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Results of Recent Tests in Aerial Triangulation*

For future improvements in accuracy, every effort must be made to correct for systematic image errors.

ABSTRACT: *The European Organisation for Experimental Photogrammetric Research (OEEPE) has set up vast experimental investigation into the accuracy of different methods of strip and block triangulation. Results of the first stage of investigation from the test-block "Oberschwaben" are reported, referring to variation of control, variation of block-size, 20 percent and 60 percent side overlap, wide-angle and super-wide-angle photographs, and different adjustment methods (bundle method, independent models, polynomials). The experimental results confirm certain theoretical accuracy expectations. However, many results contradict theory consistently. The discrepancies between experiment and simplified theory are evidently due to systematic image errors; conclusions are drawn about the treatment by refined mathematical methods.*

INTRODUCTION, THE OEEPE "OBERSCHWABEN" TEST

RECENTLY, numerical methods have greatly promoted the application and the success of aerial triangulation. Availability of sophisticated computer programs is going to make aerial triangulation still more a most economical and widely used tool of practical photogrammetry.

Various theoretical accuracy studies have revealed the great accuracy capability of blocks in particular. A number of relationships between accuracy, distribution of control points, and block-size have been established, and are increasingly relied upon for planning of aerial triangulation projects. From the excellent practical results, the practitioner has been gaining confidence in the accuracy capability and the performance of aerial triangulation methods.

However, from a scientific point of view, the state of knowledge is by far not as satisfactory as the wide and successful practical application of aerial triangulation would suggest. On one hand we have theoretical accuracy studies which are based on simplified assumptions. On the other hand we have practical results, admittedly excellent, and we face the question whether or to what degree the actual results agree with theoretical expectation. It is the general problem of the validity of simplified theoretical models for predicting the accuracy results of actual aerial triangulation. In principle, such problems can only be treated by performing experiments and assessing agreement or disagreement of experimental results with theoretical expectation by the statistical methods of hypothesis-testing.

In principle, such testing can be done on various levels, for instance:

With the single photograph, being the basic element of aerial triangulation

And/or by integral tests on adjusted strips and blocks.

COMMISSION AB (Aerial Triangulation) of the European Organisation for Experimental Photogrammetric Research (*OEEPE: Organisation Européenne d'Etudes Photo-*

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* Presented at the Annual Convention of the American Society of Photogrammetry in St. Louis, Missouri, March 1974.

grammetriques Expérimentales), having considered the scientific problems, decided in 1968 to establish a test field for experiments concerning aerial triangulation.

The "Oberschwaben" test field extends 40.0 km (east-west) × 62.5 km (north-south). It is located in southern Germany between the River Danube and Lake Constance. The region is geographically known as Upper Suebia.

From the national system of trigonometric points 548 points were selected and targeted by 80 cm × 80 cm size signals, to be used as control points and as check points. In addition, 438 standard tie points (6 per model) were signalized in the terrain, each with double signals.

Aerial photographs were taken as pin-point exposures, in the spring of 1969 by Firma Häussermann, with the cameras Zeiss RMK A 15/23 (wide-angle) and Zeiss RMK A 8.5/23. The photo scale was 1:28,000, flying height h = 4,284 m and h = 2,380 m, respectively. The photo-coverage (for what is called test Program I) amounted, with either camera, to 15 strips of 25 models each, with 60 percent side overlap. Of all exposures statoscope recordings were taken. In addition, with the same cameras photographs were taken of the Rheidt camera calibration testfield of the University of Bonn. The photographic material was divided in four parts, each forming a separate block of 20 percent side overlap, having 8 strips of 25 models (200 models), and 7 strips of 25 models (175 models) respectively. The four blocks were given to four different centers for measurement with stereocomparators:

200 models, wide-angle, Frankfurt (Institut für angewandte Geodäsie), Zeiss PSK.
175 models, wide-angle, Vienna (Technical University), Wild StK1.
200 models, super-wide-angle, The Hague (Kadaster office), Zeiss PSK.
175 models, super-wide-angle, Delft (ITC), Wild StK1.

Radial-symmetrical corrections were applied to the plate coordinates for lens distortion, refraction, and earth curvature. Such reduced plate coordinates were to be used as input for analytical strip and block adjustments (bundle method). In addition, the plate coordinates were processed to model coordinates, by analytical relative orientation, to be used for strip and block adjustment by the method of independent models.

Finally, the analytically formed models were joined by computation to form strips, to be used for strip and block adjustment by polynomials.

THE TEST PROGRAM

The wealth of data give occasion to a multitude of adjustments and tests. As far as Test Program I is concerned (Test Program II pursues different aims and is not referred to in this report), the investigations had three primary objectives:

Experimental results about the accuracy of strip and block adjustment as a function of control distribution (bridging distance) and blocksize, for wide-angle and for super-wide-angle photography, and for different side overlap (20 and 60 percent).
Experimental results concerning the comparison of different adjustment methods (bundle method, independent model method, polynomial method), for otherwise equal circumstances.
Comparison of results with theoretical expectation, as far as known; statistical tests.

The adjustments were to be done at three different levels of refinement:

No special or refined corrections for plate coordinates, except for radial-symmetrical corrections, according to conventional practice.
Application of all corrections known *a priori*, as sophisticated as possible, including geodetic map projection and, possibly, corrections derived from camera-calibration field tests.
Application of refined mathematical models for the adjustments (additional parameters) and application of collocation methods.

In addition, the test material can be used for various other investigations, such as the accuracy of aerial triangulation by analog machines (for instance with independent models) versus analytical methods, the effect of point marking and point-transfer, the effect of strong ties, the effects of gross data errors, the accuracy of statoscope measurements, the use of statoscope data in the adjustments, etc. Of course, the experimental investigations are expected to raise theoretical questions which would have to be treated in turn. COMMISSION AB of OEEPE has been working on the main test program for a considerable period of time. Although the studies are continuing, a first batch of results was presented at the OEEPE symposium in Brussels from June 12th-14th, 1973. The proceedings are being published as official OEEPE-publication[1].

In this presentation an attempt is made to summarize the main experimental results obtained up to now. They refer to stage 1 of radial-symmetrical corrections only of image coordinates. The results are presented in their own value, without thorough comparison with theoretical expectation. It is not possible, here, to give more than brief com-

TABLE 1. OEEPE OBERSCHWABEN. FRANKFURT BLOCK

(Wide-angle, 1:28,000, 8 strips with 25 models each). Absolute accuracy of different strip-adjustment methods for different control intervals. Data in μm, referring to the photo scale.

Control interval *i* (base lengths)		Adjustment with									
		bundles of rays	indep. models	2nd degree polynomials (1)	(2)	(3)	(4)	3rd degree polynomials (1)	(2)	(3)	(4)
i=1	μ_x	–	6.6	16.6	15.3	16.7	16.7	13.1	12.0	13.3	13.3
	μ_y	–	6.7	19.9	18.7	19.4	19.3	12.6	10.5	11.9	11.6
	μ_z	–	10.1	21.0	21.0	21.0	21.0	16.4	16.4	16.4	16.4
	σ_o	–	8.8	–	–	–	–	–	–	–	–
i=2	μ_x	9.7	9.1	16.8	16.0	16.9	16.9	13.8	12.8	14.0	13.9
	μ_x	8.4	9.5	21.4	20.2	20.9	20.6	14.0	12.2	13.3	12.7
	μ_z	16.2	14.0	23.1	23.0	23.0	23.0	17.9	17.8	17.8	17.8
	σ_o	5.1	8.4	–	–	–	–	–	–	–	–
i=4	μ_x	10.1	10.0	17.5	16.9	17.7	17.7	14.3	13.8	14.4	14.4
	μ_y	9.9	10.2	21.6	20.5	21.0	20.8	14.0	12.7	13.4	12.8
	μ_z	18.8	14.7	24.4	24.2	24.2	24.2	19.0	19.0	19.0	19.0
	σ_o	4.6	8.4	–	–	–	–	–	–	–	–
i=6	μ_x	12.0	10.5	18.5	17.9	18.6	18.6	14.7	14.3	14.9	14.9
	μ_y	11.1	10.2	23.1	21.8	22.6	22.6	13.9	12.9	13.3	13.0
	μ_z	21.7	16.2	25.6	25.5	25.5	25.5	20.4	20.4	20.4	20.4
	σ_o	4.6	8.4	–	–	–	–	–	–	–	–
i=8	μ_x	12.9	11.2	18.9	18.5	19.0	19.1	15.3	16.5	15.5	16.5
	μ_y	11.2	11.6	23.9	23.1	23.3	23.1	14.7	14.3	14.1	13.7
	μ_z	22.8	19.0	27.8	27.7	27.7	27.7	22.2	22.2	22.2	22.2
i=12.5	μ_x	4.4	8.4	22.0	22.5	22.2	22.2	22.6	31.6	22.4	22.8
	μ_y	17.5	15.4	27.7	27.3	27.2	27.0	17.8	20.1	17.0	16.8
	μ_z	41.1	30.9	30.8	30.8	30.8	30.8	–	–	–	–
	σ_o	4.3	8.4	–	–	–	–	–	–	–	–

i=1 means all control points are used.

(1) *x*, *y*, *z* in common.
(2) *x*, *y*, *z* separately.
(3) *x*, *y* – z, planimetry and elevation separately.
(4) same as (3), *x*, *y* conformal.

ments. Some preliminary conclusions will be drawn at the end.

In the tables the following notations will be used:

σ_o: standard error of unit weight of the adjustments (block adjustment by independent models, due to plan-height iteration, gives separate values for σ_p (planimetry) and σ_{OH} (height)).

μ_x, μ_y, μ_z root mean square values of the ***true*** errors at check points; $\mu_{xy} = \sqrt{[(\mu_x^2 + \mu_y^2)/2)]}$, = RMS values of planimetric coordinate errors.

The various control arrangements will be indicated in the tables by the letter *i*, indicating bridging distance or distances between control points in terms of base lengths. Planimetric control is restricted to perimeter control.

RESULTS OF STRIP ADJUSTMENTS

Strip adjustments were performed with all strips (30). They all refer to strips of 25 models each, for bridging distances of 2, 4, 6, 8 and 12.5 base lengths. Three different adjustment methods were applied: bundle method, independent-model method, and polynomial methods. Polynomial adjustment formulas of second and third degrees were used, with four different versions each. Table 1 shows the mean results for the 8 strips of the 200-model block Frankfurt (wide-angle). The conclusion can be drawn that, within each group, the four different polynomial versions give practically equal results.

Tables 2 and 3 show the summarized comparison of the different methods of strip adjustment. Of the polynomials, only Version 1 (*xyz* dependent) is included, for second and third degree each.

TABLE 2. OEEPE OBERSCHWABEN. WIDE-ANGLE BLOCKS
Absolute accuracy of different strip-adjustment methods for different control intervals.
Data in μm, referring to the photo scale 1:28,000.

Control interval i (base) lengths)		Frankfurt block (8 strips)				Wien block (7) strips)		
		bundles	indep. models	2nd deg. polynom.	3rd deg. polynom.	indep. models	2nd deg. polynom.	3rd deg. polynom.
i=1	μ_x	–	6.6	16.6	13.1	6.4	22.6	16.7
	μ_y	–	6.7	19.9	12.6	6.8	28.1	15.8
	μ_z	–	10.1	21.0	16.4	10.0	21.7	16.2
	σ_0	–	8.8	–	–	8.9	–	–
i=2	μ_x	9.7	9.1	16.8	13.8	7.5	22.8	17.1
	μ_y	8.4	9.5	21.4	14.0	9.6	27.4	15.4
	μ_z	16.2	14.0	23.1	17.9	14.7	23.9	18.6
	σ_0	5.1	8.4	–	–	8.6	–	–
i=4	μ_x	10.1	10.0	17.5	14.3	8.9	23.9	17.8
	μ_y	9.9	10.2	21.6	14.0	11.1	31.2	17.4
	μ_z	18.8	14.7	21.4	19.0	14.7	24.4	19.1
	σ_0	4.6	8.4	–	–	8.6	–	–
i=6	μ_x	12.0	10.5	18.5	14.7	11.4	24.6	18.5
	μ_y	11.1	10.2	23.1	13.9	13.9	34.2	18.4
	μ_z	21.7	16.2	25.6	20.4	20.4	27.4	21.3
	σ_0	4.6	8.4	–	–	8.6	–	–
i=8	μ_x	12.9	11.2	18.9	15.3	15.4	27.2	20.3
	μ_y	11.2	11.6	23.9	14.7	16.1	35.9	20.3
	μ_z	22.8	19.0	27.8	22.2	20.4	27.5	22.6
	σ_0	4.4	8.4	–	–	8.6	–	–
i=12.5	μ_x	17.1	16.8	22.0	22.6	19.7	31.9	28.9
	μ_y	17.5	15.4	27.7	17.8	26.0	41.5	27.0
	μ_z	41.1	30.9	30.8	–	39.3	32.5	–
	σ_0	4.3	8.4	–	–	8.6	–	–

i=1 means all control points are used.

The most interesting results of Tables 2 and 3 need no special comment here. The conclusions to be drawn are essentially the same as from the results of the block-adjustments.

Results of Block-Adjustments by the Method of Independent Models

The majority of the block adjustments with the Oberschwaben blocks, up to now, refers to the method of independent models (with the PAT-M-43 computer program). This rather complete series of adjustments investigate the accuracy of adjusted blocks as depending on four groups of parameters:

Variation of control, for given blocksize.
Variation of blocksize, for given control pattern.
Type of photography (wide angle (w.a.) and super-wide angle (s.w.a.))
Side-overlap (20 and 60 percent).

Preliminary tests had shown that different height control had practically no influence on the planimetric accuracy, and vice versa (contrary to the bundle adjustment).

Tables 4 and 5 display the results of the block-adjustments for 20-percent side overlap, with wide-angle and super-wide-angle photographs. Each of the four blocks was treated with five different control versions for planimetry and heights apart from the 0-version in which all known points were used as control points (for detecting gross data errors).

The planimetric control points were always located on the perimeter of the block at various distances *i*, up to the extreme situation of four control points only in the corners of a block. The height control points are arranged in chains across the block for given bridging distances *i*. One additional height control point was always placed at the open sides of the block where the maximum errors

TABLE 3. OEEPE OBERSCHWABEN. SUPER-WIDE-ANGLE BLOCKS
Absolute accuracy of different strip-adjustment methods for different control intervals.
Data in μm, referring to the photo scale 1:28.000.

Control interval i (base lengths)		Den Haag block (8 strips)				Delft block (6 strips)		
		bundles	indep. models	2nd deg. polynom.	3rd deg. polynom.	indep. models	2nd deg. polynom.	3rd deg. polynom.
i=1	μ_x	–	8.4	27.8	16.2	7.1	17.8	13.6
	μ_y	–	9.9	28.6	22.4	8.2	19.1	16.8
	μ_z	–	11.1	34.0	22.1	9.6	24.5	19.8
	σ_0	–	10.2	–	–	8.7	–	–
i=2	μ_x	9.9	11.1	29.0	17.9	9.6	18.4	14.6
	μ_y	15.7	13.9	28.4	22.2	11.1	18.9	16.6
	μ_z	17.6	17.5	37.8	25.2	15.4	26.6	22.7
	σ_0	7.1	9.6	–	–	8.1	–	–
i=4	μ_x	11.0	13.2	29.8	18.5	10.7	19.2	15.3
	μ_y	16.1	15.0	30.7	24.6	12.8	20.2	17.3
	μ_z	18.5	20.4	43.5	28.4	16.4	27.7	23.3
	σ_0	6.7	9.4	–	–	7.9	–	–
i=6	μ_x	13.6	16.1	33.2	19.5	12.1	20.6	16.1
	μ_x	17.6	17.1	33.1	25.3	14.3	22.1	19.5
	μ_z	20.3	21.3	42.8	27.8	18.2	28.6	25.4
	σ_0	6.6	9.3	–	–	7.9	–	–
i=8	μ_s	15.1	21.4	33.0	21.6	14.3	20.9	16.5
	μ_y	17.9	20.7	34.6	28.5	17.2	24.1	20.9
	μ_z	23.1	32.9	45.4	36.7	20.4	29.6	27.0
	σ_0	6.5	9.2	–	–	7.9	–	–
i=12.5	μ_x	20.5	27.1	37.8	31.7	18.9	22.6	17.5
	μ_y	23.1	28.6	41.6	36.3	19.3	26.0	22.1
	μ_z	31.7	46.4	57.1	–	33.9	37.2	–
	σ_0	6.4	9.1	–	–	7.8	–	–

i=1 means all control points are used.

TABLE 4. OEEPE OBERSCHWABEN. INDEPENDENT MODELS, WIDE-ANGLE.
Block Frankfurt, 8 × 25 = 200 models (first row).
Block Vienna, 7 × 25 = 175 models (second row).

Version	Control plan	height	σ_{OP} μm	σ_{OH} μm	μ_x μm	μ_y μm	μ_{xy} μm	μ_z μm	$\frac{\mu_{xy}}{\sigma_{OP}}$	$\frac{\mu_z}{\sigma_{OH}}$
0	all CP	all CP								
	(510)	(452)	7.2	9.2	(6.1)	(6.9)	(6.5)	(9.8)	(0.90)	(1.06)
	(450)	(404)	6.9	8.5	(5.7)	(6.5)	(6.1)	(8.9)	(0.90)	(1.05)
1	perimeter	13 chains	6.9	8.4	8.3	14.9	12.1	12.9	1.74	1.54
	i=2	i=2	6.5	7.8	13.4	10.1	11.9	13.5	1.83	1.75
2	perimeter	7 chains	6.7	8.3	13.1	20.3	17.1	13.8	2.56	1.66
	i=4	i=4(2)	6.3	7.7	16.1	12.3	14.4	15.1	2.28	1.96
3	perimeter	5 chains	6.6	8.4	18.0	23.0	20.7	15.9	3.15	1.91
	i=6	i=6(3)	6.2	7.6	19.3	13.2	16.5	16.1	2.66	2.13
4	perimeter	4 chains	6.3	8.4	22.5	28.4	25.6	16.1	4.07	1.93
	i=8	i=8(4)	6.1	7.6	17.1	17.0	17.0	19.4	2.79	2.56
5	4 corners	3 chains	6.0	8.4	40.2	49.9	45.3	19.0	7.56	2.27
		i=12.5(6)	5.9	7.5	47.6	19.5	36.4	27.2	6.17	3.61

TABLE 5. OEEPE OBERSCHWABEN. INDEPENDENT MODELS, SUPER-WIDE-ANGLE.

Block The Hague, 8 × 25 = 200 models (first row)
Block Delft, 7 × 25 = 175 models (second row)

Version	Control plan	Control height	σ_{OP} μm	σ_{OH} μm	μ_x μm	μ_y μm	μ_{xy} μm	μ_z μm	$\frac{\mu_{xy}}{\sigma_{OP}}$	$\frac{\mu_z}{\sigma_{OH}}$
0	all CP	all CP								
	(492)	(432)	9.0	8.8	(7.5)	(8.5)	(8.0)	(9.8).	(0.89)	(1.12)
	(441)	(398)	8.8	7.8	(6.7)	(8.0)	(7.4)	(9.0)	(0.84)	(1.16)
1	perimeter	13 chains	8.7	7.7	12.6	17.8	15.4	15.1	1.77	1.97
	i=2	i=2	8.7	6.8	11.6	16.0	14.0	14.7	1.60	2.16
2	perimeter	7 chains	8.3	7.6	17.0	25.6	21.7	16.1	2.61	2.13
	i=4	i=4(2)	8.5	6.7	15.5	20.7	18.3	15.6	2.14	2.32
3	perimeter	5 chains	8.1	7.5	18.5	31.1	25.6	18.4	3.16	2.45
	i=6	i=6(3)	8.4	6.5	17.7	25.1	21.7	22.6	2.60	3.46
4	perimeter	4 chains	8.0	7.5	28.9	32.6	30.8	18.9	3.87	2.54
	i=8	i=8(4)	8.3	6.5	23.8	27.3	25.6	28.2	3.09	4.32
5	4 corners	3 chains	7.6	7.5	35.9	54.0	45.9	22.9	6.00	3.07
		i=12.5(6)	8.1	6.5	33.1	33.3	33.2	32.8	4.11	5.05

TABLE 6. OEEPE OBERSCHWABEN INDEPENDENT MODELS
Wide-angle block, 15 × 25 = 375 models, q = 60½ (first row).
Super-wide-angle block, 15 × 25 = 375 models, q = 60 ½ (second row).

Version	Control plan	Control height	σ_{OP} μm	σ_{OH} μm	μ_x μm	μ_y μm	μ_{xy} μm	μ_z μm	$\frac{\mu_{xy}}{\sigma_{OP}}$	$\frac{\mu_z}{\sigma_{OH}}$
0[1]	all CP	all CP								
	(466)	(417)	6.8	9.6	(6.0)	(6.7)	(6.4)	(9.9)	(0.94)	(1.03)
	(455)	(407)	8.5	9.7	(7.2)	(8.3)	(7.8)	(10.2)	(0.92)	(1.05)
1	perimeter	13 chains	6.4	9.2	10.6	9.0	9.8	11.6	1.52	1.27
	i=2	i=2	8.2	9.2	10.3	13.9	12.2	13.3	1.49	1.44
2	perimeter	7 chains	6.3	9.1	14.5	11.5	13.1	12.8	2.07	1.41
	i=4	i=4(2)	8.0	9.2	14.2	19.1	16.9	14.1	2.10	1.53
3	perimeter	5 chains	6.2	9.0	16.6	13.9	15.3	18.0	2.45	2.00
	i=6	i=6(3)	7.9	9.1	17.9	22.3	20.2	25.4	2.55	2.80
4	perimeter	4 chains	6.1	9.0	18.0	17.9	18.0	23.4	2.92	2.60
	i=8	i=8(4)	7.8	9.0	20.7	27.2	24.1	35.6	3.09	3.94
5	4 corners	3 chains	6.0	8.9	24.8	24.4	24.6	54.7	4.10	6.17
		i=12.5(6)	7.6	8.8	26.8	44.0	36.4	79.2	4.79	9.02
3*	perimeter	grid	6.2	9.0	16.6	13.9	15.3	16.0	2.45	1.78
	i=6	i=4/4	7.9	9.0	17.9	22.3	20.2	18.7	2.55	2.07
4*	perimeter	grid	6.1	9.0	18.0	17.9	18.0	20.2	2.92	2.25
	i=8	i=6/5	7.8	8.9	20.6	27.2	24.1	28.7	3.09	3.22
5*	4 corners	grid	6.0	8.7	24.7	24.4	24.6	48.8	4.09	5.57
		i=8/7	7.6	8.7	26.8	43.8	36.3	64.3	4.77	7.35

[1] Versions 0 - 5: double overlap areas only.

were to be expected, reducing there the bridging distance to one half.

Table 6 summarizes in the same way the results concerning blocks with 60 percent side-overlap. Here, three additional versions are included, with a grid pattern of height control, rather than chains. The accuracy figures refer to the area of double overlap, and also the perimeter control points are arranged along the perimeter of the double overlap area.

THE RESULTS as summarized in the Tables 4 through 6 display the effect of control variation, for constant (maximum) blocksize. On

TABLE 7. OEEPE OBERSCHWABEN. BLOCK FRANKFURT (W.A.), INDEPENDENT MODELS
Mean accuracy of subblocks.
A,B (8×17 = 136 models). *I-VI* (4×8=32 models). *1-12* (2×4=8 models)

	Control plan	 height	σ_{OP} μm	σ_{OH} μm	μ_x μm	μ_y μm	μ_{xy} μm	μ_z μm	$\frac{\mu_{xy}}{\sigma_{OP}}$	$\frac{\mu_z}{\sigma_{OH}}$
A,B	4 corners (per. i=16)	9 chains i=2	6.0	8.3	26.3	43.1	35.7	12.7	5.93	1.52
	perimeter i=8	5 chains i=4(2)	6.2	8.2	23.1	25.5	24.3	13.3	3.91	1.61
	perimeter i=4	3 chains i=8	6.7	8.2	13.4	19.8	16.9	19.4	2.53	2.36
	perimeter i=2	3 chains i=8(4)	7.0	8.2	8.6	14.5	11.9	19.3	1.71	2.35
I-VI	4 corners (per. i=8)	5 chains i=2	6.0	8.4	15.7	15.9	15.8	13.1	2.63	1.57
	perimeter i=4	3 chains i=4(2)	6.5	8.3	10.4	12.4	11.4	13.8	1.77	1.66
	perimeter i=2	3 chains i=4	6.7	8.3	7.6	10.1	8.9	14.1	1.32	1.71
1-12	4 corners (per. i=4)	3 chains i=2	6.2	8.5	8.6	12.0	10.5	14.4	1.68	1.69
	perimeter i=2	3 chains i=2	6.6	8.5	7.0	9.4	8.2	14.4	1.26	1.69

the contrary, Table 7 shows the effects of blocksize on accuracy for given patterns of control.

COMPARISON OF ADJUSTMENT METHODS

One of the most interesting questions in aerial triangulation is how the accuracy capabilities of different adjustment methods compare if all other circumstances are kept constant. For instance, it has always been taken for granted and has been confirmed by theoretical studies[2] that the analytical (bundle) method will give considerably more accurate results than any other method. Also there still is considerable disagreement among experts about the accuracy performance of polynomial methods of block-adjustments.

Up until now, three different methods of block adjustment have been applied to the Oberschwaben test-material:

Analytical method or bundle adjustment (program PAT-B).
Independent-model method (program PAT-M-43).
Polynomial method, 2nd degree (Schut's program).

The comparative adjustments have been restricted to different control versions, for constant (maximum) blocksize. The results, as available up until now, are displayed in Table 8.

TABLE 8. OEEPE OBERSCHWABEN

Comparison of adjustment methods.
Block Frankfurt, wide-angle (first row).
Block The Hague, super-wide-angle (second row).

Perimeter Control Planimetry	bundle method σ_o	 μ_{xy}	indep. models σ_{OP}	 μ_{xy}	polynom. 2nd degr. μ_{xy}	height control chains	bundle method μ_z	indep. models σ_{OH}	 μ_z	polynom. 2nd degr. μ_z
i=2	5.7	14.7	6.9	12.1	25.8	i=2	18.2	8.4	12.9	23.8
	8.1	23.9	8.7	15.4	–		17.2	7.7	15.1	–
i=4	5.0	22.0	6.7	17.1	30.0	i=4(2)	19.1	8.3	13.8	24.0
	7.4	32.8	8.3	21.7	–		17.4	7.6	16.1	–
i=6	4.7	27.9	6.3	20.7	31.3	i=6(3)	22.2	8.4	15.9	26.4
	7.0	37.3	8.1	25.6	–		21.7	7.5	18.4	–
i=8	4.3	30.6	6.3	25.6	34.6	i=8(4)	19.0	8.4	16.1	27.5
	6.8	42.3	8.0	30.8	–		21.1	7.5	18.9	–
4 corners	4.0	47.1	6.0	45.3	62.6	i=12.5(6)	22.2	8.4	19.0	32.8
	6.4	54.3	7.6	45.9	–		27.8	7.5	22.9	–

Data in μm.

CONCLUSIONS

The first aim of this paper is only to present the actual results. It is neither intended nor possible to go through a thorough theoretical and statistical evaluation; that will be the future task of the Oberschwaben working group.

Nevertheless, the results can be summarized in a number of factual statements the implications of which are likely to reach far beyond the test-material as such.

FIRSTLY, it should be mentioned that the overall results of the Oberschwaben tests are considered very satisfactory. They confirm the high accuracy capability of aerial triangulation in general. For example, with the adjustment by independent models were obtained:

> Standard errors of unit weight between 5.9 μm and 9.0 μm for planimetry, and between 6.5 μm and 9.6 μm for heights.
>
> Planimetric coordinate accuracies of blocks of 200 (175) models with dense perimeter control of 12.1 (11.9) μm for wide-angle photographs, and of 15.4 (14.0) μm for super-wide-angle photographs.
>
> Height accuracies of blocks of 200 (175) models of 15.9 (16.1) $\mu\text{m} = 0.1$ part per thousand of h, for wide-angle photographs, with bridging distance of 6 models. Referred to the terrain, the height accuracy is 45 (45) cm, with 15 km bridging distance.

Similarly good results were obtained by strip-adjustments.

FOR some general accuracy features the test results are in satisfactory agreement with theoretical expectation. This is especially so for the independent-model method and for wide-angle photographs:

> The planimetric accuracy of perimeter-controlled blocks shows only weak dependence on blocksize.
>
> The height accuracy turns out to be independent of blocksize for a given bridging distance. Conversely the height accuracy as function of bridging distance is in fair agreement with theory.

Fortunately, these two characteristics are the ones that are most important in practical application and are relied upon very much in project planning.

A NUMBER of results must be classified as unexpected, surprising, or plainly disagreeing with general expectation or with theory proper. For instance:

- ★ The planimetric results of super-wide-angle blocks are, as expected, poorer than of wide-angle blocks by about 20 percent. It is highly surprising, however, that also the absolute height accuracy turned out to be poorer by about the same percentage. In terms of relative height accuracy the difference is even more marked. It seems that some caution or re-orientation of thoughts concerning the qualities of super-wide-angle photographs can be suggested.
- ★ The use of 60 percent side overlap did not improve the accuracy as much as expected, compared with 20 percent side overlap, neither in planimetry nor in heights. Also the relaxation of height control into a grid pattern did not at all meet expectations. Thus, under the conditions of the test, some caution seems to be justified against relying too much on the theoretical merits of 60 percent side-overlap.
- ★ Relaxation of planimetric control at the perimeter of a block causes the planimetric errors to increase considerably more than theory would suggest.
- ★ The wide-angle blocks and the super-wide-angle blocks differ between themselves more than might have been expected. Similarly, within a block the x- and y-accuracies differ considerably in many instances. Most likely, however, those effects are not truly significant.

THE comparison of methods gave, by all conventional standards, most surprising results, with respect to the analytical or bundle method:

- The independent-model results show consistently better accuracy than the results from bundle adjustments, which is quite contrary to general expectation and to theoretical accuracy studies.[2] (The results have been checked and confirmed by another independent computer program for bundle adjustment).
- The results of the bundle adjustments as such differ very considerably from theoretical expectation, in terms of multiples of the standard error of unit weight.[2]
- The strip and block adjustments by polynomials give consistently poorer results compared with the more rigorous methods. This is in agreement with the general theoretical expectation, the polynomial methods being clearly approximate adjustment methods. Nevertheless, one can say that the polynomials maintain themselves rather well, their results being perhaps closer to the rigorous methods than might have been expected.

BEFORE jumping to conclusions it should be kept in mind that the results refer to two block missions only. Statistically speaking, the sample size is small. Statistical tests will be needed to show which of the observed disagreements between test-results and theoretical expectation are significant.

Nevertheless, the disagreements with theory are obviously consistent enough to suggest statistical significance, at least to some extent, and to ask for an explanation.

It is not possible to treat such questions here. However, concluding this paper it is briefly summarized how the situation was judged. It can be stated that obviously the current theories on the accuracy of aerial triangulation, which are based on independent random errors only, are not capable of assessing realistically the accuracy behavior of strips and blocks on accuracy levels for σ_0 of better than 10 μm. The predominance of systematic and/or highly correlated image errors seemingly render the simplified accuracy models not applicable any more. The strong evidence for systematic image errors if supported by the analysis of the photographs from the camera calibration test field.[1]

The next steps are clearly outlined: Every effort must be made to correct systematic image errors by *a priori* corrections and/or by applying refined mathematical models for the photogrammetric image and the stereopair. Preliminary adjustments of the Oberschwaben material by a bundle adjustment program which applies additional parameters for image deformation correction confirm that such efforts will be highly successful, yielding very considerable improvements of accuracy.[1] Theoretical studies[3] have already pointed out the importance of correction terms for systematic image errors. They also predicted that the bundle method is more sensitive against the presence of systematic errors than the less rigorous methods, an effect for which the Oberschwaben results have shown clear evidence.

Thus it will be most interesting to continue the investigations of the Oberschwaben test material through stage 2 and 3 of refined corrections and with refined mathematical models. The results are expected to prove that the practical performance of aerial triangulation can be pushed considerably beyond the high level of accuracy which it has reached at the present time.

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Alaska Convention

THE 1975 Alaska Surveying and Mapping Convention will meet in Anchorage on February 5, 6 and 7 at the Anchorage Westward Hotel. The theme this year is "Alaska Now".

Two short courses will be offered preceding the convention. The first course, "Retracement Seminar", will be presented on January 31 and February 1 by Walter Robillard, Regional Cadastral Surveyor for the Southern Region, U.S. Forest Service. He will include presentation on retracement of the U.S. Government rectangular surveys, urban subdivisions, and lot surveys.

The second course, offered on February 3 and 4, will be a "Coordinate Workshop" under the direction of Mr. Joe Dracup who is affiliated with the National Geodetic Survey, NOAA. He will present instruction in determination of geodetic and state plane coordinates.

This is the 10th such convention sponsored by the Alaska Society of Professional Land Surveyors, the Alaska Region of the American Society of Photogrammetry and the Alaska Section of the American Congress on Surveying and Mapping.

Advance programs and short course registration forms will be available in December. For additional information, please write to Mrs. Rita Ihly, Convention Coordinator, P.O. Box 2164, Anchorage, Alaska 99510.