adjustments by independent models the planimetric correction terms p_1, p_2, p_3, p_4, p_8 and the height terms h_2, h_3, h_4, h_5 have proved as significant. Their common effect on the model points represents the systematic model deformations and is shown in figure 7 (b = 92 mm). The maximum values amount to 10 µm in x, 7 µm in y and 11 µm in z. As was to be expected the results of figure 7 agree well with the model deformations being computed from the systematic image errors of figure 6.

- Figure 7 -

The individual results of the planimetric block adjustments by independent models are summarized in table 2. Because the model coordinates x, y were treated with weight 1, the standard deviation of unit weight σ_{op} directly represents the mean accuracy of the planimetric model coordinates. With simultaneous self calibration this accuracy figure decreases to σ_{op} = 4.3 µm. The corresponding figure $\mu_{x,y}$, which estimates the mean accuracy of the adjusted block coordinates in x and y, is improved by a factor 1.6 to 2.9.

- Table 2 -

Table 3 shows the results of the height block adjustments by independent models. σ_{oh} here represents the standard deviations of the model heights and μ_z describes the mean accuracy of the adjusted heights of the block. The accuracy improvement, attained by self calibration, is much smaller than in planimetry. This is true for σ_{oh} as well as for μ_z. The only one exception appears with the extreme control distribution i = 25, where the rather poor accuracy μ_z = 65.0 µm is reduced to the reasonable value μ_z = 26.7 µm.

- Table 3 -

The results, listed in table 1 and in tables 2 and 3 were obtained from the same data material. Therefore they can be used for an accuracy comparison between bundle and independent model adjustment. Without self calibration most of the bundle results $\mu_{x,y}$ and μ_z are larger than the corresponding figures of the block adjustments by independent models. Obviously this is caused by the systematic data errors (see also |3|).

As soon as the systematic errors are compensated adequately, which is guaranteed by simultaneous self calibration, the situation changes and the bundle results prove as superior, as expected by theory. Table 4 shows the accuracy figures $\mu_{x,y}$ and μ_z, obtained with both adjustment methods and the accuracy ratios $\mu_{models} / \mu_{bundles}$. The maximum ratio is 1.2 in planimetry and

1,4 in height. Because of the limited accuracy of the control points and check points, by which the accuracy figures μ are distorted the more, the smaller they are, the real accuracy advantage of bundle block adjustment can even be expected as higher.

- Table 4 -

Altogether the test results demonstrate, that by simultaneous self calibration excellent accuracies can be obtained, even if systematic errors of considerable size are existing. As an example we cite the RMS values $\mu_{x,y}$ obtained with the extreme control distributions i = 8 and i = 11. Here, both adjustment methods lead to amounts of 7 µm to 8 µm at the photo scale or 20 cm to 22 cm in the terrain. If we compare these accuracies with the control spacings of 20 km to 31 km we obtain ratios which are better than $1:10^5$.

Finally, the important statement can be made, that the test results, obtained with simultaneous self calibration meet the theoretical expectations in a twofold way. Firstly the standard deviations of unit weight σ_0, σ_{0p} and σ_{0h} are practically independent of the control distribution and secondly the empirical ratios μ/σ_0, representing the error propagation with the block adjustment, are in well agreement with the corresponding theoretical predictions, being based on random errors only. These facts indicate, that the systematic deformations of the image and model coordinates are extensively compensated and that the remaining errors can be considered as random.

5. CONCLUDING REMARKS

This reduction of the data errors to the purely random component is the most important result of the test. It confirms, that the used strategy for self calibrating block adjustment is fully effective in the present case. For a real conclusive valuation of the suggested strategy however, further and more generally drafted tests with block variable systematic errors and different overlap configurations still have to be performed.

APPENDIX

Self calibrating block adjustment with additional parameters, treated as a collocation problem.

Let us first formulate the functional model of block adjustment as:

$$A\bar{x} - \bar{f} = 0 \qquad (2)$$

f is the observation vector, containing the measured image or model coordinates and x is the vector of the unknown terrain coordinates and transformation parameters. By $\bar{x}$ an $\bar{f}$ the theoretical values of x and f are ment.

The mathematical model of collocation supposes, that the actual observation vector f differs from the theoretical one $\bar{f}$ due to two random vectors n and s with the statistical expectations zero (see Moritz |16|).

$$f = \bar{f} + n + s \qquad (3)$$

$$E\,[n] = 0 \qquad (4)$$

$$E\,[s] = 0 \qquad (5)$$

n is called noise or uncorrelated component and s is the signal or correlated part or f. Correspondingly, the noise covariance matrix C_{nn} (usually) is a diagonal matrix, whilst the signal covariance matrix C_{ss} is rather packed. In general, noise and signal are not correlated with each other ($C_{ns} = 0$).

In the present case of self calibrating block adjustment the noise represents the purely random errors of the photogrammetric data. The signal s however, is interpreted as the effect of the additional parameters p on the image or model coordinates:

$$s = Bp \qquad (6)$$

The additional parameters p themselves are assumed as random variables with the expectation zero.

$$E\,[p] = 0 \qquad (7)$$

With (7) the signal (6) meets the requirement (5). If the covariance matrix of the additional parameters p is denoted by C_{pp}, the signal covariance matrix C_{ss} follows from (6) as:

$$C_{ss} = B\,C_{pp}\,B^T \qquad (8)$$

The problem is solved by least squares. For that purpose the residual vectors v_1 and v_2 are attached to n and p. Considering (3) and (6) we then obtain:

$$Ax - (\bar{f} + (n+v_1) + B\,(p+v_2)) =$$

$$Ax - v_1 - Bv_2 - f = 0 \qquad (9)$$

The minimum condition to be satisfied reads:

$$v_1^T C_{nn}^{-1} v_1 + v_2^T C_{pp}^{-1} v_2 = \min \qquad (10)$$

Equation (9) represents a conditioned adjustment with unknown parameters, which however, is equivalent to the observation equation system (1a), (1b): This can be shown by back substitution of (1b) into (1a) which directly leads to formula (9). For a detailed proof see Schwarz |17|. Because of (4) and (7) the obtained results x, v_1 and v_2 are unbiased.

The additional parameters usually are common to many images or models occasionally. In this case formulation (1a), (1b) is superior to formulation (9), because it leads to normal equation matrices of more favourable structure (banded bordered system) and of better numerical condition (see |18|).

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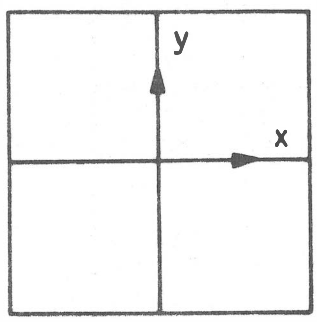
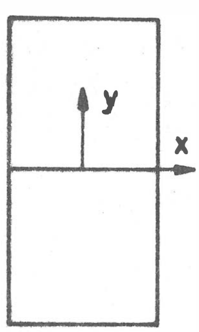


$$\begin{bmatrix} 1 & y & y^2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ x \\ x^2 \end{bmatrix} \begin{bmatrix} 1 & y & y^2 \\ x & xy & xy^2 \\ x^2 & x^2y & x^2y^2 \end{bmatrix}$$

Figure 1



$$\begin{bmatrix} 1 & y & y^2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ x \end{bmatrix} \begin{bmatrix} 1 & y & y^2 \\ x & xy & xy^2 \end{bmatrix}$$

Figure 3

ADDITIONAL IMAGE PARAMETERS FOR SELF CALIBRATION

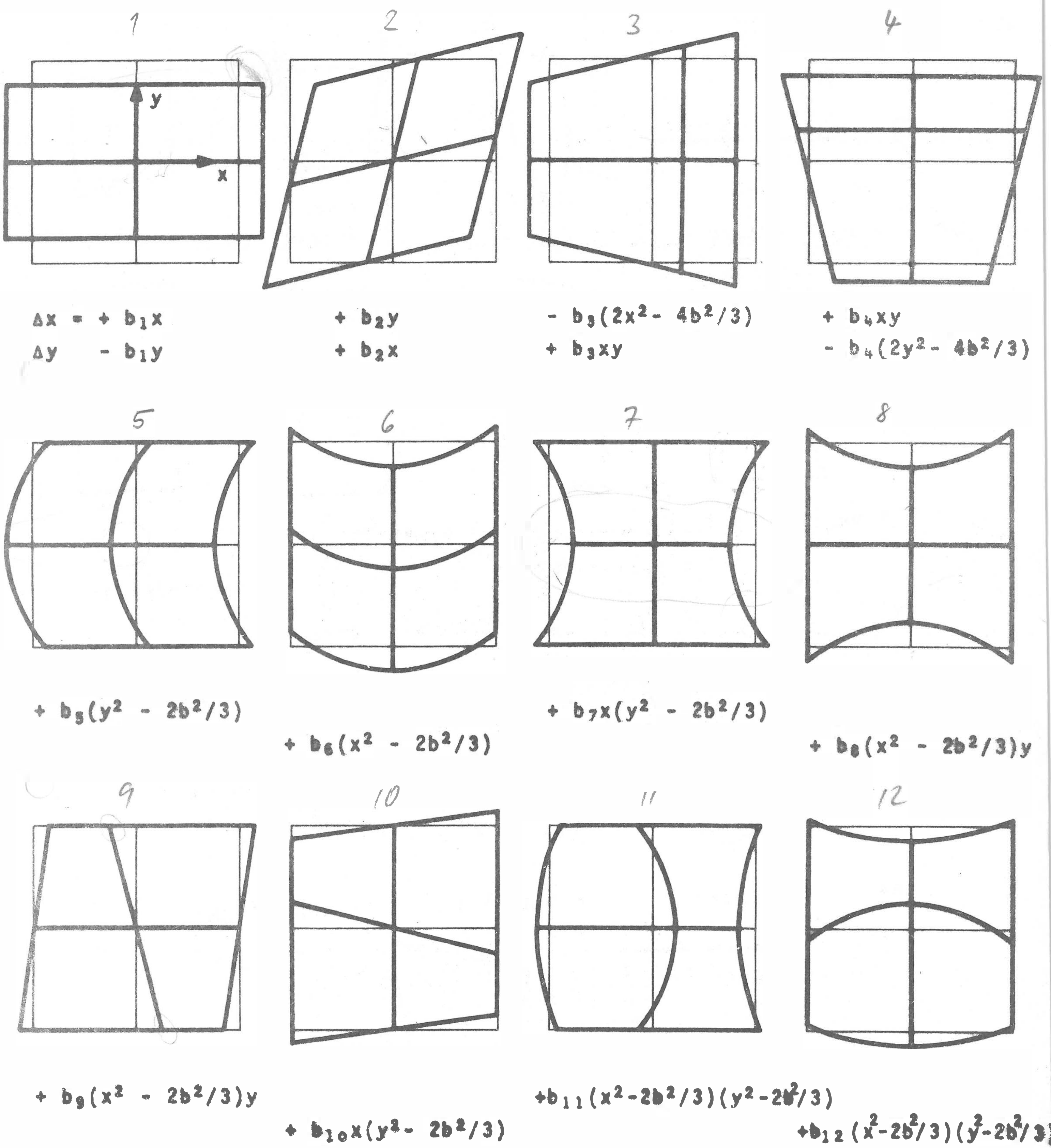


Figure 2

ADDITIONAL MODEL PARAMETERS FOR SELF CALIBRATION

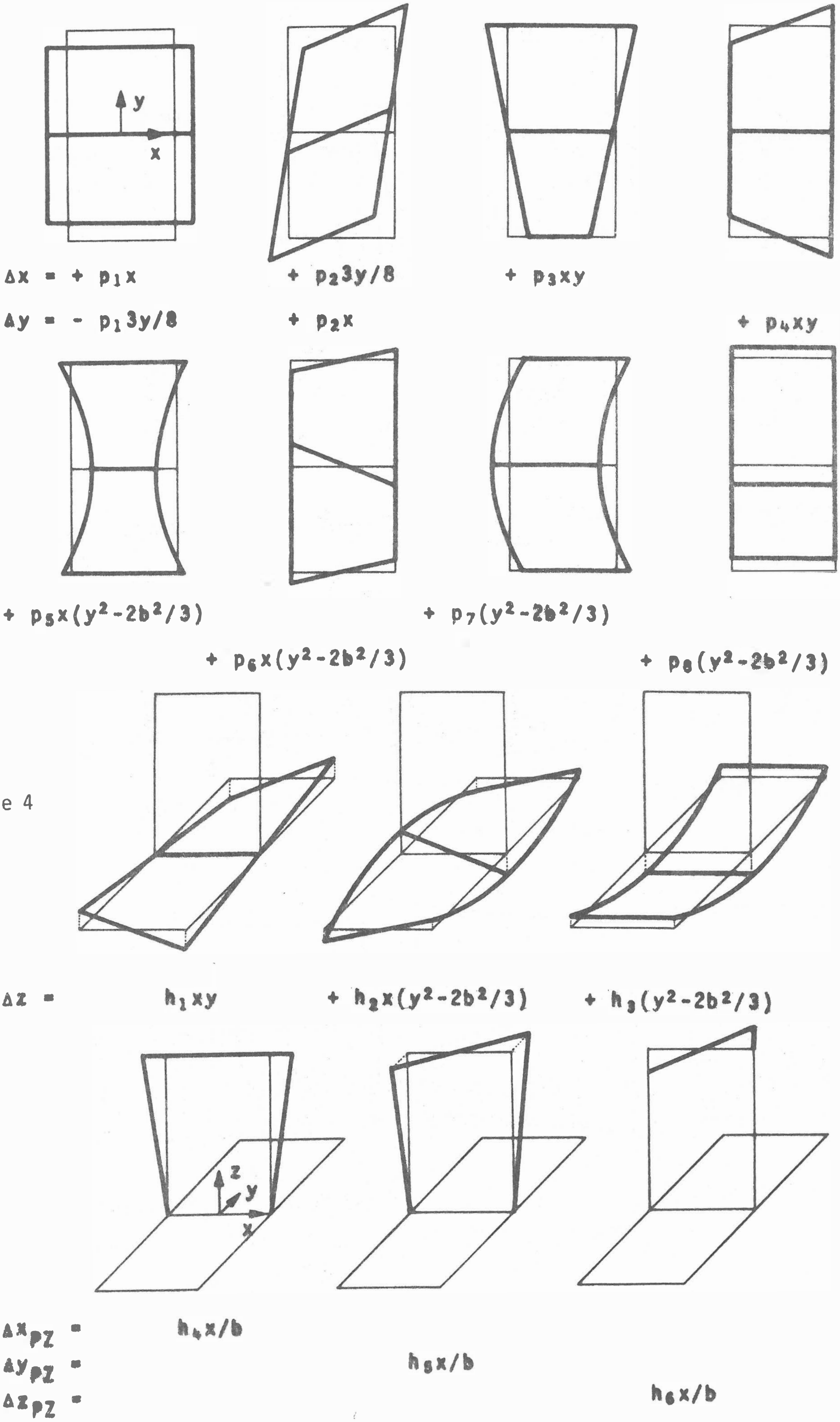


Figure 4

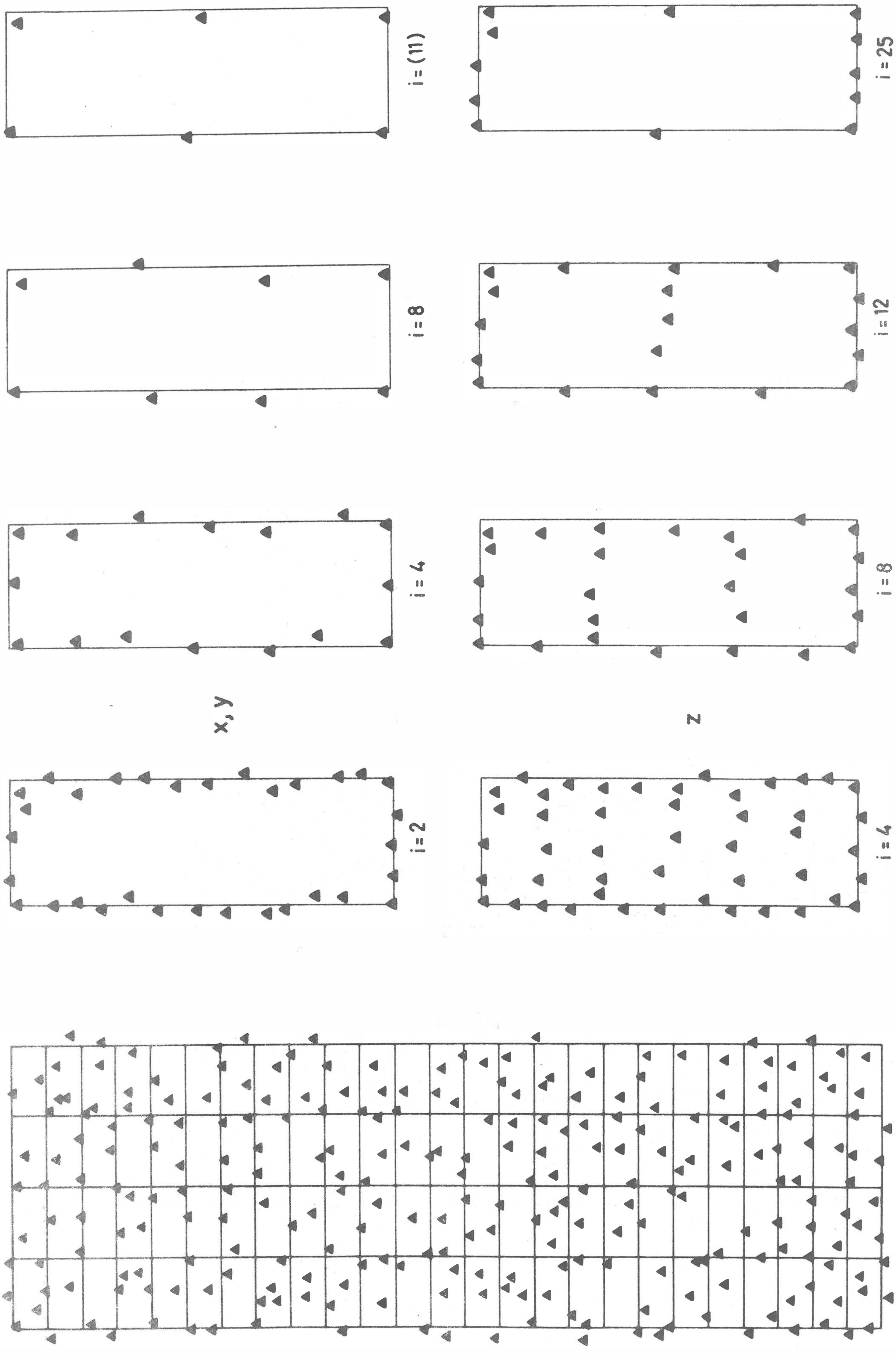


Figure 5

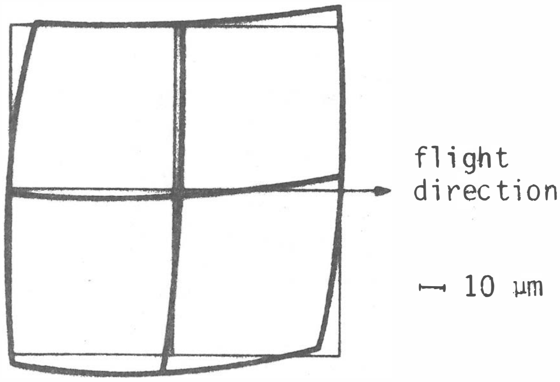


Figure 6

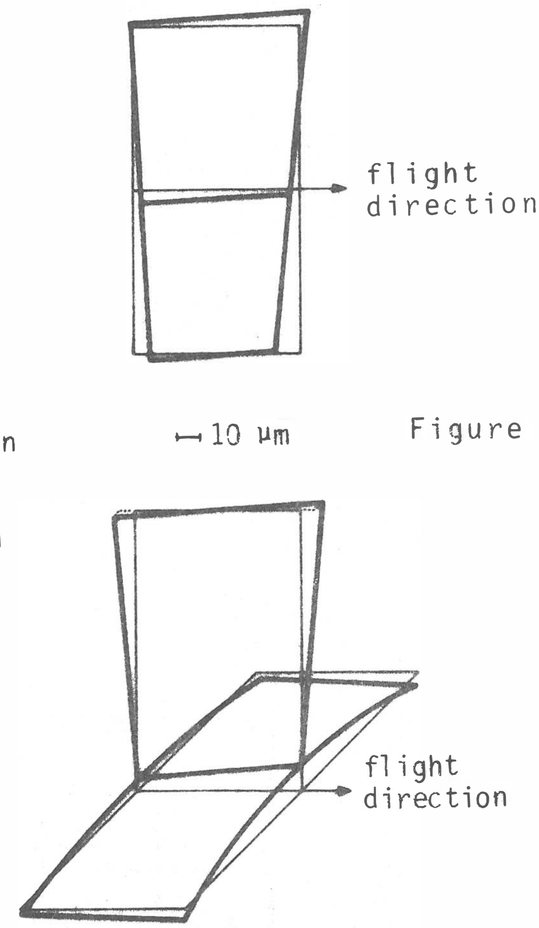


Figure 7

BUNDLE BLOCK ADJUSTMENT

Oberschwaben, 1:28 000, wide angle
Block Frankfurt, 104 photos, q = 20 %

control version		without self calibration			with self calibration			accuracy improvement		
x,y	z	σ_0 |µm|	$\mu_{x,y}$	μ_z |µm|	σ_0 |µm|	$\mu_{x,y}$	μ_z |µm|	σ_0	$\mu_{x,y}$	μ_z
i=2	i=4	5.3	8.8	15.8	3.3	5.2	12.2	1.6	1.7	1.3
i=4	i=8	4.7	14.0	22.2	3.2	5.6	14.6	1.5	2.5	1.5
i=8	i=12	4.1	24.1	28.4	3.2	7.2	16.5	1.3	3.3	1.7
(i=11)	i=25	4.0	24.9	44.2	3.2	8.0	18.9	1.3	3.1	2.3

Table 1

PLANIMETRIC BLOCK ADJUSTMENT BY INDEPENDENT MODELS
Oberschwaben, 1:28 000, wide angle
Block Frankfurt, 100 models, q = 20 %

control version	without self calibration		with self calibration		accuracy improvement	
	σ_{op} |µm|	$\mu_{x,y}$ |µm|	σ_{op} |µm|	$\mu_{x,y}$ |µm|	σ_{op}	$\mu_{x,y}$
i=2	6.8	9.9	4.4	6.3	1.5	1.6
i=4	6.5	13.4	4.3	6.6	1.5	2.0
i=8	6.2	20.0	4.3	7.1	1.4	2.8
(i=11)	6.1	22.1	4.3	7.7	1.4	2.9

Table 2

HEIGHT BLOCK ADJUSTMENT BY INDEPENDENT MODELS
Oberschwaben, 1:28 000, wide angle
Block Frankfurt, 100 models, q = 20 %

control version	without self calibration		with self calibration		accuracy improvement	
	σ_{oh} |µm|	μ_z |µm|	σ_{oh} |µm|	μ_z |µm|	σ_{oh}	μ_z
i=4	8.4	14.7	7.6	14.1	1.1	1.0
i=8	8.3	19.0	7.6	17.1	1.1	1.1
i=12	8.3	22.1	7.6	18.9	1.1	1.2
i=25	8.3	65.0	7.6	26.7	1.1	2.4

Table 3

SELF CALIBRATING BLOCK ADJUSTMENT
Oberschwaben, 1:28 000, wide angle
Block Frankfurt, 100 models, q = 20 %

control version		bundles		models		ratio	
x,y	z	$\mu_{x,y}$ |µm|	μ_z |µm|	$\mu_{x,y}$ |µm|	μ_z |µm|	$\mu_{x,y}$	μ_z
i=2	i=4	5.2	12.2	6.3	14.1	1.2	1.2
i=4	i=8	5.6	14.6	6.6	17.1	1.2	1.2
i=8	i=12	7.2	16.5	7.1	18.9	1.0	1.1
(i=11)	i=25	8.0	18.9	7.7	26.7	1.0	1.4

Table 4