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Processing of Very Large Networks in Photogrammetry and Photographic Astrometry (*)

DR. HEINRICH EBNER

Institute of Photogrammetry – University of Stuttgart

Summary. — The paper gives a rewiew on the use of very large networks in photogrammetry and photographic astrometry.

A general photogrammetric program package for aerial triangulation called PAT and the basic routine HYCHOL for the solution of linear equation systems of any size are discussed. The largest network adjusted up to now was containing 2 193 photogrammetric models, 10 297 terrain points and 29 366 unknowns.

A similar computer program for astrometric applications, being based again on the routine HYCHOL is under development. It will be applied first to a network covering whole the northern celestial hemisphere. This network will contain 180 000 stars and 390 000 unknowns approximately.

ELABORAZIONE DI RETI MOLTO GRANDI IN FOTOGRAMMETRIA E ASTROMETRIA FOTOGRAFICA.

Sommario. — L'articolo tratta dell' impiego di reti molto grandi in fotogrammetria e astrometria fotografica.

Vengono discussi un insieme generale di programmi fotogrammetrici per la triangolazione aerea, chiamato PAT, e il sottoprogramma fondamentale HYCHOL per la soluzione di sistemi lineari di equazioni di qualsiasi dimensione. La più grande rete della quale finora è stata fatta la compensazione conteneva 2.193 modelli fotografici, 10.297 punti di appoggio a terra e 29.366 incognite.

È in corso di sviluppo un analogo programma di calcolo per applicazioni astrometriche basato sempre sul sottoprogramma HYCHOL. Sarà applicato dapprima ad una rete che copre l' intero emisfero celeste settentrionale. Questa rete conterrà approssimativamente 180.000 stelle e 390.000 incognite.

L'ÉLABORATION DES RÉSEAUX TRÈS GRANDS DANS LA PHOTOGRAMMÉTRIE ET L'ASTROMÉTRIE PHOTOGRAPHIQUE.

Résumé. — L'étude traite de l'emploi des réseaux très grands dans la photogrammétrie et l'astrométrie photographique.

On discute un ensemble général de programmes photogrammétriques pour la triangulation aérienne, appelé PAT, et le sous-programme fondamental HYCHOL pour la solution des systèmes linéaires d'équations de dimension quelconque. Le plus grand réseau duquel on a jusqu'à présent

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fait la compensation contenait 2.193 modèles photogrammétriques, 10.297 points d'appui à terre et 29.366 inconnues.

Un programme de calcul similaire pour applications astrométriques, encore fondé sur le sous-programme HYCHOL, est en cours de développement. On l'appliquera d'abord à un réseau couvrant tout l'hémisphère céleste septentrional. Ce réseau contiendra à peu près 180.000 étoiles et 390.000 inconnues.

AUSGLEICHUNG SEHR GROSSER NETZE IN DER PHOTOGRAMMETRIE UND IN DER PHOTOGRAPHISCHEN ASTROMETRIE.

Zusammenfassung. — Der Artikel gibt einen Überblick über die sehr grosser Netze in der Photogrammetrie und in der photographischen Astrometrie.

Ein allgemeines photogrammetrisches Programm-Paket PAT für die Aerotriangulation und die zugehörige Basis-Routine HYCHOL für die Lösung linearer Gleichungssysteme beliebiger Grösse werden diskutiert. Die bisher grösste Ausgleichung enthielt 2193 photogrammetrische Modelle, 10297 Geländepunkte und 29366 Unbekannte.

Ein ähnliches Rechenprogramm für astrometrische Ausgleichungen, das ebenfalls auf der Routine HYCHOL aufbaut, ist gegenwärtig in Entwicklung. Es soll zuerst auf ein Netz angewendet werden, das die gesamte nördliche Himmelskugel überdeckt. Dieses Netz wird etwa 180 000 Sterne und 390 000 Unbekannte enthalten.

ELABORACIÓN DE REDES MUY GRANDES EN FOTOGRAMETRÍA Y ASTROMETRÍA FOTOGRÁFICA.

Sumario. — El artículo trata del empleo de redes muy grandes en fotogrametría y astrometría fotográfica.

Se debaten un conjunto general de programas fotogramétricos para la triangulación aérea, llamado PAT, y el subprograma fundamental HYCHOL para la solución de sistemas lineales de ecuaciones de cualsiquiera dimensión. La más grande red de la que hasta hoy se haya efectuado la compensación comprendía 2.193 modelos fotográficos, 10.297 puntos de apoyo en tierra y 29.366 incógnitas.

Se encuentra en curso de desarrollo un análogo programa de cálculo para aplicaciones astrométricas fundado siempre sobre el subprograma HYCHOL. Se aplicará antes a una red que cubre el entero emisferio celeste septentrional. Esta red comprenderá approximadamente 180.000 estrellas y 390.000 incógnitas

1. — Today very large networks are applied not only in Geodesy but in photogrammetry and photographic astrometry too. As opposed to Geodesy those networks are not determined absolutely but require a certain number of predetermined reference points. In photogrammetry such points are called control points. In photographic astrometry the name reference stars is used. Processing of photogrammetric and astrometric networks therefore may be compared with the break down of geodetic networks.

2. — Photogrammetric networks (or blocks) are based on highly accurate overlapping photographs of the terrain. Each terrain point to be determined is imaged at two or more photos. With the processing of the network either of two efficient methods are applied today.

2.1 — The bundle block triangulation starts directly from the image coordinates of the terrain points measured by comparators. The image coordinates are treated as observations in the adjustment procedure. Unknowns are the x, y, z coordinates of the terrain points and the transformation parameters of the image bundles. These are 3 rotations and 3 translations per each bundle.

2.2 — With independent model block triangulation photogrammetric models are produced from overlapping photographs computationally or by using a stereo plotter instrument. Here the x, y, z model coordinates are treated as observations. Unknowns are the x, y, z coordinates of terrain points and the transformation parameters of the models. Because a model is transformable into the terrain system by a three dimensional similarity transformation here 3 rotations, 3 translations, and a scale factor are used for each model.

2.3 — The accuracy properties of photogrammetric block triangulation are very favourable so far as planimetry is concerned. By extensive theoretical investigations it was proved that control points along the block perimeter are sufficient. Additional control points inside the block improve the planimetric accuracy only by a few percent [1]. The most favourable error propagation is obtained with the so called dense perimeter control, where control points are used every two models along the perimeter. Here the root mean square value of the standard deviations of the adjusted planimetric coordinates increases only with the logarithm of the number of models used [2].

With respect to heights the error propagation of blocks is less promising. A satisfying accuracy can only be obtained from a rather dense grid of height control points covering the whole block [2].

2.4 — Presently only a few powerful computer programs for photogrammetric block triangulation are existing (for instance [3]). The only one general system applicable to both triangulation methods mentioned above was developed at the

Institute of Photogrammetry at Stuttgart University between 1969 and 1972 [4], [5]. The system which is called PAT is practically unlimited as far as the number of photos (the terrain area respectively), the number of points and the overlap pattern is concerned.

2.5 — The basis of the package PAT is a highly optimized routine for a direct solution of linear symmetric and positive definite equation systems of any size. The coefficient matrix and the right hand side matrix (many right hand side vectors are allowed at the same time) are considered as hyper matrices and divided into submatrices of arbitrary size. The solution procedure is a generalization of the well known Cholesky method applied to those submatrices. Therefore we called the routine HYCHOL = Hyper Cholesky [6]. At the beginning all submatrices are stored on external memory. During the Cholesky decomposition and the back substitution process the corresponding submatrices are transferred to the central memory for transformation after which they are stored again externally. At the end the submatrices of the solution matrix (or solution vector in case of one right hand side only) are available at the external memory. During the whole procedure at the maximum three submatrices are stored in the central memory. So the capacity of the central memory doesn't restrict the size of the equation system.

2.6 — With the system PAT approximately a hundred practical block adjustments were performed up to now. The largest network was the canadian block COED [7]. Here aerial triangulation was used for the determination of minor control points which are necessary for the photogrammetric plotting of the 1 : 50 000 map of Canada. The main data of the project COED are :

block area	97 000 km²
photo scale	1: 60 000
photogrammetric models	2 193
terrain points	10 297
observations	49 754
unknowns	29 366
CPU time at Control Data CDC 6600	2 937 sec

When aerial triangulation is applied to small scale mapping a planimetric accuracy of some meters is sufficient. With COED, where only 12 planimetric control points at distances of about 130 km were used the mean coordinate accuracy at 185 planimetric check points was 11 m approximately. Here, it should be mentioned that the accuracy of the check points themselves was poor (in the order of 7 m in x and in y).

With larger photo scales and using signalized terrain points the planimetric accuracy of blocks can be improved up to a few cm [8]. In Stuttgart presently we are working on a block adjustment experiment for the photogrammetric break down of the third order network in the state of Baden-Württemberg. Applying four-fold photo-coverage and a photo scale of 1 : 8000 we expect a mean coordinate accuracy of 3 cm in planimetry.

3. — Photographic astrometry is a method for the determination of star positions α, δ from photographic images. The cameras used are called astrographs. Usually the focal length is 2 m and photographic plates are used. The image coordinates are measured by comparators. The accuracy in the image plane is in the order of 2 μm in x and y which corresponds to 0″2.

In the past for each single plate a sufficient number of reference stars, determined by meridian circles, was used. The more modern method is the simultaneous adjustment of a network of overlapping plates. Usually each star to be determined is imaged at two or more plates. In that way the accuracy can be improved and a lot of reference stars can be saved. The method became known under the name plate overlap technique. Recently the name block adjustment is used as in photogrammetry [9].

3.1 — In the adjustment procedure the image coordinates are treated as observations. Unknowns are the star positions α, δ and the transformation parameters of the plates. The type of transformation used depends on the geometric quality of the astrograph mainly.

3.2 — Most favourable accuracy properties can be expected from simultaneous adjustments of networks covering a hemisphere or even the whole sphere. Theoretical investigations have shown that here a positional accuracy of 0″1 can be obtained using a small number of reference stars only or even no reference stars at all [10], [11].

3.3 — Presently a general computer program for the rigorous adjustment of astrometric networks is under development. This is a cooperation between the Hamburg Observatory and the Stuttgart Institute of Photogrammetry [12]. The basis of the system again is the routine HYCHOL for the solution of the big normal equation systems. The computer program will allow for different network structures and overlap patterns as well as for different mathematical models of the plate parameters.

3.4 — As soon as the computer program will be finished it may be applied first to a network covering the whole northern hemisphere. The plates and the necessary reference stars are available from the AGK2/AGK3 catalogue material. Of course this leads to an extremely large adjustment problem. Therefore the fastest computers available today are required. The corresponding project data are :

plates	1 939
exposures	3 878
stars	180 000
observations	1 440 000
unknowns	390 000
estimated CPU time	
Control Data CDC 6600	18 hours
Control Data Cyber 76	3 hours

Later a new network of plates covering the whole sphere is planned. The photography of the southern hemisphere is finished already. From the rigorous adjustment of this network we expect a positional accuracy of 0″1 as predicted by theory.

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