# RESULTS FROM THE SOUTH-WESTERN ONTARIO APR-TEST BLOCK

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## SUMMARY

A new adjustment program has been developed at the Institute of Photogrammetry in Stuttgart: PAT-M 43-APR; a program for the combined simultaneous block adjustment of photogrammetric models with APR and'or statoscope data. As a first application the APR test block South-western Ontario was adjusted, in order to prove theory and to check handling of the program in practice.

The results of several test groups with different control distribution, different bridging distances for APR cross flights and a varied density of APR points on the profiles show the advantage of a rigorous combined adjustment.

## THE APR TEST BLOCK

The APR test block South-western Ontario was provided by the Canadian Ministry of Energy, Mines and Resources for the purpose of evaluating the results of the combined block adjustment of independent photogrammetric models with APR data and of testing the APR version of the computer program PAT-M43 |1|, |2|. The test area is located in Canada between Lake Huron, Lake St Clair, Lake Erie and Lake Ontario as shown on the attached map. It is a rectangle flown in 5 strips between Lake St Clair and Lake Ontario, each strip about 76 models or 250 km long.

Block description :

| | |
|---|---|
| Date of photography | July 1972 |
| Camera | class A, wide-angle, f = 6" |
| Flying height of photography | 5250 m |
| Scale of photography | 1/33.000 |
| Strips with simultaneous APR | 5 |
| Number of models per line | appr. 76 |
| Total number of models | 380 |
| Area | 6750 $km^2$ |
| East-west extension | 250 km |
| North-south extension | 25 km |
| Number of APR cross flights | 10 |
| Flying height of cross flights | appr. 2000 m |
| APR instrumentation | Radar APR |

The area is hilly with heights above sea-level between 75 m and 370 m. The density of APR points is 1/2 base length or 5 points across a photograph in both directions, the number is 988. The total number of vertical control points is 440; they are very evenly distributed across the block.

## MATHEMATICAL APPROACH

The APR block program is based on the PAT-M43 program of block adjustment with independent models |1|. The program iterates horizontal and vertical adjustments, applying 4-parameter and 3-parameter transformations in successive steps.

The 4-parameter horizontal adjustment is the wellknown "Anblock"-method. Because of the small correlation between horizontal and vertical accuracy only the vertical adjustment is discussed in this paper. As described in |1| the vertical block adjustment makes use of the following linearized observational equations for the height $z_{ij}$ of a point i within a model j when $Z_i$ means the unknown height of the terrain point i :

for model points:
$$\left[v_z\right]_{ij} = \left[-y \quad x\right]_{ij} \begin{bmatrix} da \\ db \end{bmatrix}_j - \left[dz\right]_j + \left[Z\right]_i - \left[z\right]_{ij} \qquad (1a)$$

for perspective centers:
$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}_{ij} = \begin{bmatrix} 0 & -z \\ z & 0 \\ -y & x \end{bmatrix}_{ij} \begin{bmatrix} da \\ db \end{bmatrix}_j - \begin{bmatrix} 0 \\ 0 \\ dz \end{bmatrix}_j + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i - \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{ij} \qquad (1b)$$

for vertical control:
$$\left[v_z\right]_i = \left[Z\right]_i - \left[z\right]_i^{terr} \qquad (1c)$$

The increments da, db are two of the three independent rotation parameters of a modified Rodrigues-Caylay matrix |1|, $dz_j$ is the z-shift of the model j.

Points of APR profiles are used as observational data for the combined adjustment. They are treated, in a way, as additional vertical control with appropriate weighting. However, the isobaric surface to which the APR recordings refer is not known. Therefore additional unknown parameters are needed. We allow a constant shift and a tilt correction of the isobaric surface along each profile.

For the APR height $z_{ik}$ of a point i in profile k we obtain the following observational equation:

for APR heights of model points:
$$\left[v_z\right]_{ik} = -\left[a_k + b_k \cdot t_{ik}\right] + \left[Z\right]_i - \left[z\right]_{ik} \qquad (2a)$$

for vertical control:
$$\left[v_z\right]_i = \left[Z\right]_i - \left[z\right]_i^{terr} \qquad (2b)$$

The coefficient $t_{ik}$ represents the distance of APR measurement i on profile k from an arbitrary starting point, in practice the elapsed time is used.

The essential difference between (2b) and (1c) is the fact that the point i in (2b) is not necessarily measured in a photogrammetric model, for instance it can be the height of a lake level on which the APR-line closes. Therefore a connection of a photogrammetric block to control outside of the block is possible using APR profiles.

HENRY CORRECTION

The ideal situation for APR measurements would be to fly on a geopotential surface. Unfortunately the noise level is prohibitive for the delicate gravimeter instrumentation needed to measure deviations from this surface.

Another possibility is to fly on an isobaric surface and calculate the slope of the isobaric surface with respect to the geopotential surface from wind information. On the assumption of constant atmospheric pressure and a balance between the pressure force and the Coriolis force we obtain the following correction formula for APR measurements, recorded on an isobaric surface; the so called Henry-correction formula |3|:

$$\Delta z = \frac{2\,\omega}{g} \int_{s_1}^{s_2} A \cdot \sin L \cdot \sin\delta \cdot ds \qquad (3)$$

$\omega$ = angular velocity of the earth, g = local value of gravity ($g_{45}$ = 980.665 cm/sec$^2$), A = true air speed, L = latitude, $\delta$ = drift angle, ds = line element along the APR-line. In practice g, A, $\omega$ and L are assumed constant on the track, the drift angle $\delta$ is measured about every five minutes with an accuracy of 1/2 degree. Therefore the Henry correction is used as a linear correction between two changes of the drift angle. Because we allow a linear correction of the isobaric surface in our adjustment program, we have the possibility to use non-corrected APR-measurements while cutting the profiles at the points of change of the drift angle.

THE STOCHASTICAL MODEL

In our combined adjustment program all observations can be weighted. Because it is impractical to weight all observations individually, the program allows only for different sets of weight matrices to different groups of observations. In order to obtain realistic weights the APR test block South-western Ontario was first adjusted with all given 440 vertical control points and all 988 APR measurements. The following table shows the chosen weights for the vertical adjustment and their agreement with the r.m.s. values of the residuals.

| Group of observations | | weights (related) to the ground) | r.m.s. values of residuals |
|---|---|---|---|
| model points | z | 1 | 0.347 |
| perspective centers | x | 0.1 | 1.209 |
| | y | 0.1 | 1.119 |
| | z | 0.0 | 0.469 |
| control points in the model | z | 1 | 0.303 |
| APR points in the model | z | 1 | 0.330 |
| APR profile points (≈ 5200 m) | z | 0.25 | 0.913 |
| APR profile points (≈ 2000 m) | z | 0.5 | 0.572 |
| vertical control | z | 0.5 | 0.521 |

Sigma-nought of vertical adjustment = 0.43

Because of unexplained large z-residuals of the perspective centers at the beginning and the end of each strip the weight 0 was given to the z-coordinate of the perspective centers, which is possible without detrimental effects to the adjustment.

Looking at the root mean square values of the residuals and sigma-nought we already can point out the following results:

1. The accuracy of observations before adjustment is about 0.43 m.
2. The accuracy of APR heights depends on the flying height. APR heights from profiles flown at lower altitude have a better accuracy.
3. The accuracy of APR cross profiles and the accuracy of vertical control happens to be about the same.

RESULTS OF THE COMBINED ADJUSTMENT

Up to now 10 different series of adjustments have been performed with the test-block South-western Ontario, always using minimum control. Each series consists of 7 adjustments, varying the distance between cross profiles from 76 models to 5 models.

The last 4 adjustments of each group were done only with the western half of the block because in the other part not enough cross profiles were flown. Absolute accuracy is obtained from 170 check points.

The first 4 testgroups were adjusted with 2 chains of control at the front sides of the block and without any closing of the profiles on control.

Accuracy results of the testblock Southwestern Ontario
absolute accuracy in m obtained from 170 check points.

combined adjustment:

| △ b [1] | full block 76 | 38 | 19 | half block 38 | 19 | 10 | 5 | control 2 chains | simult. profiles | cross profiles | sub-divided profiles | space of APR points |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| group 1 | 2.45 | 1.66 | 1.46 | 1.57 | 1.32 | 1.20 | 1.21 | yes | no | no | no | 0.5b [2] |
| group 2 | 4.39 | 3.43 | 2.42 | 2.52 | 1.89 | 1.80 | 1.76 | yes | no | no | yes | 0.5b |
| group 3 | 2.77 | 1.91 | 1.72 | 1.85 | 1.56 | 1.36 | 1.37 | yes | no | no | no | 1 b |
| group 4 | 3.61 | 3.02 | 1.79 | 2.37 | 1.68 | 1.53 | 1.49 | yes | no | no | yes | 1 b |
| group 5 | 2.05 | 2.67 | 2.11 | 2.67 | 2.11 | 1.90 | 1.96 | no | yes | yes | no | 0.5b |
| group 6 | 2.14 | 2.54 | 2.16 | 2.70 | 2.62 | 2.05 | 2.14 | no | yes | yes | yes | 0.5b |
| group 7 | 1.96 | 2.38 | 1.98 | 2.33 | 1.93 | 1.63 | 1.77 | no | yes | yes | no | 1 b |
| group 8 | 1.86 | 1.69 | 1.63 | 1.67 | 1.61 | 1.54 | 1.55 | no | yes | no | no | 1 b |
| group 9 | 1.38 | 2.35 | 1.56 | 2.31 | 1.51 | 1.22 | 1.14 | no | yes | yes | no | 0.5b |
| group10 | 1.45 | 2.13 | 1.53 | 2.06 | 1.45 | 1.22 | 1.17 | no | yes | yes | no | 1 b |

[1] △ b = bridging distance of cross profiles in base length    [2] b = base length

two step method:
absolute accuracy with all APR points
and all closing points 3.24 m.

Test 5 - 10 used no height control points within the block. Instead 2 cross profiles were used and the longitudinal APR profiles closed on the lakes. It was anticipated that the overall absolute height accuracy of the combined block adjustment would reach about 2 m [2]. The results now show that the expectation was not too optimistic.

Before going into a more detailed discussion it is perhaps expedient to point to some accuracy results first:

- Bridging the full length of the block (~ 250 km or 76 models) the height accuracy can be about 2 m.
- The accuracy is increased when cross profiles are used. When bridging 19 models or about 60 km 1.60 m is reached.
- The best results, obtained with cross profiles all 5 models or 15 km, are around 1.20 m. This is obviously the inherent limit of the test material, given by flying height, photo-scale, APR equipment, terrain and control.

Up to now the adjustment of APR profiles and their use for aerial triangulation was in most cases much simplified. The profiles starting and closing over known areas were adjusted by applying linear corrections. From the adjusted profiles, vertical control points were drawn to be used for the subsequent block adjustment.

With this two step method the absolute accuracy reached with our test material was 3.24 m. Due to the more general mathematical and stochastical approach of the combined adjustmeht the height accuracy has been considerable increased.

Thus, the combined adjustment has proven its effectiveness. There are a number of details to be commented upon. There is one point in particular to mention: Absolute accuracy is steadily increased with shorter distances between cross profiles up to an inherent limit (testgroup 1 - 4). This effect is disturbed when the cross profiles are closed on known water surfaces. The results become irregular and absolute accuracy is not as much increased as with free cross profiles (testgroup 5 - 7). Using less control absolute accuracy even is sometimes better (compare testgroup 7 with testgroup 8).

This results show that control is only necessary to fix in a way the isobaric surface. More control points do not result in a better accuracy.

Using control only on known water surfaces the ratio of the maximum error and the average absolute accuracy is less than 2. While using this closing points as check points the residuals on each lake are close to a certain positive value. There is obviously a systematic effect of the APR recordings on lakes. After correction of the APR measurements with this values we obtained a much better result. The absolute accuracy has been increased to 1.14 m (testgroup 9 - 10).

In the test material the density of APR points is 1/2 base length. Using APR points all 1 base length only (without the APR points in the middle of the models) absolute accuracy become sometimes better (compare testgroup 2 against 4) and sometimes worse (compare testgroup 1 against 3). Nevertheless the r.m.s. values of APR measurements of longitudinal profiles are decreasing in both cases from 91 cm to 85 cm. It seems to be a systematic effect of the APR points on the sides of the models against the points in the centers.

It is an advantage of the combined adjustment that profiles which can not be closed on known water surfaces or other vertical control can nevertheless be used very efficiently. The system automatically provides the interconnection of different profiles via the photogrammetric models. Particularly free cross profiles are controling tilts and twists of long strips. By using free cross profiles the absolute accuracy of the testblock South-western Ontario has been increased by more than a factor 2 (testgroup 1 - 4).

Between two changes of the drift angle the Henry correction is used in practice as a linear correction. Because we allow a linear transformation of each profile we can do the combined adjustment using non-corrected profiles. The profiles have to be subdivided at the points of change of the drift angles. Each part then is treated as a separate computational profile. It has to be tested in further investigations whether it is possible to use non-subdivided profiles in a first adjustment and then cut the profiles on the strength of the obtained residuals for a second adjustment.

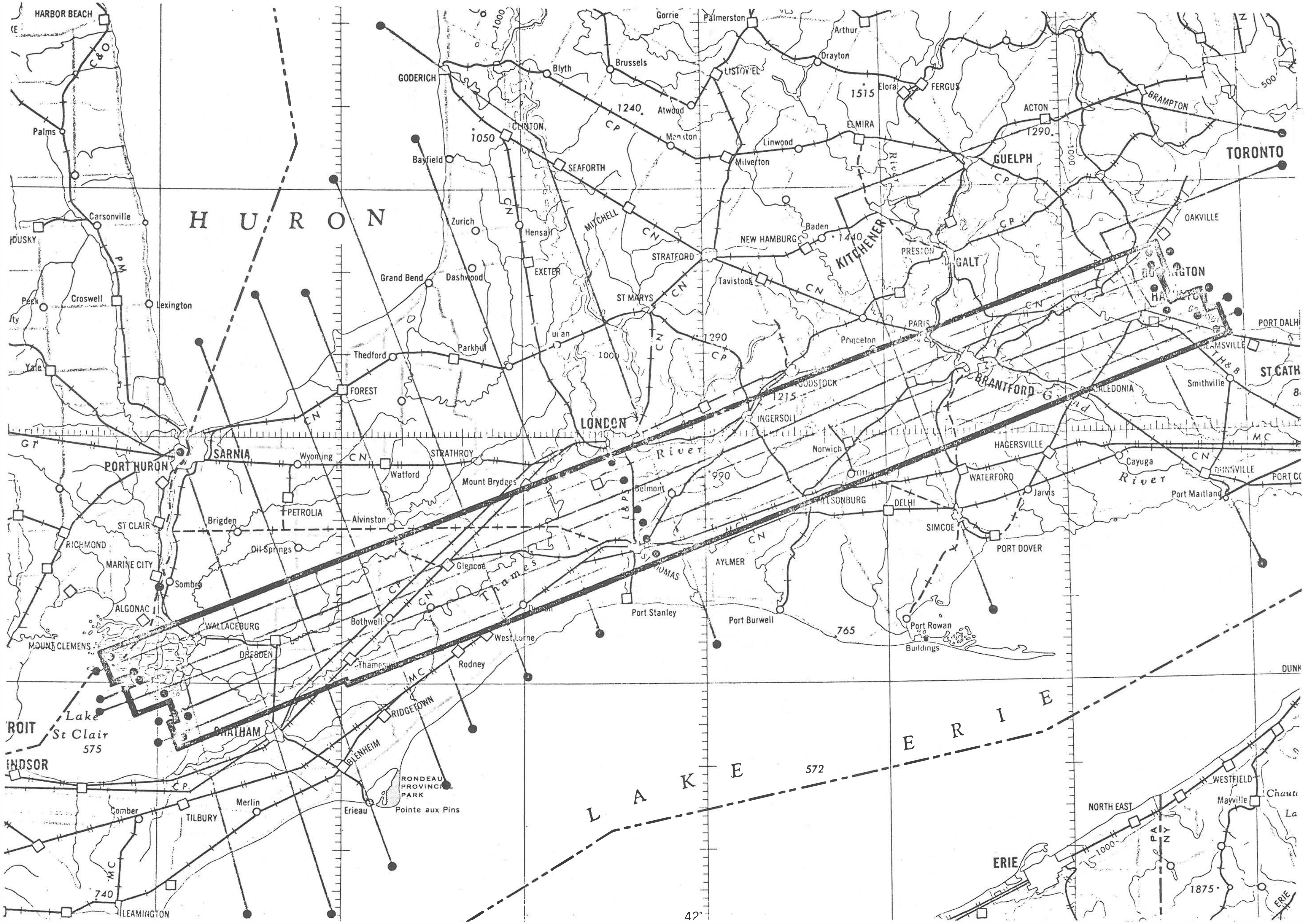
Also the problems which are due to local irregularities or disturbances of the isobaric surface can be solved in that way. However, in all these cases we have to take care of the geometric stability. Additional control or cross profiles could be necessary.

If we compare the results of testgroup 1, 3 and 5 against 2, 4 and 6 respectively we see that the non-subdivided profiles are always better. The largest differences were obtained without cross profiles, whilst the results get closer with increasing number of cross profiles and control points.

Errors of terrain coordinates or APR measurements of control points are hard to detect with the two step method. Because of the interconnection of different APR profiles via the photogrammetric models such erroneous measurements are recognized by large residuals in the combined adjustment. Within the material of the testblock South-western Ontario it was possible to detect 4 grossly erroneous APR measurements of control points. With weight zero they obtained residuals of more than 7 meters.

## CONCLUSION

The results of the combined adjustments show that an absolute accuracy of 1.30 m (~ 0.25 ‰ of flying height) can be obtained even with bridging distances between control points from 125 km to 250 km at photo scale 1 : 33 000. In future the combined adjustment will give results sufficient for mapping with contour intervals of 5 m and less and very long bridging distances of control points, even with control points up to 50 km or 100 km outside the block. The use of APR measurements with simultaneous block adjustment represents a similar break through for height accuracy as perimeter control did for planimetry.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