

## SYSTEMATIC MODEL DEFORMATION OF THE OEEPE-TESTBLOCK "OBERSCHWABEN"

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### 1. INTRODUCTION

Recent theoretical and empirical investigations have been concerned very much with systematic image errors and their influence on the accuracy of aerial triangulation. Several methods have been suggested for considering systematic errors during block adjustment both by the bundle and the independent model method (see [1], [2], [3], [4], [5]). There is, however, only very limited information available about the systematic errors occurring in practical cases, in particular about the types and magnitudes of such errors and their dependence on various factors.

In this paper the results are presented of an empirical investigation into the systematic model deformations of the testblock Oberschwaben, as appearing after block adjustment by independent models. It will be investigated whether systematic model deformations depend on the camera (wide-angle, super-wide-angle), flight direction, date of the flight mission and measuring instrument. Other points of interest are the influence of different distributions of control points on systematic model deformations and the question of how constant such errors are within certain areas of a block.

### 2. THE TEST "OBERSCHWABEN"

During spring 1969 aerial photographs were taken of 1 : 28 000 scale of the test area Oberschwaben which is located in southwestern Germany. The test area of 40 km x 62.5 km was photographed with a wide-angle camera Zeiss RMK 15/23 and a super-wide-angle camera Zeiss RMK 8.5/23, each coverage resulting in a block of 60 % longitudinal and lateral overlap with 15 strips of 25 models each. The strips run in north-south direction or vice versa.

For the restitution each of the two blocks was subdivided in two blocks of 20 % lateral overlap. The four separate blocks were given the names of the 4 centres which undertook the stereocomparator measurements:

- Wide-angle blocks: block Frankfurt, uneven strip numbers (1,3,5,...13,15),  
8 x 25 = 200 models; block Vienna, even strip numbers (2,4,6,...12,14),  
7 x 25 = 175 models.
- Super-wide-angle blocks: block The Hague, uneven strip numbers (1,3,...13,15),  
8 x 25 = 200 models; block Delft, even strip numbers (2,4,...12,14),  
7 x 25 = 175 models.

Aerial photography was taken on five different days by pin-point flying. Table 1 for wide-angle and Table 2 for super-wide-angle display date and direction of the flight strips. Within the test area 548 trigonometric points were signalized, to be used as control-points and check-points. For 480 of them also the heights are given. Pin-point flying was necessary because also the tie-points were signalized in the terrain by double targets, for test purposes. Therefore the models contain 6 x 2 signalized tie-points located in the 6 standard positions.

Stereocomparator measurements of the photographs were performed by 4 different photogrammetric centres (Zeiss PSK: Frankfurt, The Hague; Wild StK 1: Vienna, Delft). The image coordinates were corrected for radial symmetrical distortion, refraction, and earth curvature. Then independent models were computed, by analytical relative orientation. The independent models went into a number of block adjustments with the PAT-M-43 program the results of which have been published in [6]. The present study for systematic errors is based on the residual errors at tie-points and perspective centres as appearing after such block adjustments.

### 3. METHOD OF INVESTIGATION

The systematic errors of the independent models are represented by the average residual vectors at the 6 standard positions and at the two perspective centres of the models after block adjustment. In order to obtain the systematic errors the models, as appearing after the block adjustments, were transformed onto a nominal reference model, specified by giving the perspective centres of each model

the coordinates  $x_1 = 0$ ,  $y_1 = 0$ ,  $x_2 = 2500$  m,  $y_2 = 0$  (2500 m was the base length according to the flight plan).

The residual errors of the tie-points were referred respectively to the ideally located standard positions of the reference model, see fig. 1. Then for each of the standard positions of tie-points and of the perspective centres the arithmetical means of all residual errors in  $x$ ,  $y$ ,  $z$  of all models were computed. The mean values represent the systematic planimetric and vertical model deformations.

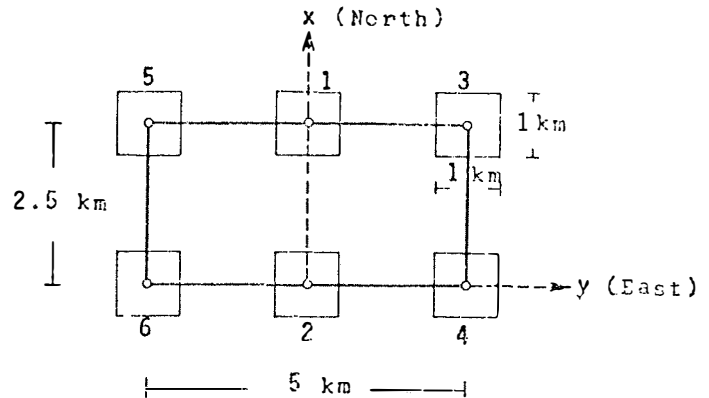


Figure 1: Reference model for representation of systematic model deformations.

The empirical investigation on systematic errors of independent models refers to the four separate Oberschwaben blocks of 20 % lateral overlap which include 200 and 175 models respectively. The following adjustments with different control versions are included:

Block Frankfurt (w.a.) and block The Hague (s.w.a.):

- version 0: all planimetric and vertical control points
- version 1: Planimetric control: perimeter, spacing  $i=2$  base-lengths  
Vertical control: chains, bridging distance  $i=2$  base-lengths
- version 5: Planimetric control: 4 corner points  
Vertical control: 3 chains, bridging distance  $i=12.5(6)$  base-lengths, with 4 additional perimeter points (see [6]).

Block Vienna (w.a.) and block Delft (s.w.a.):

- version 0: all given planimetric and vertical control points.

Version 0 gives 2 - 3 control points per model. Therefore the adjustment is more or less equivalent to absolute orientation of single models. Version 0 will give the best estimation of the actual systematic model deformations. The versions 1 and 5 of the blocks Frankfurt and The Hague are intended to show the influence of control distribution on the apparent systematic errors.

#### 4. RESULTS

The results of the investigation are summarized in the tables 1 - 4 and figures 2 - 5. The tables 1 and 2 present, separately for each of the 15 wide-angle and 15 super-wide-angle strips, the average residual coordinate errors at the 6 standard model tie-point positions, referring to the adjustment version 0 (all control points used). Table 3 presents the results of all adjustment versions treated of all 4 blocks, by summarizing the strips in groups by common flight direction. Table 4 displays accordingly the systematic residual errors at the perspective centres. In addition, the essential results are graphically represented in figures 2 - 5.

Table 1. OEEPE-Oberschwaben  
Systematic model deformations after block adjustment by independent models.  
Wide-angle strips: Control version 0; in units of  $\mu\text{m}$

strip no.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
flight direction		NS	SN	NS	SN	NS	SN	NS	SN	SN	NS	SN	SN	NS	NS	SN
flight mission		8.4.	8.4.	8.4.	8.4.	8.4.	8.4.	9.4.	12.5.	9.4.	12.5.	12.5.	12.5.	12.5.	12.5.	12.5.
	standard position															
x	1	+1.6	+0.6	+2.1	+2.1	+1.7	+1.0	+2.1	+1.6	+ 3.3	+2.7	+ 2.4	+2.0	+1.9	+0.3	+2.0
	2	-1.7	-0.8	-2.1	-2.1	-1.7	-1.0	-2.1	-1.7	- 3.3	-2.7	- 2.4	-2.0	-1.9	-0.3	-2.0
	3	-0.8	+7.7	+0.0	+6.4	-2.4	+6.7	+0.1	+4.0	+10.3	-4.2	+11.4	+5.8	-0.6	-5.4	+7.9
	4	+4.1	-3.7	+4.1	-2.7	+5.1	-4.5	+2.6	-4.5	-6.2	+3.9	- 6.3	-6.1	+3.9	+5.4	-7.9
	5	+6.0	-3.4	+4.6	-4.1	+4.6	-4.4	+5.0	-2.7	- 3.5	+9.4	- 5.7	-2.1	+4.7	+7.1	-3.9
	6	-6.0	+3.9	-8.5	+0.0	-8.7	+1.5	-7.9	+0.7	+ 0.6	-9.2	+ 1.9	+2.4	-8.0	-6.2	+1.2
y	1	+2.8	+2.1	+3.4	+2.2	+2.7	+2.5	+3.0	+0.5	+ 2.0	-1.1	+ 2.6	-0.2	+1.5	-2.1	+1.7
	2	-2.7	-2.3	-3.4	-2.2	-2.7	-2.5	-3.0	-0.6	- 2.0	+1.1	- 2.6	+0.2	-1.5	+2.1	-1.7
	3	+0.4	+3.0	+0.8	+1.8	+0.4	+1.4	-0.6	+0.7	- 1.5	-2.8	+ 2.4	+0.4	-1.5	+0.5	+2.3
	4	-2.8	-5.6	-2.1	-4.8	-3.3	-3.4	-7.2	-1.5	- 2.8	-5.0	- 3.6	-0.7	-5.7	-0.3	-2.3
	5	+2.5	-0.1	+5.9	+3.8	+3.3	+3.3	+2.7	-1.0	+ 7.2	-0.2	+ 4.2	+1.5	+2.5	+0.6	+3.9
	6	-2.2	-0.2	-2.6	-0.7	-1.8	+0.2	-0.1	+3.0	+ 0.3	+0.6	+ 0.6	+6.3	-1.4	-0.2	+2.5
z	1	+2.6	+0.5	+3.5	-3.4	+3.4	-1.8	+3.8	-1.4	- 3.5	+2.7	- 0.8	-3.1	+0.2	+3.3	-1.1
	2	-2.4	-0.0	-3.5	+3.3	-3.4	+1.8	-3.8	+1.6	+ 3.5	-2.7	+ 0.8	+3.1	-0.2	-3.3	+1.1
	3	-1.2	-3.1	+0.0	-1.8	+0.1	+1.7	-1.5	-1.0	+ 2.8	-3.8	+ 0.1	-1.2	+0.0	-3.9	-0.1
	4	+0.1	+2.0	+1.3	-0.3	-0.0	-0.9	+2.6	+1.9	- 1.3	+3.8	- 0.9	-0.1	+0.9	+3.6	+0.1
	5	-2.1	+1.0	-0.9	+2.8	-3.3	+1.9	-2.3	+1.7	+ 1.6	+0.7	- 1.6	+1.8	+0.7	+1.7	-0.0
	6	+2.7	-0.7	+1.5	-2.2	+2.2	-0.1	+2.3	-2.4	- 2.2	-1.3	+ 0.3	-1.8	+0.1	-0.4	-0.6
r.m.s. differences of model-residuals against mean of strip																
	$\sigma_x$	2.3	2.8	3.1	2.9	2.7	3.9	3.1	2.9	2.8	2.6	2.9	2.6	2.6	2.5	1.9
	$\sigma_y$	3.3	3.0	4.5	5.1	3.3	4.1	3.8	2.8	4.1	2.8	3.7	2.8	2.9	3.3	3.2
	$\sigma_z$	5.2	4.1	4.7	4.6	4.6	5.2	4.5	3.9	4.5	3.7	4.1	3.4	3.2	2.4	3.6

Table 2. OEEPE-Oberschwaben  
Systematic model deformations after block adjustment by independent models.  
Super-wide-angle strips: Control version 0; in units of  $\mu\text{m}$

strip no.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
flight direction		NS	SN	NS	SN	NS	SN	NS	SN	NS	SN	NS	SN	NS	SN	NS
flight mission		10.4.	10.4.	10.4.	10.4.	10.4.	10.4.	10.4.	10.4.	26.4.	26.4.	26.4.	26.4.	26.4.	26.4.	26.4.
	standard position															
x	1	+3.1	+1.4	+3.0	+ 3.5	+3.7	+2.9	+4.8	+ 4.3	+4.3	+1.8	+ 4.3	+2.8	+2.3	+1.2	+1.7
	2	-3.1	-1.4	-3.0	- 3.5	-3.7	-2.9	-4.8	- 4.3	-4.3	-1.8	- 4.3	-2.8	-2.3	-1.2	-1.7
	3	+6.3	-0.3	+8.5	+ 0.9	+6.9	+0.7	+5.9	+ 0.0	+8.7	-1.4	+ 8.8	-1.7	+8.2	-5.1	+5.1
	4	-4.9	+3.8	-5.3	+ 1.1	-4.4	+1.9	-4.3	+ 1.9	-4.0	+5.0	- 4.9	+4.8	-5.2	+4.8	-5.1
	5	-1.1	+7.1	-1.0	+ 5.8	-0.1	+5.6	+1.6	+ 8.0	+1.0	+6.0	- 2.8	+4.9	-3.9	+4.7	-2.4
	6	+1.1	-7.1	-0.9	-10.0	-2.5	-7.7	-3.0	-10.5	-2.6	-7.8	- 2.1	-8.0	-0.3	-8.3	+0.9
y	1	+1.0	+0.7	+3.0	+ 1.0	+2.0	+1.1	+0.7	+ 1.0	+1.9	+2.0	+ 4.5	+2.2	+1.8	+1.6	-0.0
	2	-1.0	-0.7	-3.0	- 1.0	-2.0	-1.1	-0.7	- 1.0	-1.9	-2.0	- 4.5	-2.2	-1.8	-1.6	+0.0
	3	-2.8	+1.9	-5.5	+ 1.4	-5.9	-1.1	-2.8	+ 0.5	-4.8	+2.9	- 3.4	+5.6	-2.2	+4.9	+0.7
	4	-2.0	-6.1	+1.2	- 7.6	-0.2	-4.5	-2.8	- 5.4	-0.8	-6.9	- 0.8	-8.6	-0.4	-4.8	-0.7
	5	+4.0	+0.4	+7.4	+ 4.2	+8.2	+5.0	+7.4	+ 2.7	+8.6	+4.1	+10.8	+0.4	+7.0	-0.2	+5.3
	6	-4.0	-0.4	-1.6	+ 0.6	-3.7	+0.9	+0.4	+ 2.8	-2.1	+0.6	- 3.7	+3.1	-2.6	+2.4	-2.8
z	1	+4.2	-4.3	+1.9	- 2.3	+2.2	-3.5	+3.5	- 2.9	+4.4	-3.8	+ 4.7	-3.0	+3.5	-1.9	+2.3
	2	-4.2	+4.3	-1.9	+ 2.3	-2.2	+3.5	-3.5	+ 2.9	-4.4	+3.8	- 4.7	+3.0	-3.5	+2.1	-2.3
	3	-1.5	+0.1	+0.9	- 1.2	+1.1	+1.5	-3.0	+ 0.6	-0.8	+1.6	- 1.0	+1.4	-1.3	+1.1	-2.2
	4	+1.4	+0.3	-0.1	+ 1.1	-0.5	-1.9	+2.4	- 0.1	+1.6	+0.2	- 0.3	-1.0	+0.5	-1.2	+2.2
	5	-0.8	+2.9	-0.8	+ 1.7	-0.4	+0.9	+0.8	+ 0.2	-0.2	+1.3	- 1.1	-0.7	+0.4	-0.5	+1.7
	6	+0.8	-2.8	+1.4	- 1.8	-0.1	-0.8	-0.8	+ 0.2	+0.8	-1.7	- 0.1	-0.4	+1.1	+0.6	-1.2
r.m.s. differences of model-residuals against mean of strip																
	$\sigma_x$	2.8	3.3	4.0	3.6	3.8	3.9	3.8	3.2	3.4	3.9	3.8	3.6	3.7	3.3	3.5
	$\sigma_y$	4.9	3.7	5.3	4.3	4.9	4.6	5.7	4.6	5.1	4.7	4.3	6.1	4.1	3.7	5.1
	$\sigma_z$	3.6	3.5	3.9	3.2	4.5	3.3	3.9	3.2	4.4	3.3	4.0	4.6	3.1	3.2	3.0

Table 3. OEEPE-Oberschwaben  
 Systematic model deformations after block adjustment by independent models.  
 Dependence on flight direction and control version (in units of  $\mu\text{m}$ )

block	Frankfurt (w.a.)						Vienna (w.a.)		The Hague (s.w.a.)			Delft (s.w.a.)
	0	0	1	1	5	5	0	0	0	1	5	0
flight direction	NS	SN	NS	SN	NS	SN	NS	SN	NS	NS	NS	SN
average of n strips	n =											
x	5	3	5	3	5	3	2	5	8	8	8	7
standard position												
1	+1.9	+2.6	+1.6	+1.6	+0.1	-0.1	+1.5	+1.5	+3.4	+2.7	+0.2	+2.6
2	-1.9	-2.6	-1.6	-1.6	-0.1	+0.1	-1.5	-1.5	-3.4	-2.7	-0.2	-2.6
3	-0.7	+9.8	-1.2	+8.5	-3.9	+5.6	-4.8	+6.1	+7.3	+6.3	+3.0	-0.9
4	+4.0	-6.8	+4.0	-6.0	+3.9	-5.2	+4.6	-4.3	-4.8	-4.3	-2.8	+3.3
5	+5.0	-4.3	+4.9	-5.0	+4.3	-5.1	+8.3	-3.3	-1.2	-1.7	-2.9	+6.0
6	-7.9	+1.2	-7.1	+2.3	-4.4	+4.8	-7.7	+1.7	-1.2	-0.1	+2.6	-8.5
y	1	2	3	4	5	6	1	2	3	4	5	6
1	+2.6	+2.1	+2.2	+1.8	+0.3	+0.1	-1.6	+1.4	+1.8	+1.3	+0.0	+1.4
2	-2.6	-2.1	-2.1	-1.8	-0.3	-0.1	+1.6	-1.5	-1.8	-1.3	-0.0	-1.4
3	-0.1	+1.0	+0.4	+1.0	-0.3	+0.2	-1.2	+1.4	-3.4	-3.1	-2.6	+2.2
4	-4.2	-2.9	-2.8	-2.6	+0.3	-0.3	-2.7	-3.0	-0.8	+0.1	+2.7	-6.3
5	+3.4	+5.1	+2.9	+2.9	+0.4	-0.1	+0.2	+1.6	+7.4	+6.1	+3.2	+2.4
6	-1.6	+1.1	-1.1	-0.2	-0.3	+0.2	+0.2	+1.8	-2.5	-2.5	-2.9	+1.4
z	1	2	3	4	5	6	1	2	3	4	5	6
1	+2.7	-1.8	+2.7	-1.7	+2.8	-1.8	+3.0	-1.9	+3.4	+3.5	+3.4	-3.1
2	-2.7	+1.8	-2.7	+1.7	-2.7	+1.8	-3.0	+1.9	-3.4	-3.5	-3.4	+3.1
3	-0.5	+1.0	-0.4	+0.7	-0.3	+0.6	-3.8	-1.0	-1.0	-0.9	-0.9	+0.7
4	+1.0	-0.7	+0.9	-0.7	+0.8	-0.6	+3.7	+0.5	+0.8	+0.8	+0.8	-0.3
5	-1.6	+0.1	-1.7	+0.4	-1.8	+0.5	+1.2	+1.8	-0.1	-0.2	-0.3	+0.8
6	+1.8	-0.8	+1.7	-0.8	+1.6	-0.9	-0.9	-1.4	+0.2	+0.2	+0.3	-1.0

Table 4. OEEPE-Oberschwaben  
 Systematic errors of perspective centres after block adjustment by independent models.  
 Wide-angle-strips: Control version 0, in units of  $\mu\text{m}$

strip no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
flight direction	NS	SN	NS	SN	NS	SN	NS	SN	SN	NS	SN	SN	NS	NS	SN
flight mission	8.4.	8.4.	8.4.	8.4.	8.4.	8.4.	9.4.	12.5.	9.4.	12.5.	12.5.	12.5.	12.5.	12.5.	12.5.
x	+7.7	+2.0	+2.1	+1.1	+7.6	+6.3	+6.2	+3.9	+4.3	+0.7	+4.6	+2.8	+5.1	+3.5	+5.0
y	-6.4	-2.0	-2.7	-1.9	-5.4	-2.2	-5.8	-1.6	-3.5	-0.3	-1.5	-0.3	-3.8	+0.9	-3.0
z	+1.5	-4.2	+2.7	-3.0	+0.8	-1.3	-0.1	-2.5	+0.1	+0.7	-2.1	-4.1	-0.3	+0.9	-2.4

Super-wide-angle strips: Control version 0; in units of  $\mu\text{m}$

strip no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
flight direction	NS	SN	NS	SN	NS	SN	NS	SN	NS	SN	NS	SN	NS	SN	NS
flight mission	10.4.	10.4.	10.4.	10.4.	10.4.	10.4.	10.4.	10.4.	26.4.	26.4.	26.4.	26.4.	26.4.	26.4.	26.4.
x	+0.4	+0.7	-1.5	+1.3	+1.8	+3.5	-0.0	-1.7	-0.7	-0.2	-0.3	+1.8	+2.6	+3.3	+3.0
y	+0.7	-1.1	-0.8	-1.6	-2.2	-2.0	+0.7	-3.9	-1.2	-3.7	+0.1	-1.2	-0.7	-2.4	+1.7
z	+3.0	-3.2	+4.5	-3.0	+4.7	-0.6	+2.8	-2.4	+3.6	-3.1	+5.8	-4.5	+4.0	-3.3	+1.8

Dependence on flight direction and control version (in units of  $\mu\text{m}$ )

block	Frankfurt (w.a.)						Vienna (w.a.)		The Hague (s.w.a.)			Delft (s.w.a.)		
	0	0	1	1	5	5	0	0	0	1	5	0		
flight direction	NS	SN	NS	SN	NS	SN	NS	SN	NS	NS	NS	SN		
average of n strips	n =													
x	5		3		5		3		2		5			7
1	+5.8	+4.6	+5.9	+5.0	+6.5	+5.9	+2.1	+3.2	+0.7	+1.4	+3.8	+1.3		
2	-4.8	-2.7	-4.2	-2.8	-2.4	-1.3	+0.3	-1.6	-0.2	+0.3	+1.6	-2.3		
3	+0.9	-1.4	+0.9	-1.6	+1.1	-1.8	+0.8	-3.0	+3.8	+3.6	+3.6	-2.9		

Figure 2: OEEPE Oberschwaben - Systematic model deformations after block adjustment by independent models  
Dependence on flight direction  
Planimetry: CP-version 0; 1 mm  $\hat{=}$  1  $\mu$ m

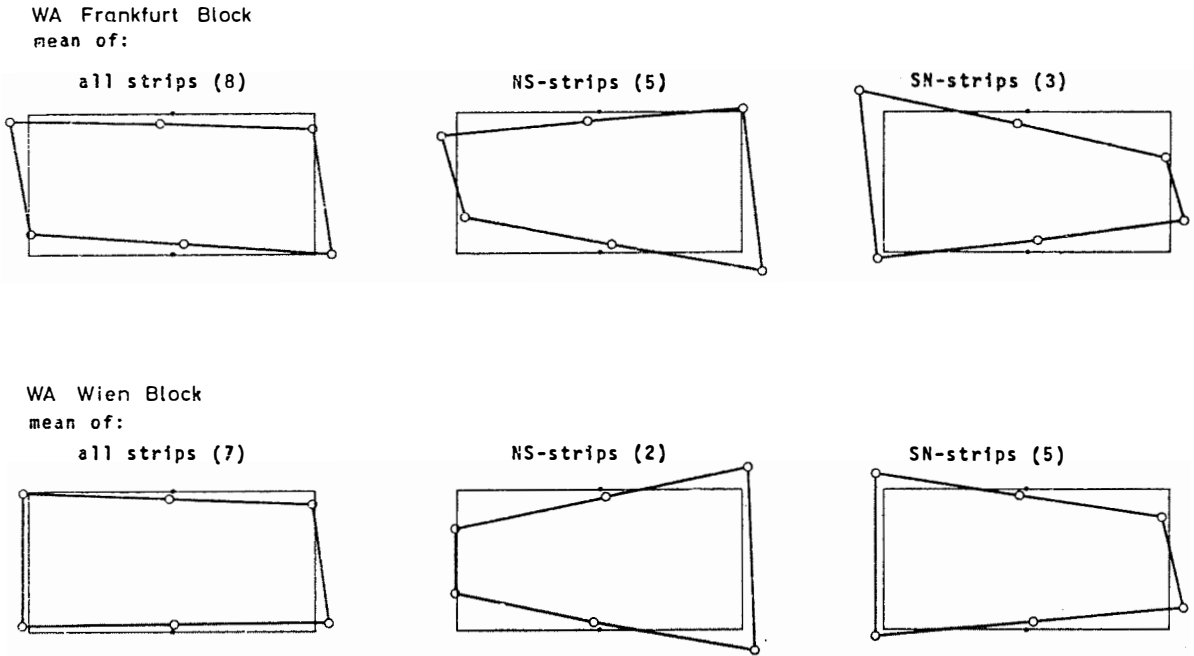


Figure 3: OEEPE Oberschwaben - Systematic model deformations after block adjustment by independent models  
Comparison of wide-angle and super-wide-angle  
Planimetry: CP-version 0; 1 mm  $\hat{=}$  1  $\mu$ m

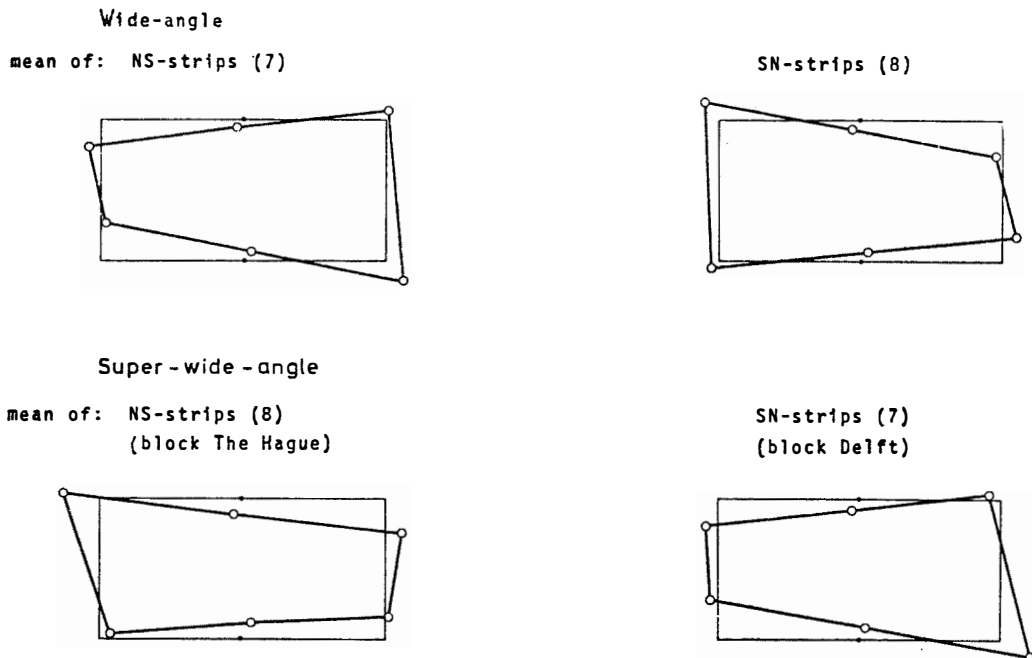


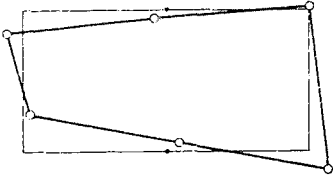
Figure 4 : OEEPE Oberschwaben - Systematic model deformations after block adjustment by independent models

Dependence on control versions:

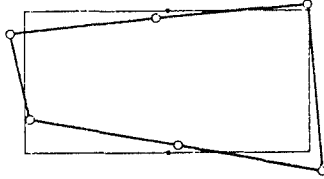
Planimetry 1 mm  $\hat{=}$  1  $\mu$ m

WA Frankfurt Block (mean of 5 NS-strips)

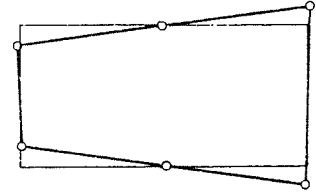
CP-version 0:  
all control points



CP-version 1:  
perimeter control

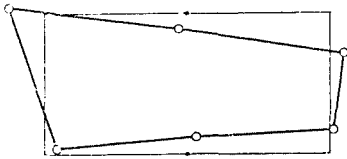


CP-version 5:  
4 corner points

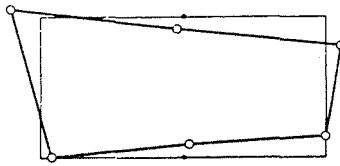


SWA Den Haag Block (mean of 8 NS-strips)

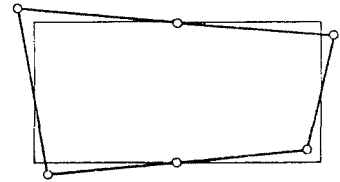
CP-version 0:  
all control points



CP-version 1:  
perimeter control



CP-version 5:  
4 corner points



## 5. DISCUSSION OF THE RESULTS

### 5.1 Magnitude and type of systematic model deformations

The results show clearly the presence of systematic errors, of considerable magnitude, at model points and perspective centres.

The maximum planimetric errors occur at model corners, the central points (1,2) being less affected. When judging the strips separately the maximum systematic coordinate errors are: Wide-angle 11.4  $\mu$ m in x, 7.2  $\mu$ m in y; super-wide-angle 10.5  $\mu$ m in x, 10.8  $\mu$ m in y. The respective maximum values at the central points 1 and 2 are only: w.a. 3.3  $\mu$ m and 3.4  $\mu$ m, s.w.a. 4.8  $\mu$ m and 4.5  $\mu$ m.

The average systematic errors of the separate strips are only slightly larger than the average systematic errors of groups of strips of common flight direction. Thus the systematic errors are, to a most remarkable degree, constant.

The root mean square values  $m_s$  of the planimetric systematic errors of the standard points are presented in table 5. They represent noticeable percentages of the values of  $\sigma_o$  of the block adjustments.

Table 5: Comparison of standard errors of unit weight ( $\sigma_o$ ) and r.m.s. values  $m_s$  of systematic errors; planimetry, control version 0.

block	Frankfurt w.a.	Vienna w.a.	The Hague s.w.a.	Delft s.w.a.
$\sigma_o$	20.3 cm 7.2 $\mu$ m	19.2 cm 6.9 $\mu$ m	25.1 cm 9.0 $\mu$ m	24.6 cm 8.8 $\mu$ m
$m_s$	11.1 cm 4.0 $\mu$ m	9.7 cm 3.5 $\mu$ m	10.9 cm 3.9 $\mu$ m	11.1 cm 4.0 $\mu$ m

The systematic vertical model deformations are, altogether, considerably smaller than the planimetric deformations. The maximum errors, occurring often at the central points 1 and 2, amount to 3.9  $\mu$ m and 4.7  $\mu$ m for wide-angle and super-wide-angle, respectively. The vertical errors at model corners are unexpectedly small.

Figure 5:

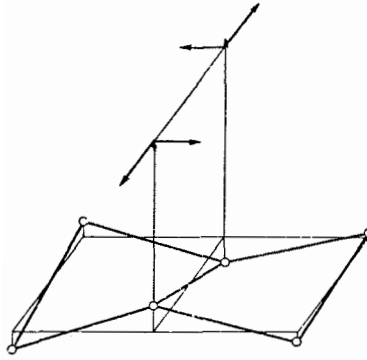
OEPE Oberschwaben - Systematic vertical model deformations and systematic errors of perspective centres

CP-version 0; 1 mm  $\hat{=}$  1  $\mu$ m

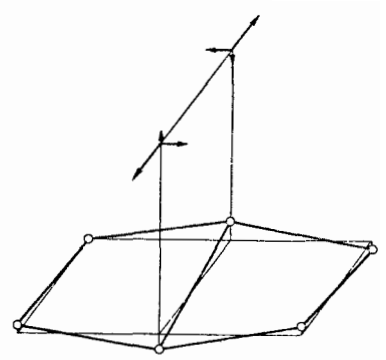
The systematic errors at the perspective centres are collected in table 4 and presented graphically in fig. 5. The values refer always to p.c. No. 1, the values of p.c. No. 2 being of exactly equal magnitude and opposite sign. In general, the planimetric errors at the perspective centres are smaller than of the model points, whilst the magnitude of vertical errors of the perspective centres compares with the vertical model deformations. The maximum systematic errors per strip, of the perspective centres amount in x, y, z, respectively, to 7.7  $\mu$ m, 6.4  $\mu$ m, 4.2  $\mu$ m (w.a.) and 3.5  $\mu$ m, 3.9  $\mu$ m, 5.8  $\mu$ m (s.w.a.), for control version 0.

Apart from the magnitude also the types of model deformations are of special interest. They are common for both w.a. and s.w.a. models as displayed in fig. 2 - 5.

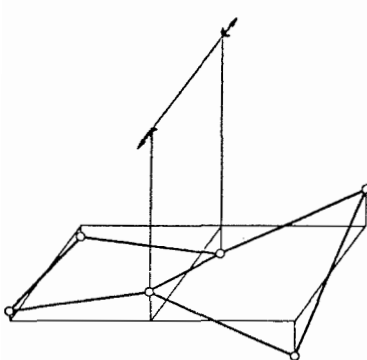
WA Frankfurt Block  
mean of: NS-strips (5)



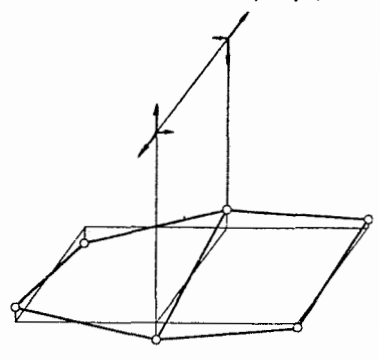
mean of: SN-strips (3)



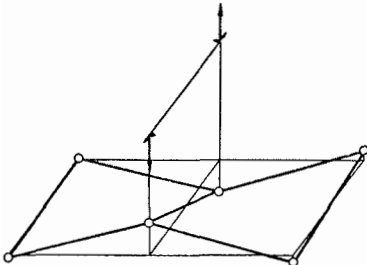
WA Wien Block  
mean of: NS-strips (2)



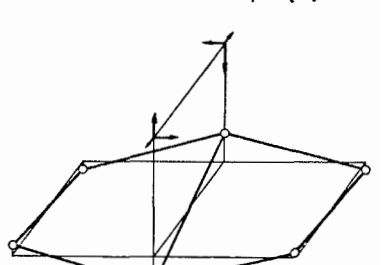
mean of: SN-strips (5)



SWA Den Haag Block  
mean of: NS-strips (8)



SWA Delft Block  
mean of: SN-strips (7)



The planimetric model deformations are trapeziform. Characteristic features are: Considerable scale difference between the short sides of the models; points 3-1-5 and 4-2-6, respectively, form straight lines; errors of points 1 and 2 have equal magnitude, reversed sign.

The typical features of vertical model deformations are: Little or no warping of the models; maximum height errors, of opposite sign, at central points 1 and 2 (arching and sagging).

It is only at the perspective centres that w.a. and s.w.a. behave somewhat different. In the first case (w.a.) symmetrical enlargement of the base and azimuthal rotation is predominant, in the second case (s.w.a.) the base is mainly tilted.

## 5.2 Dependencies of systematic model errors

With regard to possible corrections of systematic model deformations it is most important to investigate how constant they are and on what parameters they depend.

Flight direction, camera: The tables and graphs show clearly that the model deformations, as referred to the state coordinate system, depend in first instance on the flight direction. This is true for w.a. and s.w.a., see figures 2 and 3. After rotation by 180° the planimetric model deformations of the NS-strips match very well with those of the SN-strips. The dependence on flight direction is equally confirmed by table 3 and fig. 5 for vertical model deformations. It also holds for the perspective centres.

Fig. 2 shows that by taking the mean deformation of all models, without taking the flight direction into account, an entirely misleading type of model deformation would result. Only an affine deformation would be the result, the trapeziform deformation, related to the flight direction, being eliminated.

The important conclusion is, that the main systematic model deformations originate from the camera (all exposures were taken from the same airplane - Aero Commander 560 F; all strips were triangulated by stereocomparator in direction north-south; it is not known, how many film magazines were used during the flight missions). This is confirmed by the different planimetric model deformations of w.a. and s.w.a., although being of the same type. (The vertical deformations are, however, very similar in both cases, except for the perspective centres). It is also confirmed by the remarkable constancy of the deformations within each group, independent of the date of the flight missions which has no traceable effect.

The overall r.m.s. deviations of the individual residual errors of the models against the average systematic errors are for wide-angle  $2.9 \mu\text{m}$  in x and  $3.8 \mu\text{m}$  in y, for super-wide-angle  $3.8 \mu\text{m}$  in x and  $4.9 \mu\text{m}$  in y.

It is evident, therefore, that any method of correction of systematic errors by additional parameters, see [3], [5], has to consider the flight direction of strips. A different set of parameters per strip might be advisable.

Control version: According to table 3 and fig. 4, there is some influence of the Control version of the block adjustment on the apparent systematic model deformations. The effect is noticeable in planimetry only. The type of the effect is most interesting. With poor planimetric ground control the trapeziform type of planimetric model deformation is correctly indicated, with, however, additional affine deformation (scale and shear) superimposed. The additional affine deformation is rather small, however. The systematic planimetric model deformations differ with perimeter control (version 1) by less than  $2 \mu\text{m}$ , with 4 control points only (version 5) by less than  $5 \mu\text{m}$  against the "true" model deformations as determined with all available control points (version 0). This effect is independent of camera or other flight parameters.

Restitution centre: There is a small systematic effect related to the 4 different centres of restitution, traceable again in planimetry only. The average planimetric model deformations, after duly considering the flight directions, differ for w.a. between block Frankfurt (PSK) and block Vienna (StK1) by  $2.0 \mu\text{m}$ , for s.w.a. between block The Hague (PSK) and block Delft (StK1) by  $2.4 \mu\text{m}$ . The respective maximum average differences are  $4.2 \mu\text{m}$  and  $2.2 \mu\text{m}$ . Such systematic differences can be explained by the different sets of diapositives and systematic errors of the stereocomparators used.

## 6. SUMMARY AND CLOSING REMARKS

This paper presents the results of an empirical investigation into the systematic model deformations of the OEEPE test block Oberschwaben, as they are apparent from residual errors at tie-points and perspective centres after block adjustment by independent models. The results are most interesting with regard to magnitude and type of the model deformations and their relation to flight parameters:

- The magnitude of the systematic coordinate errors is considerable. In planimetry the deformations are trapeziform. The mean values range between  $3.5 \mu\text{m}$  and  $4.0 \mu\text{m}$ , which amounts to a considerable percentage of the  $\sigma_0$  values of the block adjustments (without correction of systematic errors), between  $6.9 \mu\text{m}$  and  $9.0 \mu\text{m}$ . The vertical model deformations are rather small but also typical, with maximum values of opposite sign at the central points 1 and 2 of the models.
- The model deformations, as referred to the state coordinate system, are predominantly related to the flight directions. Thus, the main cause of the deformation is clearly the camera. Small systematic effects in planimetry have been found to be related to the density of ground control used for the adjustment and to the centre (instrument) of restitution. Other flight parameters have had no traceable influence on the model deformations.



The systematic model deformations have turned out, for a given camera, to be constant to a remarkable degree. Therefore it can be expected that suitable procedures for correcting the systematic errors during block adjustment will be highly effective and will extraordinarily increase the resultant accuracy of the adjusted block, possibly by a factor 2 or 3. It will be essential, however, to consider the flight direction when introducing additional parameters in a refined mathematical model.

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