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A GENERAL COMPUTER PROGRAM FOR THE APPLICATION OF THE RIGOROUS BLOCKADJUSTMENT SOLUTION IN PHOTOGRAPHIC ASTROMETRY

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SUMMARY

The evaluation of stellar positions from overlapping plates by rigorous blockadjustment methods leads to a very large system of normal equations containing both, stellar positions and plate constants as unknowns. These general normal equations can be reduced to a subsystem with banded-bordered coefficient matrix, containing only the plate constants as unknowns. In the case of a large photographic net covering a hemisphere or sphere with multiple overlap of the plates, the actual number of unknown plate constants may exceed 20 000, depending on the adopted reduction model.

The general structure of a computer program, now under development at the Hamburg observatory, is described in detail. The program, entirely written in FORTRAN IV language, is based on a very fast subroutine, developed at the University of Stuttgart, for the solution and inversion of large symmetric and positive-definite linear systems of equations with banded-bordered coefficient matrix. It provides a direct non-iterative solution of the block-adjustment problem with, at least, partial inversion of the normal equation matrix for evaluation of the standard deviations of the computed star positions at selected blockpoints. The choice of suitable reduction models and the application of the program for theoretical accuracy studies using inversion and simulation methods are discussed. Estimates of computing effort and external storage capacity as a function of the number of unknowns and the actual matrix-structure are obtained. For a variety of overlap patterns and blockstructures, occurring more frequently in photographic zone-work, simple analytic expressions for estimates of computing effort are quoted. In addition, ordering schemes for an optimal arrangement of the single plates according to minimum bandwidth of the reduced normal equation matrix are provided.

Based on these results and adopting a CDC 7600 computer system, computing times and necessary external storage capacities are obtained for several large photographic nets. It is shown that a blockadjustment of the full sphere with multiple overlap can be provided within 3 hr computing time for the solution of the reduced normal equations containing 48 000 unknown plate constants.

1. INTRODUCTION

The rigorous blockadjustment formulation of the problem of evaluating stellar positions from overlapping plates lead to a very large system of normal equations containing both stellar positions and plate constants as unknowns. Although the general system can be reduced to a subsystem leaving only the plate constants as unknowns, the remaining number of unknowns can go beyond 20 000. Even today a rigorous direct solution of extensive blockadjustment problems is only possible with the largest existing computer systems like IBM 360/91 and 360/195 or the CDC CYBER 76 and requires additionally the development of a very fast subroutine for the solution of linear systems of equations which takes fully into account the special band-structure of the coefficient matrix.

The following computer program which is now under development at the Hamburg observatory will consider these special requirements and allows a considerable flexibility both in the selection of the functional and stochastic model and the structure of the overlap pattern.

2. MATHEMATICAL MODELS FOR BLOCKADJUSTMENT APPLICATION IN PHOTOGRAPHIC ASTROMETRY

The formulation of indirect unconditional measurements is used in setting up the (non-linear) observation equations. For a star i appearing on a plate k, the measured coordinates $x_{i,k}$, $y_{i,k}$ depend on the spherical coordinates $\bar{\alpha}_i$, $\bar{\delta}_i$ and the plate constants $\bar{a}_k$, $\bar{b}_k$, $\bar{c}_k$, . . . , so one gets the following equations:

$$\begin{aligned} x_{i,k}+vx_{i,k} &= f_1(\bar{\alpha}_i, \bar{\delta}_i, \bar{a}_k, \bar{b}_k, \bar{c}_k, \ldots) \\ y_{i,k}+vy_{i,k} &= f_2(\bar{\alpha}_i, \bar{\delta}_i, \bar{a}_k, \bar{b}_k, \bar{c}_k, \ldots) \end{aligned} \quad (1)$$

where $vx_{i,k}$, $vy_{i,k}$ denote the residuals, which are minimized in the least squares adjustment.

If the star i is a reference star with (measured) known catalogue-positions α_i, δ_i, two additional equations have to be added:

$$\begin{aligned} \alpha_i+v\alpha_i &= \bar{\alpha}_i \\ \delta_i+v\delta_i &= \bar{\delta}_i. \end{aligned} \quad (2)$$

Both groups of equations will be treated as uncorrelated with proper weights chosen.

Often for some of the plate constants *a priori* estimates with certain accuracy properties obtained from preceding studies will exist. In those cases an additional observational equation will be set up for each of the plate constants, for instance, $\bar{c}_k$.

$$c_k+vc_k = \bar{c}_k. \quad (3)$$

The *a priori* estimates c_k have to be treated in those observation equations with proper weights according to their accuracy.

According to this most general concept constants and free unknowns of the adjustment problem can be interpreted as special cases of *a priori* estimates with infinite weight and weight zero, respectively. With realistic weight assumptions in addition an optimal error propagation can be provided. (A more detailed discussion of these questions can be found in de Vegt & Ebner 1972; Ebner 1973.)

Blockadjustment techniques will provide maximum advantage with respect to error-propagation and minimum number of reference stars if the number of unknown parameters is small. The functional models quoted below seem to be the most adequate for a treatment of very large nets. With special view of new projects to be carried out with improved optics and observing techniques, the question of incorporating magnitude and general higher-order terms in the functional relationship has not been considered here. In uncertain cases, from a pilot adjustment, taken over a part of the whole block area (say 200 plates), the most realistic reduction model could be established.

Denoting the measured coordinates by $x; y$ and the standard coordinates by $\xi; \eta$, respectively, the following models are set up:

8 *parameters*

$$\begin{aligned} \xi &= ax+by+c+px^2+qxy \\ \eta &= a'x+b'y+c'+pxy+qy^2. \end{aligned} \tag{4a}$$

It frequently will occur, that *a priori* estimates for the tilt terms p and q can be provided. In that case, additional observation equations according to (3) have to be added with suitable weights.

6 *parameters* (*general formulation*)

$$\begin{aligned} \xi &= ax+by+c+ex+fy \\ \eta &= ay-bx+c'-ey+fx. \end{aligned} \tag{4b}$$

This formulation is obtained by splitting up the linear terms of equation (4a) into an orthogonal part (unknowns a, b) and an affine part (unknowns e, f), the latter allow for an explicit incorporation of different scale values in both coordinates and assumed non-orthogonality of the axes. If *a priori* estimates for the unknowns e and f are available, due to this approach, the additional observation equations can be set up directly. Further on, if all plate constants are treated as free unknowns either (4b) can be used, treating the additional observation equations with zero-weight, or the already quoted linear relationship

$$\begin{aligned} \xi &= ax+by+c \\ \eta &= a'x+b'y+c' \end{aligned} \tag{4c}$$

could be adopted.

4 *parameters*

$$\begin{aligned} \xi &= ax+by+c \\ \eta &= ay-bx+c'. \end{aligned} \tag{4d}$$

Optimal error propagation with minimum number of reference stars will be expected from the orthogonal relationship (4d). However, this approach requires a careful orthogonal measurement of the rectangular star coordinates x, y on each plate and an *a priori* correction of the measured x, y for all non-orthogonal terms due to differential refraction and aberration. In addition the astrograph has to be monitored continuously for tilt and tangential point changes.

3. A SHORT DESCRIPTION OF THE BLOCKADJUSTMENT PROGRAM

The main item of this program is to provide a direct solution of the rigorous blockadjustment problem with special emphasis on very large photographic nets, such as a hemisphere or the whole sphere which will be of considerable interest in future astrometric work. In addition, an at least partial inversion of the normal equation-matrix for evaluation of the standard deviations of the computed star positions can be performed. The program will entirely be written in FORTRAN IV language so that a relatively easy exchange to other observatories and computer-systems should be possible. However, the practical application to very large nets is restricted to a small number of very fast computer systems.

As the program has to match a number of very different tasks with respect to I/O requirements and central processor-time, it has been decided to split up the program in several segments which could be executed independently. This has the advantage that the required central storage capacity is smaller and the probability of computer breakdowns during the running phase is minimized because already segment 2 of the program requires execution times of several hours. In the following text program subsegments are denoted by [ ], in accordance with Fig. 1.

Search and sort program

[1.1] The input tape [A] has to contain all available data for each star and plate, in addition the plate constants from a previous classical solution of every single plate and the positions of each field star, computed from this previous solution. It is further assumed that either the proper motion effect on the star positions arising from different epochs of every single plate is negligible, or all plates have been updated to the same epoch.

[1.2] At present the program will consider the following plate networks and overlap structures:

(0) A (smaller) star field with arbitrary overlap pattern.
(1) A zonal pattern with centre-edge overlap or 4-fold overlap Examples are shown in Figs 3 and 4.
(2) A hemisphere, 2-fold overlap (each star on at least 2 plates) or 4-fold overlap (each star on at least 4 plates).
Examples are shown in Figs 5 and 6.
(3) A full sphere, the same overlap patterns as in (2) are adopted.

Other patterns can in principle be added to the program if necessary.

[1.3] According to the (1), (2), (3) overlap patterns, the plates are reordered into a sequence which allows for a favourable banded-bordered structure of the coefficient matrix of the reduced normal equations [N_{red}]. Examples are given in Sections 4.1–4.3 of the present paper. At the same time a connexion matrix [CM] is built up which contains the information whether the plate number j, ($j = 1, \ldots n$) has common star numbers with other plates.

[1.4] The re-ordered plate-data and the connexion matrix [CM] are stored on a second tape [B] which contains now all necessary information with proper data arrangement for a direct formation of [N_{red}].

Formation and solution of the reduced normal equations

[2.1] The functional and stochastic reduction models will be set up along the outlines of Section 2 of the present paper. The above quoted functional models (4a–4d) are incorporated *a priori*. Other relationships, including magnitude terms can be added if necessary, although the handling of terms of that kind should be performed according to the already quoted remarks.

[2.2], [2.3] In all cases of blockadjustment applications, especially pattern. (1), (2), (3), which we are discussing here, the number of unknown positions (or positional corrections) $p = (\overline{\Delta\alpha_1}, \overline{\Delta\alpha_2} \ldots \overline{\Delta\alpha_n}; \overline{\Delta\delta_1}, \ldots \overline{\Delta\delta_n})^T$ will be much

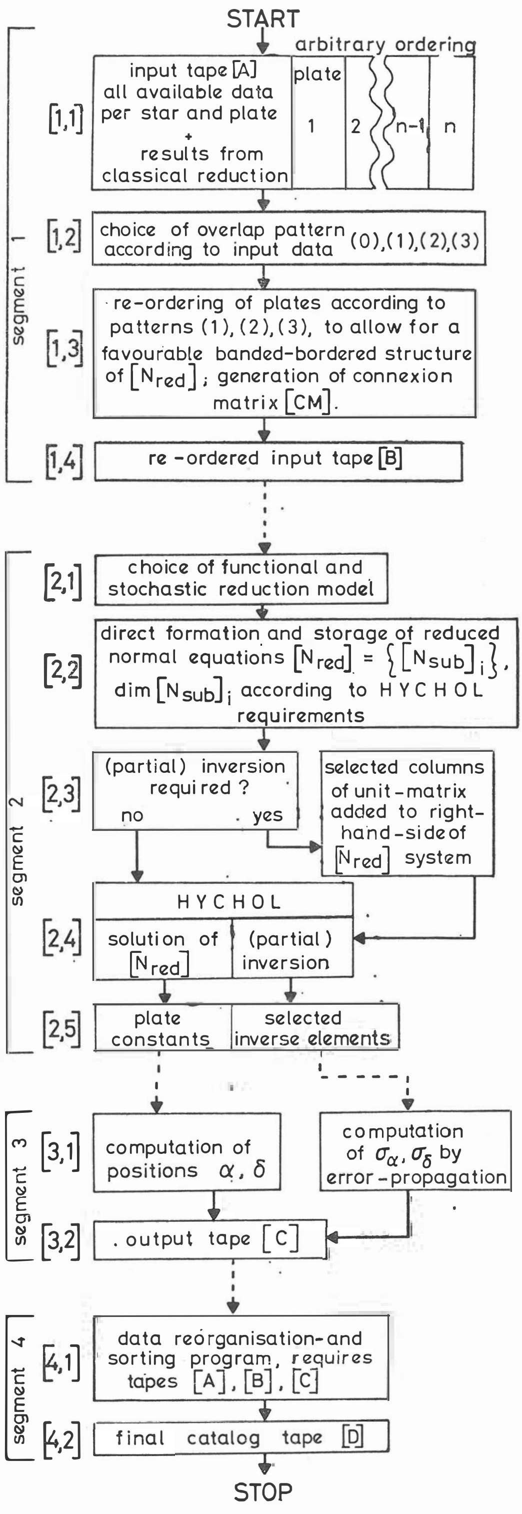

FIG. 1. *General structure of the blockadjustment program.*

larger than the number of unknown plate constants k, so that a reduction of the normal equations to a subsystem, containing only the plate constants as unknowns, will provide a real advantage. From the (linearized) system of observation equations (Section 2)

$$v = Ap + Bk - f \tag{5}$$

one gets the following normal equations, which are split up according to the unknowns p and k.

$$\begin{bmatrix} A^T G^{-1} A & A^T G^{-1} B \\ B^T G^{-1} A & B^T G^{-1} B \end{bmatrix} \cdot \begin{bmatrix} p \\ k \end{bmatrix} = \begin{bmatrix} A^T G^{-1} f \\ B^T G^{-1} f \end{bmatrix}. \tag{6}$$

The associated covariance matrix of all observations is denoted by G. After having eliminated the unknowns p from the general normal equations (6), the following reduced system for the desired plate constant unknowns is obtained:

$$[N_{red}].k = (B^T G^{-1} f - B^T G^{-1} A (A^T G^{-1} A)^{-1} A^T G^{-1} f) \tag{7}$$

The coefficient matrix $[N_{red}]$ of the reduced normal equations is given by the expression

$$[N_{red}] = (B^T G^{-1} B - B^T G^{-1} A (A^T G^{-1} A)^{-1} A^T G^{-1} B). \tag{8}$$

Because the matrix $A^T G^{-1} A$ is a 2×2 hyperdiagonal matrix, an inversion can easily be performed. This matrix is in addition needed in subsegment [3.1] of the program.

During the calculation of all matrix terms in equations (6)–(8), containing the large coefficient matrix B, zero submatrices are suppressed automatically by the program.

The HYCHOL subroutine

[2.4] The reduced normal equation system (7) is in general too large to allow for a direct storage and solution in the central computer memory. Because the access-time to external storages is large as compared with performing a multiplication in the core memory it would be not feasible to consider a transfer of single coefficients of the system of equations. Therefore an algorithm has been developed and programmed which is based on a direct transfer of submatrices instead of single coefficients. For that purpose a suitable hyper-structure (Fig. 2) is imposed on the matrix $[N_{red}]$.

The reduced normal equation matrix is split up into submatrices A_{ik} of arbitrary size—the generated diagonal matrices A_{ii} are square matrices—and the vectors of the unknowns and the right-hand side are divided into corresponding subvectors x_k, a_i (Klein 1971). This leads to the following expression:

$$\sum_{k=1}^{n} A_{ik} x_k = a_i \quad i = 1, n. \tag{9}$$

The optimal size of the A_{ik} submatrices can be chosen in accordance with the available central store capacity, in addition the algorithm takes account of the special band-structure of the coefficient matrix. Only non-zero submatrices and subvectors are stored in the external storage.

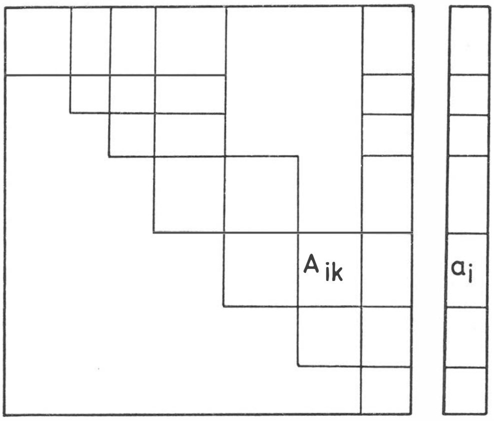

FIG. 2. *Hyperstructure of reduced normal equations with banded-bordered coefficient matrix.*

The solution procedure is a generalized Cholesky method transforming the hyper-system (9) into:

$$\sum_{k=i}^{n} R_{ik} x_k = r_i \quad i = 1, n. \tag{10}$$

The hyper-elements R_{ik} and r_i are computed according to the algorithm

$$R_{ik} = R_{ii}^{T-1} \left(A_{ik} - \sum_{l=1}^{i-1} R_{li}^{T} R_{lk} \right) \quad i = 1, n \quad k = i, n$$

$$r_i = R_{ii}^{T-1} \left(a_i - \sum_{l=1}^{i-1} R_{li}^{T} r_l \right) \quad i = 1, n. \tag{11}$$

Performing the Cholesky decomposition of the hyper-system and the subsequent re-substitution starting from the last hyper-row, the corresponding submatrices and subvectors needed are transferred into the central memory for transformation and are stored again externally. Each time only three submatrices have to be stored in the central memory simultaneously.

The HYCHOL routine provides the simultaneous solution of linear, symmetric and positive-definite systems of equations of any size with several right-hand sides. In this way a partial inversion is possible introducing certain columns of the unit matrix as additional right-hand sides, although for a complete inversion of banded systems a special algorithm can be constructed. The computation of those elements of the inverse matrix $[N_{\text{red}}]^{-1}$ being situated within the band could then be performed in about $3 . t_{\text{h}}$, where t_{h} denotes the computing time necessary for a single solution of the $[N_{\text{red}}]$ system by HYCHOL.

The necessary computing effort for the solution of a symmetric and positive-definite system of equations by the HYCHOL routine with n unknowns and r right-hand sides, assuming a bandwidth m and a borderwidth l for the $n \cdot n$ coefficient matrix, can be estimated with an uncertainty of a few per cent from the following simple expressions:

$$n_{\text{mult}} \approx nm^2/2 + nml + 2nmr + nlr$$

$$t_{\text{h}} \approx 0.2 \cdot 10^{-9} \, n_{\text{mult}} \quad [\text{hr}] \tag{12}$$

where n_{mult} is the number of multiplications during the solution process and t_{h} denotes the total computing time on a CDC CYBER 76 computer system.

The necessary external storage capacity n_{s} [full CDC-words] is given by the expression

$$n_{\text{s}} \approx n(m+l+r). \tag{13}$$

Computation of positions and final data arrangement [3.1]–[4.2]

[3.1] Having obtained the solution of the reduced normal equations, the plate constant unknowns k can now be substituted into the original normal equation system and the positions are computed according to the expression

$$p = (A^{T}G^{-1}A)^{-1}(A^{T}G^{-1}f - A^{T}G^{-1}Bk). \tag{14}$$

As can be verified by the general error propagation law or direct computation of the partial inverse matrices of the original normal equations, the following relation for the desired covariance matrix of the positions holds:

$$Q_{p} = (A^{T}G^{-1}A)^{-1} + (A^{T}G^{-1}A)^{-1}\, A^{T}G^{-1}B[N_{\text{red}}]^{-1}\, B^{T}G^{-1}A(A^{T}G^{-1}A)^{-1}.$$

This general expression requires the full inverse of $[N_{\text{red}}]$, however, for practical applications it seems to be sufficient to obtain the standard deviations of the positions only at selected points of the block area (details see Section 5) which reduces the problem to a partial inversion.

[3.2] The output tape [C] contains the final positions and additional necessary data from tape [B] together with the standard deviations at selected block points.

The final data arrangement according to program segments [4.1] and [4.2] has not been specified in detail at present.

4. EXAMPLES OF PROGRAM APPLICATION TO VARIOUS ASTROMETRIC BLOCKSTRUCTURES AND ESTIMATES OF COMPUTING EFFORT

The following examples will be restricted to those blockstructures which will be met in practice more frequently or seem to be of special interest with respect to recently planned astrometric activities. Further examples and a more detailed discussion of special reduction models can be found in a previous paper (de Vegt & Ebner 1972). The below quoted examples of estimates for necessary computing effort are given in a semi-analytical form which easily enables one to perform the necessary modifications for similar overlap problems. The number of unknown parameters per plate is denoted by p. In all cases a centre-edge overlap pattern and a plate size of $5° \times 5°$ has been assumed.

4.1 *A zonal pattern, 2-fold and 4-fold overlap*

The assumed overlap patterns are shown in Figs 3 and 4. To obtain an optimal band-structure, the plates have to be ordered transverse to the zonal extension. According to the expressions (12), (13) of Section 3, we have the following results:

(a) *2-fold overlap*

$$n = 648\,p;\; m = 6\,p;\; l = 4\,p.$$

The border-width arises from the fact that the four last plates (Nos 645–648) are connected with the first plates.

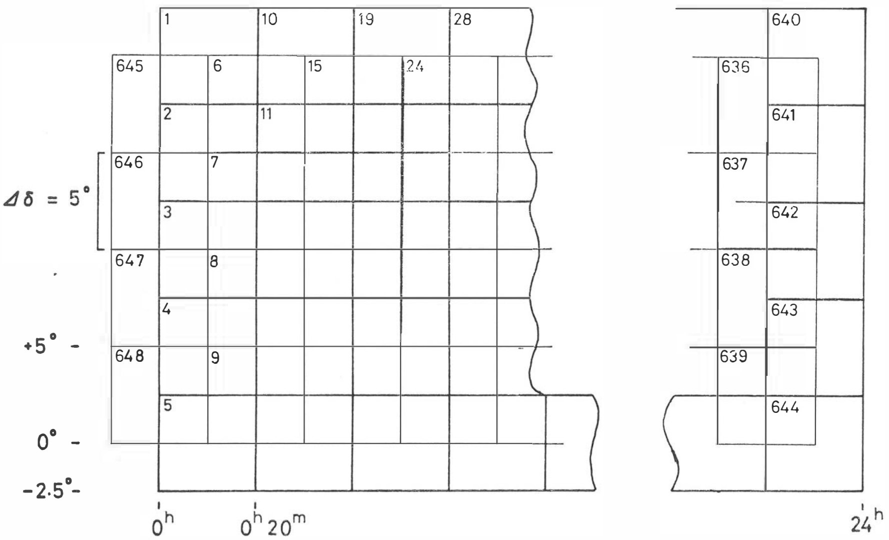

FIG. 3. *Zonal pattern ordering scheme, 2-fold net.*

$$n_{\text{mult}} \approx 648{\cdot}36\,p^3/2 + 648{\cdot}6{\cdot}4\,p^3 \approx 27{\cdot}10^3\,p^3$$

$$t_{\text{h}} \approx 5.4{\cdot}10^{-6}\,p^3;\; n_{\text{s}} \approx 648(6+4)\,p^2 \approx 6.5{\cdot}10^3\,p^2$$

(b) 4-*fold overlap*

$$n = 1296\,p;\; m = 11\,p;\; l = 9\,p.$$

Due to the strengthened overlap, bandwidth and borderwidth are increased.

$$n_{\text{mult}} \approx 1296{\cdot}121\,p^3/2 + 1296{\cdot}11{\cdot}9\,p^3 \approx 207{\cdot}10^3\,p^3$$

$$t_{\text{h}} \approx 42{\cdot}10^{-6}\,p^3;\; n_{\text{s}} \approx 1296(11+9)\,p^2 \approx 26{\cdot}10^3\,p^2.$$

With eight parameters, the 4-fold net requires less than 2 min for the solution of the $[N_{\text{red}}]$ system with 10 368 unknowns. So, even with more parameters, block areas of this size can easily be treated as a representative pilot program for larger nets and the most realistic reduction model can be established along the outlines of Section 2.

4.2 *A hemisphere, 2-fold and 4-fold overlap*

For this overlap pattern typical examples for a 2-fold net would be the AGK2 or AGK3 plate material. A 4-fold coverage of the Southern Hemisphere has been provided recently by the Cape Observatory (Clube 1970), but the smaller field-size ($4° \times 4°$) and the therefore increased number of plates (6000) are different from the present example. An estimate for the required computing effort is given at the end of this section.

(a) 2-*fold overlap*

$$n = 2000\,p;\; m = \text{variable},\; m_{\max} = 74\,p \text{ at the celestial equator};\; l = 0.$$

Due to the convergence of the meridians, the number of plates necessary to cover a RA-belt decreases to the pole, therefore the bandwidth is variable.

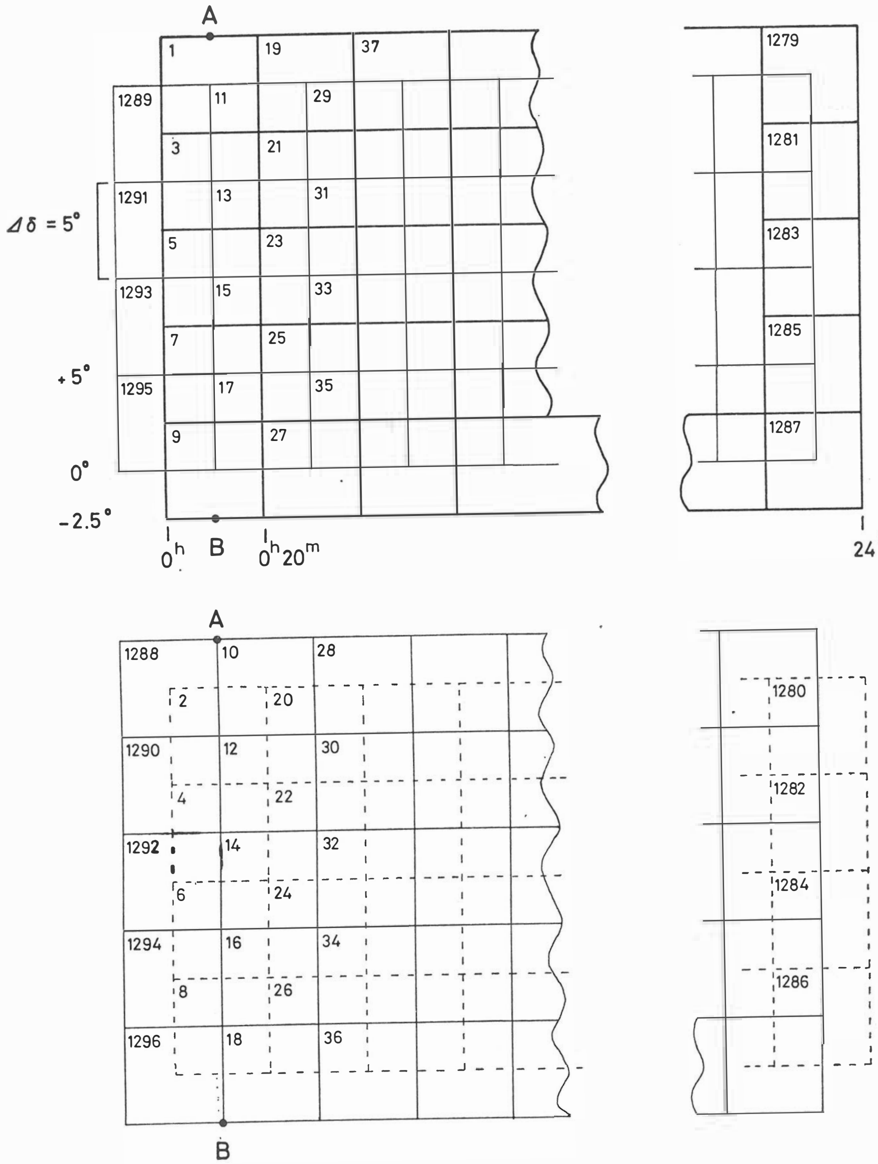

FIG. 4. *Zonal pattern ordering scheme, 4-fold net. Both nets have to be superposed on points AB.*

The adopted ordering scheme is shown in Fig. 5. To obtain an optimal band-structure, the plates have been arranged in a 'spiral' ordering scheme starting from the equator. In contrast to the previous zonal pattern, the borderwidth l is zero. The resulting computing effort is estimated by

$$n_{\text{mult}} \approx 4\cdot10^6\, p^3;\ t_{\text{h}} \approx 0.8\cdot10^{-3}\, p^3;\ n_{\text{s}} \approx 0.12\cdot10^6\, p^2$$

(b) 4-*fold overlap*

$n = 4000\,p$; m = variable, $m_{\max} = 147\,p$ at the celestial equator; $l = 0$

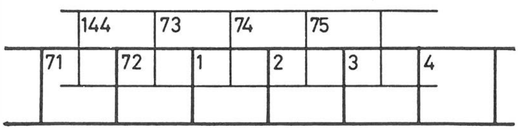

FIG. 5. *Overlap pattern and ordering scheme for a hemisphere, 2-fold net.*

The overlap pattern and ordering scheme for a 4-fold net is shown in Fig. 6. All plates in the same RA-belt are numbered consecutively.

The resulting computing effort is given by

$$n_{\text{mult}} \approx 32 \cdot 10^6\, p^3;\; t_{\text{h}} \approx 6.4 \cdot 10^{-3}\, p^3;\; n_{\text{s}} \approx 0.48 \cdot 10^6\, p^2.$$

Comparing these results with the 2-fold net the computing time is increased by a factor 8 and the necessary external store capacity by a factor 4.

The required computing effort for a blockadjustment of the already quoted 4-fold net of the Cape Observatory is larger, due to increased bandwidth and number of plates.

If t_{h} and n_{s} denote the results from our adopted 4-fold net, one gets the following result:

$$t_{\text{h}}' \approx 2.4 \cdot t_{\text{h}};\; n_{\text{s}}' \approx 1.9\, n_{\text{s}}.$$

Therefore a larger field size, with the same focal length adopted, will reduce the computing effort considerably without loss of accuracy, provided the larger field does not require the introduction of additional parameters in the reduction model. A field size of $5° \times 5°$ and a focal length of about 2 m seems to be the best compromise with respect to present technical limitations in the performance of zone-astrographs.

4.3 *A full sphere, 2-fold and 4-fold overlap*

These overlap patterns are obvious extensions of the previous hemisphere examples.

(a) *2-fold overlap*

Numbering of the plates has to be started at the celestial equator with 2000, decreasing to the North Pole and increasing to the South Pole. The internal plate ordering scheme according to Fig. 5 is preserved. This is equivalent to a pole-to-pole spiral ordering (Brown 1968).

$$n = 4000\, p;\; m = \text{same as 2-fold hemisphere};\; l = 0$$

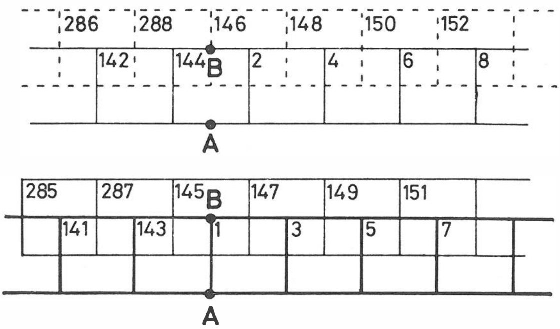

FIG. 6. *Overlap pattern and ordering scheme for a hemisphere, 4-fold net. Both nets have to be superposed on points AB.*

The computing effort is then given by

$$n_{\text{mult}} \approx 8\cdot10^6\, p^3;\ t_{\text{h}} \approx 1.6\cdot10^{-3}\, p^3;\ n_{\text{s}} \approx 0.25\cdot10^6\, p^2$$

(b) 4-*fold overlap*

In analogy to the previous example the numbering here starts at the equator with 4000. The internal ordering scheme is in accordance with Fig. 6.

$$n = 8000\,p;\ m = \text{same as 4-fold hemisphere};\ l = 0$$

The computing effort is given by

$$n_{\text{mult}} \approx 64\cdot10^6\, p^3;\ t_{\text{h}} \approx 13\cdot10^{-3}\, p^3;\ n_{\text{s}} \approx 1\cdot0.10^6\, p^2.$$

Note that in both cases the bandwidth is not increased as compared with the hemisphere.

The following table summarizes the main numerical results of the present section.

TABLE I

Estimation of computing effort for various overlap patterns according to Section 4. Assumed number of plate parameters p = 6

Overlap pattern		Number of plates	Number of unknown plate-constants	Computing effort CDC 7600	External storage [10^6 words]
Zone	2-fold	648	3888	5·0 s	0·23
	4-fold	1296	7776	33·0 s	0·94
Hemisphere	2-fold	2000	12000	11·0 min	4·3
	4-fold	4000	24000	1·4 hr	17·3
Sphere	2-fold	4000	24000	21·0 min	9·0
	4-fold	8000	48000	2·8 hr	36·0

5. APPLICATION OF THE BLOCKADJUSTMENT PROGRAM TO THEORETICAL ACCURACY STUDIES

The computer program as described in the preceding sections has been established with special view to be applied to theoretical accuracy studies, too. In contrast to the field of analytical photogrammetry (Ackermann 1967; Ebner 1970) theoretical accuracy studies of large astrometric nets on a rigorous basis are not available at present.

The main aim of these theoretical studies is to investigate the dependence of the positional accuracy obtained from the blockadjustment on the number, accuracy and distribution of reference stars and the adopted reduction model. This seems to be of considerable importance to detailed plannings of future reference star observations for photographic positional work by transit circles.

As a first step the quoted blockstructures of Section 4 will be investigated under these aspects. Because the program contains a partial inversion routine [2.4] these investigations can be performed by the simulation technique as well as by the inversion method, for details see de Vegt & Ebner (1972). While the simulation technique will determine the mean accuracy of all adjusted star positions with a sufficient high level of confidence, the partial inversion can be used to determine the standard deviations of the positions at selected blockpoints, being of special interest. Here, for instance, the expected maximum standard deviations with respect to the whole blockarea of the adjusted star coordinates can be obtained. As a pilot investigation, accuracy studies of a smaller net (400 plates, $5° \times 5°$ size) will soon be available from a doctoral thesis being carried out at the Hamburg observatory under the supervision of the first author.

6. CONCLUDING REMARKS

The present efficiency of large computer configurations has opened promising aspects, concerning the application of the rigorous blockadjustment solution to very large astrometric nets. From the detailed estimates of computing effort, based on practical experience with the HYCHOL subroutine it is obvious that the adjustment of a hemisphere or sphere could be performed with available computer facilities. The critical point in a program development of this kind is the availability of a powerful subroutine for the solution of banded-bordered systems of equations which has to minimize the required large external storage traffic and computational effort for the execution of this program segment.

The application of blockadjustment techniques in the field of photographic astrometry will only supply its full potential if the treatment of actual astrometric data material is combined with a careful investigation of all questions concerning the selection of the most realistic reduction model for each special case and the associated error-propagation in the block area.

As has been outlined in Section 5 of the present paper, the described reduction program will provide in addition all means for a performance of theoretical accuracy investigations with special emphasis on large nets. The application of those techniques to a variety of widely different blockstructures will help to understand fully both the inherent power and pitfalls of the blockadjustment method.

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