SIMULTANEOUS COMPENSATION OF SYSTEMATIC ERRORS WITH BLOCK ADJUSTMENT BY INDEPENDENT MODELS

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Summary

An advanced concept of block adjustment by independent models is presented, allowing for a simultaneous compensation of certain types of systematic errors of model coordinates. To gain practical experience with this concept a corresponding computer program was written. The test results obtained up to now allow for the following conclusions:

- The practical application of the concept causes no problems.
- The accuracy of adjusted block coordinates is improved considerably (in planimetry up to a factor 3).
- The obtained accuracy corresponds very well with the accuracy as predicted by theory.

PREFACE

In modern aerial triangulation systematic errors are of central importance again. This was so already, years ago when the polynomial methods were introduced into strip and block triangulation. But during the following phase which was characterized by simultaneous least squares adjustment of all bundles or models of a block the interest concentrated on random errors whilst systematic errors were neglected most of the time.

The recent change of thinking was caused by the results of various practical block adjustments which indicate clearly that systematic errors of considerable size are present in photogrammetric data usually [1]. Systematic errors, being not compensated make the accuracy of the individual image coordinates or model coordinates worse and propagate with the adjustment.

The propagation of systematic errors depend on the type of systematic as well as on control distribution, block size and overlap pattern [2], [3]. In some cases the propagation of systematic errors is considerably less favourable than the propagation of random errors. Consequently the accuracy results can be much poorer than expected from theoretical accuracy models for block triangulation, being based on random errors only [4]. Uncompensated systematic errors can cause the following phenomena:

- A reduction of control leads to a higher decrease of accuracy than predicted by theory.
- The accuracy decrease with increasing block size is higher than expected from theory.
- Replacing 20 % sideward overlap by 60 % side lap the accuracy is improved only slightly or even not at all.
- Starting from the same data a block adjustment by independing models gives more accurate results than a bundle block adjustment

(See [2], [3], [4] and [5], [6])

To avoid the unfavourable and sometimes really dangerous effect of systematic errors we have to compensate them. If a certain systematic is known definitely it should be compensated a priori of course. But normally only a small part of the existing systematic can be caught in that way. Much more success can be expected from a concept which compensates the systematic errors by additional parameters of the block adjustment [7]. For that the proper term self calibration is used also [8]. From a correct application of this concept we may expect a considerable improvement of accuracy and a much better correspondence with theoretical accuracy predictions.

THE CONCEPT FOR COMPENSATION OF SYSTEMATIC ERRORS

A suitable mathematical model

For the compensation of the expected systematic errors we put up additional parameters of proper type. In the adjustment we treat these parameters as random variables with appropriate weights [9], [10]. This approach has two essential advantages

1. It is fully general and leads to optimal accuracy results. Random variables (or observations) are the general case of parameters. Free unknowns as well as constants are special cases of observations and can be represented by weight zero and infinite weight respectively. In aerial triangulation the systematic errors normally are rather small. Most of the time their effect on image coordinates or model coordinates is in the order of the random coordinate accuracy. Therefore it is adequate and optimal with respect to accuracy to treat the additional parameters as observations with appropriate weights and not as free unknowns.

2. Additional parameters put up as free unknowns can cause serious numerical problems. If some of the parameters are highly corre-

lated with each other the normal equations become ill conditioned. These problems are reduced considerably when the additional parameters are treated as observations with proper weights. In this case we even are allowed to introduce different parameters of identic type. Then one parameter can be common to all models of the block for instance and other parameters of equal type can be put up individually for different strips.

The adjustment can be formulated in different ways. These formulations are equivalent theoretically but they lead to normal equations of different structure and condition [10]. If the additional parameters generally are common to groups of photos or models (to whole strips for instance) the following formulation of the block adjustment is suitable.

$$v_1 = Ax + By - f \qquad (1a)$$
$$v_2 = \qquad Iy - s \qquad (1b)$$

f = vector of observations (including the constant term)
v_1 = vector of residuals belonging to f
s = vector of additional observations
v_2 = vector of residuals belonging to s
x = vector of unknowns
A = coefficient matrix belonging to x
y = vector of additional unknowns
B = coefficient matrix belonging to y
I = unit matrix

In equation (1a) the additional parameters are put up as unknowns. Equation (1b) expresses that these unknowns are observed. Usually the additional observations s will be zero. But if some of the additional parameters are known from calibrations the corresponding amounts can be introduced into (1b).

Equations (1a) and (1b) represent the functional model. The associated stochastic model is given by the weight coefficient matrix G of the common observation vector $[f \quad s]^T$:

$$G = \begin{bmatrix} G_{ff} & G_{fs} \\ G_{fs}^T & G_{ss} \end{bmatrix} \qquad (2)$$

G_{ff} = weight coefficient matrix of the observations f
G_{ss} = weight coefficient matrix of the additional observations s

The existence of the submatrix G_{fs} points out that f and s may be correlated with each other. In practice however G_{fs} will be zero and G_{ff} as well as G_{ss} usually will be chosen as diagonal matrices. The question of a proper choice of the weights of the additional observations s is treated in detail in the chapter on test results.

The formulation of the adjustment according to (1) and (2) leads to a banded bordered normal equation matrix. Most of the time the band width will be greater than the border width, given by the total number of additional parameters. Therefore the computing time normally will not be very much longer than without such a simultaneous compensation of systematic errors.

The formulation presented here fits into the approach of Generalized Least Squares [11]. This approach itself is related to the concept of Bayesian Estimation [12]. Furthermore it can be shown that the present formulation according to equations (1) and (2) fits into the mathematical model of Least Squares Collocation if we set s = 0 (additional observations of amount zero) and $G_{fs} = 0$ (no correlations between the observations f and s) [13]. This simplification is realistic because in practical applications usually s will be zero and G_{fs} will be neglected. Considering this and converting equations (1) properly we obtain:

$$Ax - v_1 + Bv_2 = f \tag{3}$$

$$G = \begin{bmatrix} G_{ff} & \\ & G_{ss} \end{bmatrix} \tag{4}$$

Ax = trend

$-v_1$ = noise

Bv_2 = signal

G_{ff} = weight coefficient matrix refering to noise

$BG_{ss}B^T$ = weight coefficient matrix refering to signal

Realization in case of independent model block adjustment

As the basic method for block adjustment by independent models we choose the planimetry height iteration used in the PAT-M43 program [14]. Concerning the additional parameters we suppose that the systematic deformations are common to a certain group of models at times but change from group to group. In addition some systematic can be common to all models. These assumptions have been proven as very realistic [1]. With the formulation of equal deformations for different models we presume that those models have the same base length and the same direction of the base line approximately. Moreover the flying height shall not vary too much. However, a further problem appears resulting from the fact that the coordinate origin is arbitrary for each model. The same formulation $\Delta x = axy$, $\Delta y = 0$ for instance leads to different model deformations, depending on the origin of x (see figure 1).

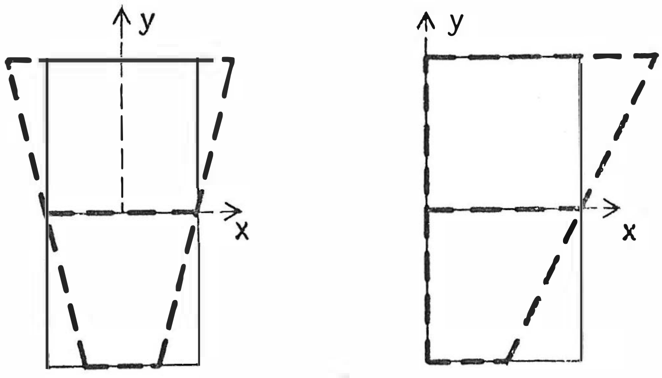

Figure 1

This problem doesn't appear in bundle block adjustment where the origin of each image is well defined by the centre point. To solve the problem also in case of independent models we search for parameters whose effects are not changed by shifts of the coordinate system in x and y direction. This condition leads to 4 planimetric parameters e, f, p, q and to 6 height parameters r, s, t, u, v, w which contribute to the observational equations for planimetry and height as follows:

Planimetric block adjustment

$$\Delta x = ex + fy + p(x^2-y^2) + q2xy$$
$$\Delta y = -ey + fx + p2xy + q(y^2-x^2)$$
for model points (5)

Height block adjustment

$$\Delta z = rx^2 + sy^2 + txy$$
for model points

$$\Delta x = -r2xz - tyz + (-)u$$
$$\Delta y = -s2yz - txz + (-)v$$
$$\Delta z = rx^2+sy^2 + txy + (-)w$$
for left hand side (right hand side) perspective centres (6)

In equations (5) and (6) the additional parameters are treated as unknowns. At the same time for each of those parameters a new observational equation is put up according to (1b).

The parameters e and f allow for a compensation of affine deformations of the planimetric model coordinates. The parameters p and q are the only one parameters of degree 2 whose effects are independent of coordinate shifts in x and y direction. They also appear in conformal polynomial strip adjustment [15]. The influence of e, f, p and q over the model coordinates x and y is plotted in figure 2.

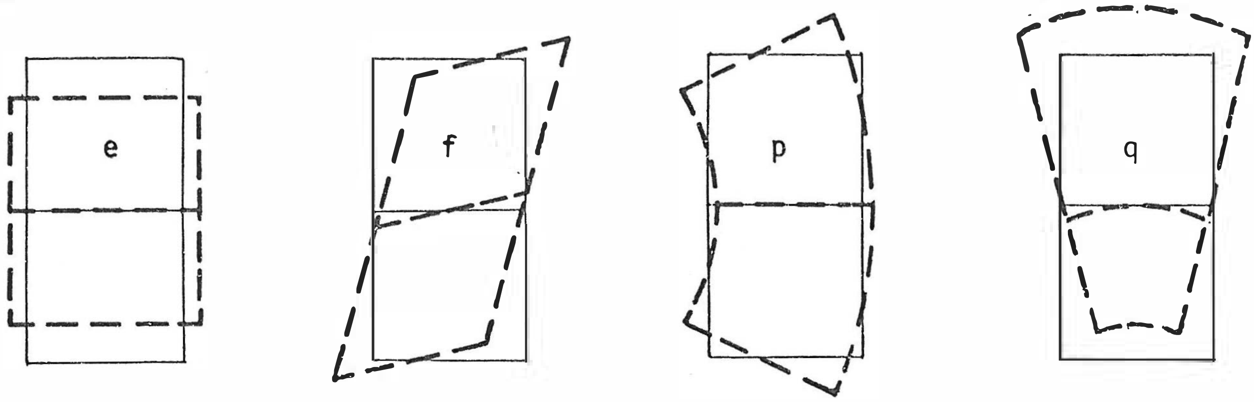

Figure 2

The parameters r and s compensate for second degree z deformations in x and y direction. The parameter t corrects for twisted models. Moreover r, s and t have influence over the perspective centre coordinates too. The formulation according to equations (6) guarantees that the effects of r, s and t are not changed by coordinate shifts in x and y direction. A plot of the corresponding effects is given in figure 3.

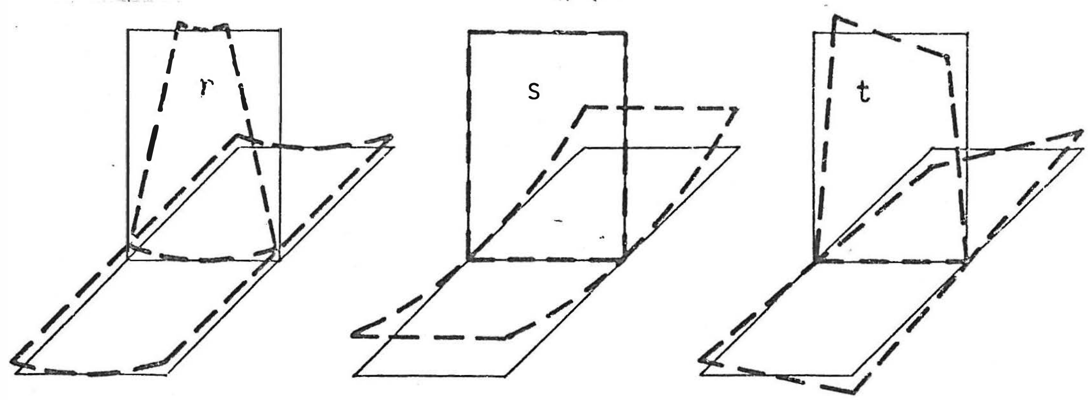

Figure 3

The parameters u, v and w put up also in equations (6) compensate for systematic errors of perspective centre coordinates. At the same time u, v and w are able to correct for deformations of perspective centres being caused eventually by the parameters r, s and t.

The nonlinear spatial block adjustment is started by 2 succeeding iteration steps using the computer program PAT-M43. With that we have proper initial values for the following iteration step with additional parameters. This step being split again into planimetry and height is performed only once normally. In that way we save computing time without loosing accuracy.

Before the planimetric block adjustment with additional parameters is performed the individual models and the control points are transformed to their centres of gravity in x and y. In accordance with the computer program PAT-M43 the perspective centres are not used in the planimetric block adjustment. With the following rigorous spatial transformation of the individual models the additional parameters e, f, p and q have no influence on the model heights and on the perspective centre coordinates.

Before the height block adjustment with additional parameters is executed the photogrammetric models are transformed to their centre of gravity in x, y and z. Corresponding with the program PAT-M43 the observational equations belonging to the planimetric model coordinates are omitted in the height block adjustment. With the following rigorous spatial transformation of the individual models the additional parameters r, s and t have no influence on the planimetric model coordinates.

The preliminary computer program

To gain practical experience with the concept suggested a preliminary computer program was written by the second author. This program allows for a rigorous block adjustment with additional parameters according to formulae (1) and (2) and is fully operational. The additional parameters as defined by equations (5) and (6) may be common to any group of models or/and to all models of the block. The weight of each of the additional parameters can be varied separately in a range between zero and infinite. The program is capable to adjust practical blocks of medium size with a reasonable computing time.

At a later time this program shall be replaced by an extended version of the PAT-M package.

TEST RESULTS

The practical tests were performed to get answers to the following questions

- For which models shall be put up common additional parameters and which weights shall be used ?
- Which accuracy improvement can be attained by an extended block adjustment with additional parameters?
- Is the accuracy obtained in agreement with the corresponding theoretical accuracy predictions ?

The test material

For the practical tests the data of the OEEPE project Oberschwaben could be used. From the comprehensive material of this project we selected a test block consisting of strips 5, 7, 9 and 11 of the block Frankfurt (for more details see [16]). Figure 4a shows the test block with all available control points. The project data of the test block are represented in table 1.

block size	20.0 km x 62.5 km
camera used	Zeiss RMK A 15/23
photo scale	1:28.000
forward overlap	60 %
sidewardoverlap	20 %
number of strips	4
number of models	100
number of model points	1662
number of control points available	258

All control points and tie points were signalized.
The image coordinates were measured by a Zeiss PSK stereo comparator. The independent models were built computationally.

Table 1

Planimetric results

In planimetry perimeter control was used exclusively and the distance between control points was varied. The different control distributions used are represented in figure 4 b. To the questions raised at the beginning of the chapter the following answers can be given.

Common additional parameters and proper weights

Most of the time the models belonging to the same flight strip and measured with the same instrument will show very similar deformations. Moreover strips flown in the same direction often are deformed quite similarly [1]. In this context it is important to know that the effect of the additional parameters e and f is independent of the flight direction whilst the effect of the parameters p and q changes in case of a turn of 180^0.

Paying regard to this at the beginning each strip was given its own set of additional parameters e, f, p and q. In the adjustment to each of those parameters the associated standard deviation σ is computed too. Considering these σ values we have learned that the parameters p and q are extremely well determined (very small σ)

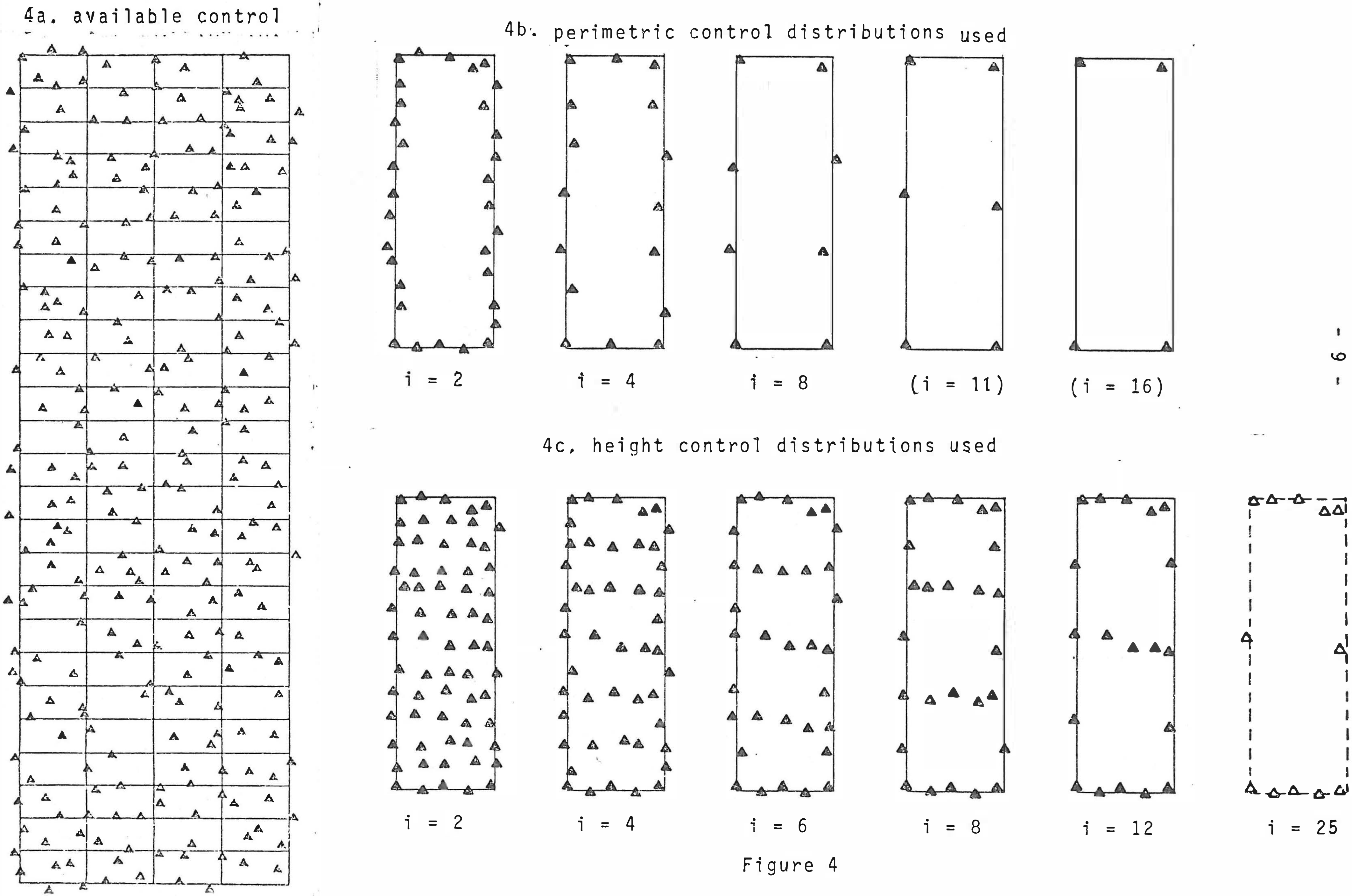
4a. available control
4b. perimetric control distributions used
i = 2
i = 4
i = 8
(i = 11)
(i = 16)
4c, height control distributions used
i = 2
i = 4
i = 6
i = 8
i = 12
i = 25

Figure 4

even if 4 control points are used only. Unfortunately the determination of the affinity terms e and f is much poorer and depends strongly on the control distribution. When 4 control points are used only the standard deviations are in the order of the amounts o the parameters. If the affinity terms e and f are common to all models of the block their determination is much better (considerabl reduced σ). Respecting these facts we recommend to put up both common and individual parameters e and f but with different weights For the parameters being common to all models relatively small weights should be used to allow for a complete compensation of the common deformation (for proper weights see later on). For the parameters e and f being common to one individual strip each we suggest to use hight weights, corresponding to an effect over the model coordinates in the order of 1 μm for instance. These assumptions ar adequate if the affine deformation is more or less constant for all models of the block. Very often this will be valid. But if some of the strips have deformations e and f differing significantl from the common onesthey will show up inspite of the high weights introduced. In this case of course the adjustment must be repeated with better (smaller) weights for the parameters e and f of those strips.

Following these suggestions we have found that the individual affinity terms belonging to the 4 strips of our test block are very small. In the further runs of the test we therefore put up common parameter e and f only. The second question in context with the additional parameters concerns the choice of proper weights. Here it was found that the amounts of the additional parameters being computed in the block adjustment are only slightly dependent on their weights. This is true also in case of poor control distributions. An example is given in table 2. The results are related to a block adjustment with 4 control points which is the most critical case. For the model coordinates weight 1 is assumed.

Weight zero used for all additional parameters

common parameters		parameters of strips 1 to 4	
e = -31(13)	f = -53(13)	p_1 = 0(1)	q_1 = 25(1)
		p_2 =-3(1)	q_2 = 18(1)
μ_{xy} = 0.377 m		p_3 = 8(1)	q_3 =-25(1)
		p_4 =-3(2)	q_4 =-32(2)

Weights roughly adapted to the amounts of the additional parameters (appropriate weights)

common parameters		parameters of strips 1 to 4	
$e = -30(12)$	$f = -51(12)$	$p_1 = 0(1)$	$q_1 = 25(1)$
		$p_2 = -3(1)$	$q_2 = 18(1)$
$\mu_{xy} = 0.375$ m		$p_3 = 7(1)$	$q_3 = -25(1)$
		$p_4 = -3(2)$	$q_4 = -32(2)$

Table 2

The unit of the amounts of the additional parameters represented in table 2 is 10^{-6}. They are related to model coordinates with the dimension km in the terrain. Table 2 confirms that the chosen weight have only a small influence on the amounts of the additional parameters. As further runs have proven this is true even when weights are used which are somewhat higher than the so called appropriate weights which correspond to the amounts of the additional parameters. Moreover table 2 shows that the RMS value μ_{xy} of the coordinate errors at check points improves slightly when adequate weights are used. Paying regard to these results it can be recommended to choose the weights of the additional parameters according to their expected amounts or somewhat smaller. With that the accuracy is optimized and problems with respect to the condition of the normal equation matrix are avoided.

Regarding the amounts of the additional parameters being represente in table 2 the following can be said. The affinity terms $e = -30$ and $f = -51$ correspond to maximum model deformations of 2.7 μm and 4.6 μm respectively (related to the photo scale). The terms p are rather small, even $p_3 = 8$ corresponds to 1.8 μm only (at the maximum). In contrast to p the parameters q are rather large. The change of sign from q_1, q_2 to q_3, q_4 is in agreement with the change of flight direction. The largest amount $q_4 = -32$ corresponds to a maximum model deformation of 7.1 μm. A positive sign of the additional parameters corresponds to the deformations represented in figure 2. The results from table 2 are in agreement with the model deformations obtained in [1].

Accuracy improvement by additional parameters

Using the control distributions represented in figure 4 b the test block was adjusted without and with additional parameters. The affinity terms were put up common to all models but individual parameters p and q were used for each strip. The corresponding results are represented in table 3. The accuracies are related to the photo scale.

control version	control points	check points	without add. param.		with add. param.		accuracy ratios	
			σ_0 [μm]	μ_{xy} [μm]	σ_0 [μm]	μ_{xy} [μm]	σ_0	μ_{xy}
i=2	32	226	6.8	9.9	4.3	6.3	1.6	1.6
i=4	16	242	6.5	13.4	4.2	6.6	1.5	2.0
i=8	8	250	6.2	20.0	4.2	7.4	1.5	2.7
(i=11)	6	252	6.1	22.1	4.2	7.3	1.5	3.0
(i=16)	4	254	5.9	32.4	4.2	13.5	1.4	2.4

Table 3

Let us start the discussion with σ_0 representing the random accuracy of model coordinates. Without additional parameters σ_0 depends significantly on the control distribution used. This is in disagreement with theory. When additional parameters are introduced into the block adjustment σ_0 becomes considerably smaller (at a factor 1.4 to 1.6) and the dependency on control distribution disappears. With 4.2 μm sigma nought is very close to the noise limit we can expect from photogrammetry today. Even more important is the comparison of the absolute accuracies expressed by μ_{xy}, the RMS value of the coordinate errors at check points. We see that the additional parameters improve the accuracy the more the poorer the control distribution is. The improvement increases up to a factor 3.0 in case of 6 control points used. In figure 5 the corresponding results are represented graphically.

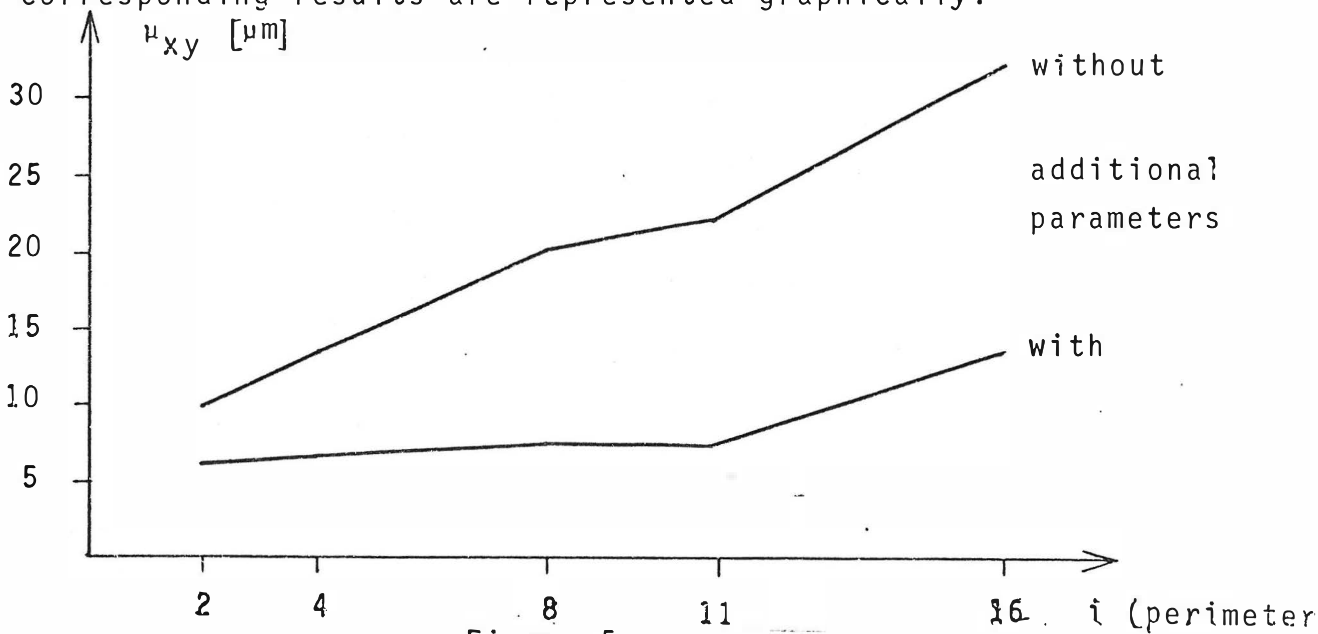

Figure 5

The test shows that absolute accuracies of about 7 μm at the photo scale can be realized today, even when the control spacing along the block perimeter is in the order of 4 to 8 base length. If we put this accuracy of 7 μm ≙ 20 cm in relation to the length of the block (62.5 km) we obtain a relative accuracy which is better than 1:300.000.

Comparison with theory

Now a comparison is made between the accuracy obtained by block adjustment with additional parameters and the corresponding theoretical accuracy being based on random errors only [4]. However, to allow for a correct comparison we have to consider that the check points used in the test are not errorfree as assumed by theory. Therefore the theoretical accuracy figures obtained from [4] are superposed by the random accuracy of check points which we assume with 10 cm in the terrain. This assumption can be considered as realistic. The result of the comparison is given graphically in figure 6.

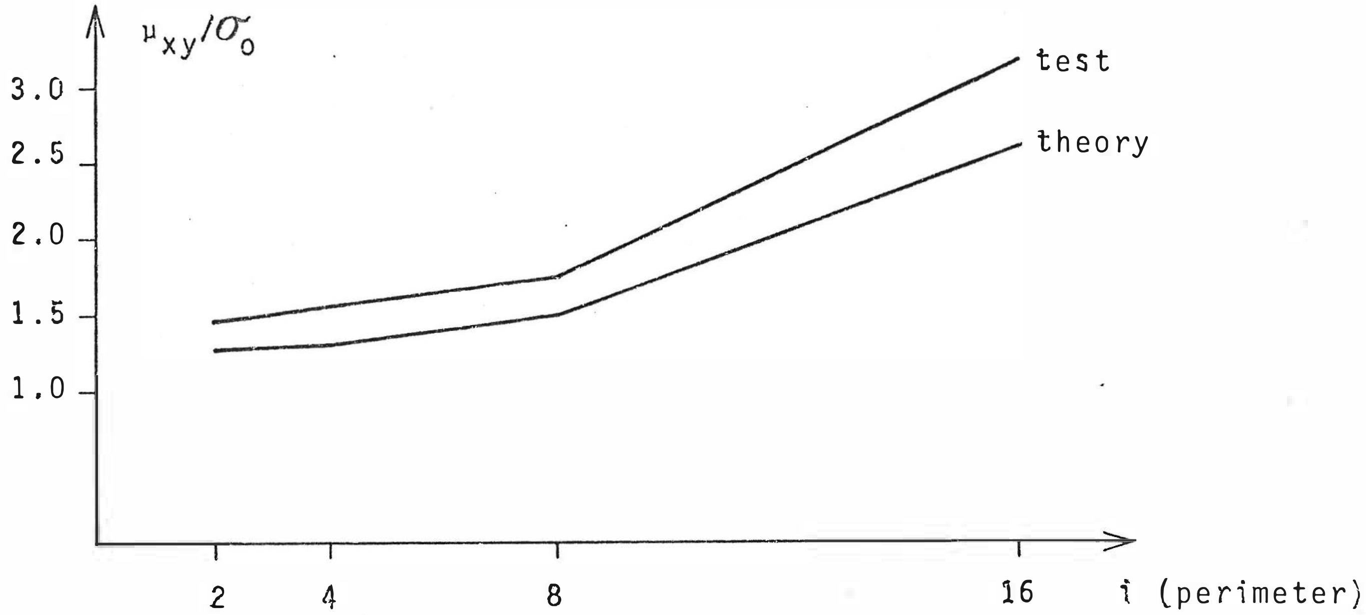

Figure 6

Figure 6 shows that the accuracy obtained in the test is close to the accuracy as predicted by theory. The discrepancies are less than 20 % and can be explained by the facts that one test is just one sample and that the test doesn't meet the premises of the theory rigorously (different block shape for instance).

Considering this we can say that the accuracy results of the test are in agreement with the corresponding theoretical predictions. This agreement is most important because it indicates that the existing systematic errors are compensated very well by the additional parameters used and that the remaining errors can be considered as random.

Comment on height

The corresponding investigation on height block adjustment with additional parameters is still at work. The test will be based on the control distributions represented in figure 4 c.

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