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Comparison of different methods of block adjustment

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1. Scope and state of the empirical investigations

The test Oberschwaben was started to investigate the accuracy of strip and block triangulation, obtainable under practical conditions. The accuracy of blocks depends on the block parameters and on the method used for block adjustment. In the test Oberschwaben the polynomial block adjustment, the block adjustment by independent models and the bundle block adjustment are investigated.

The extensive empirical results of such a test may be used in two different ways. First they give the possibility to check theoretical accuracy models for block triangulation existing so far and secondly they allow for the determination of accuracy relations of the different methods of adjustment, based exactly on the same practical material.

The topic of my lecture is the presentation of the results obtained by polynomial block adjustment and by bundle block adjustment and the comparison with the corresponding results of independent model block adjustment, presented by Prof. Ackermann this morning.

As a result of the great computational demand up to now only the block Frankfurt (wide angle, 20 % side lap) was adjusted by the polynomial method as well as by the bundle method. With this block the effect of the control distribution on the accuracy was investigated completely. Against that the accuracy dependency on the block size is'nt studied yet.

For that reasons only an intermediate report without a critical statistical valuation can be given presently. However already the results obtained up to now allow for some important conclusions and indicate very clearly in which direction further research and development activities are necessary.

2. Control point distributions

In accordance with the results of preceding theoretical investigations on most favourable control point distributions for planimetry and height the types, presented in figure 1 were used [1] . The only one degree of freedom is the bridging distance i expressed in units of the base length.

To allow for a valid comparison of the different methods of block adjustment not only the same types but even exactly the same control points were used with polynomials, independent models and bundles.

3. Polynomial block adjustment

The polynomial block adjustments were performed at the IfaG Institute in Frankfurt, using a Telefunken TR 440 computer. For that I want to thank Prof. Förstner and Mr. Nüßlein very much. The necessary strip formations started from the cleaned up model coordinates which had already been used in the independent model block adjustments.

For the polynomial adjustment Dr. Schut's iterative program, version 1966 was used [2]. The basic transformation is conformal in x and y. In planimetry as well as in height second degree polynomials were chosen. According to the independent model block adjustments weight 1 was given to the photogrammetric coordinates of tie points and of control points.

The results obtained with the five different control versions are represented in table 1. In figure 2 and figure 3 they are plotted, together with the corresponding results of the block adjustment by independent models. Table 2 contains the accuracy ratios between polynomials and independent models.

In planimetry the degree of inferiority of the polynomial results depends strongly on the number of control points used. With a dense perimeter control the ratio is 2.13. If only 4 control points are available the ratio decreases to 1.36.

In height the polynomial results are inferior too, but there is practicly no dependency on the control density. The accuracy ratio varies between 1.66 and 1.85 only.

Further computations have to show whether the accuracy of the polynomial block adjustments can be improved by different weight assumptions or by third degree polynomials.

Valuating the results two aspects have to be distinguished. On the one hand the test confirms that quite good results can be obtained by polynomial block adjustment. On the other hand the improvement of the results by application of block adjustment with independent models, being a different computational treatment only is really remarkable.

4. Bundle block adjustment

The bundle block adjustments were performed at the Institute of Photogrammetry in Stuttgart, using a Control Data CDC 6600 computer and the program PAT-B [3]. Previously the original, image coordinates were cleaned up with the same care as the model coordinates before. The weight 1 was used for all image coordinates. In accordance with the adjustment by independent models the control points were treated as errorfree. For the performance of the bundle block adjustments I want to thank Mr. Schneider heartily.

The results obtained are shown in table 3. Table 4 represents the corresponding theoretical accuracy models for bundle block triangulation. They have been derived very recently and are based on random errors only. Figure 4 and figure 5 give a comparison of the theoretical and empirical results. There is a strong disagreement between theory and test. It indicates the existence of systematic image errors being not compensated by the bundle block adjustment.

In figure 6 and figure 7 the results of the bundle block adjustments are plotted, together with the corresponding results of the block adjustments by independent models. Table 5 contains the accuracy ratios of both methods.

Here we have the most surprising results of the test: independent models give a better accuracy than bundles. The accuracy ratio varies between 1.04 and 1.35 in planimetry and between 1.17 and 1.41 in height. The denser the control the more the results of independent models are superior.

This comparison indicates that, in spite of it's principial generality the bundle block adjustment is more sentive against systematic errors than the block adjustment by independent models.

5. Conclusion

Starting from the same practical material of the block Frankfurt the highest accuracy after block adjustment was obtained by the method of independent models. With that the high efficiency of the model block adjustment is demonstrated under practical conditions.

Taking an average the bundle results are worse at 30 % and the polynomial results at 70 % approximately. Whilst the inferiority of the polynomial block adjustment was expected, the worse results of the bundle block adjustment are most surprising. Responsible for that fact seem to be the systematic errors, being present in the material. This systematic errors obviously impair the bundle block adjustment more than the block adjustment by independent models.

The test Oberschwaben demonstrates, that systematic errors can have more influence than we expected; even in accurate material. Concerning further research and developments from there it follows that we have to take into account rigorously not only random errors but systematic errors too. The most favourable way doing this today is the introduction of additional parameters into the block adjustment to compensate for the systematic errors as far as possible [4]. This concept may be applied to the bundle block adjustment as well as to the block adjustment by independent models.

The proper selection of the type and number of those parameters will become one of the topics of further research activities. Based on the results of this research general programs for block adjustment with additional parameters can and should be developed. Applying them in practive we may expect to meet the accuracy results as predicted by the theoretical investigations.

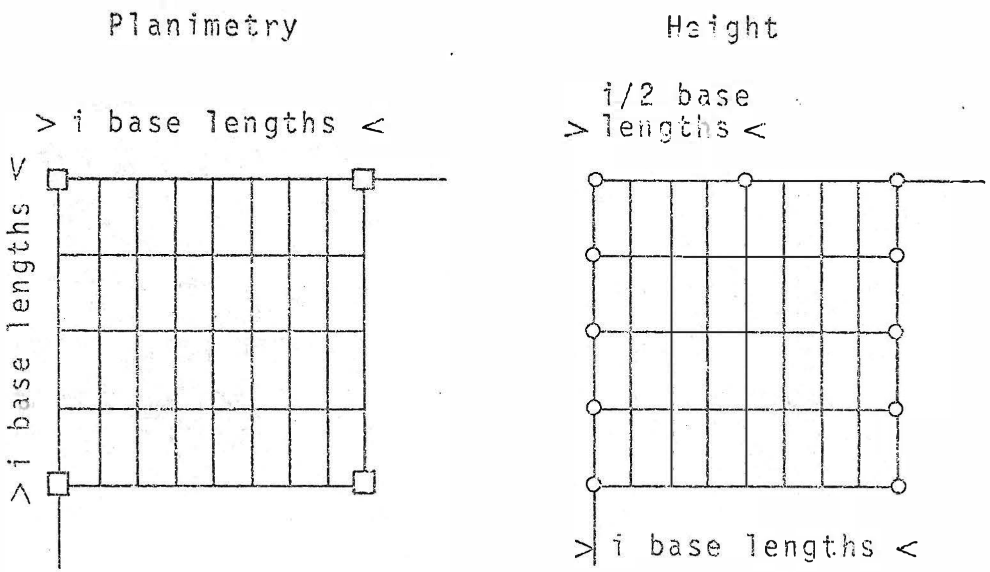
References

[1] Ebner, H.: Theoretical accuracy models for block triangulation. Invited paper of Commission III, 12th Congress of the International Society of Photogrammetry, Ottawa. BuL, 214 - 221, 1972.

[2] Schut, G.H.: A fortran program for the adjustment of strips and of blocks by polynomial transformations. National Research Council of Canada, NRC - 9265, 1966.

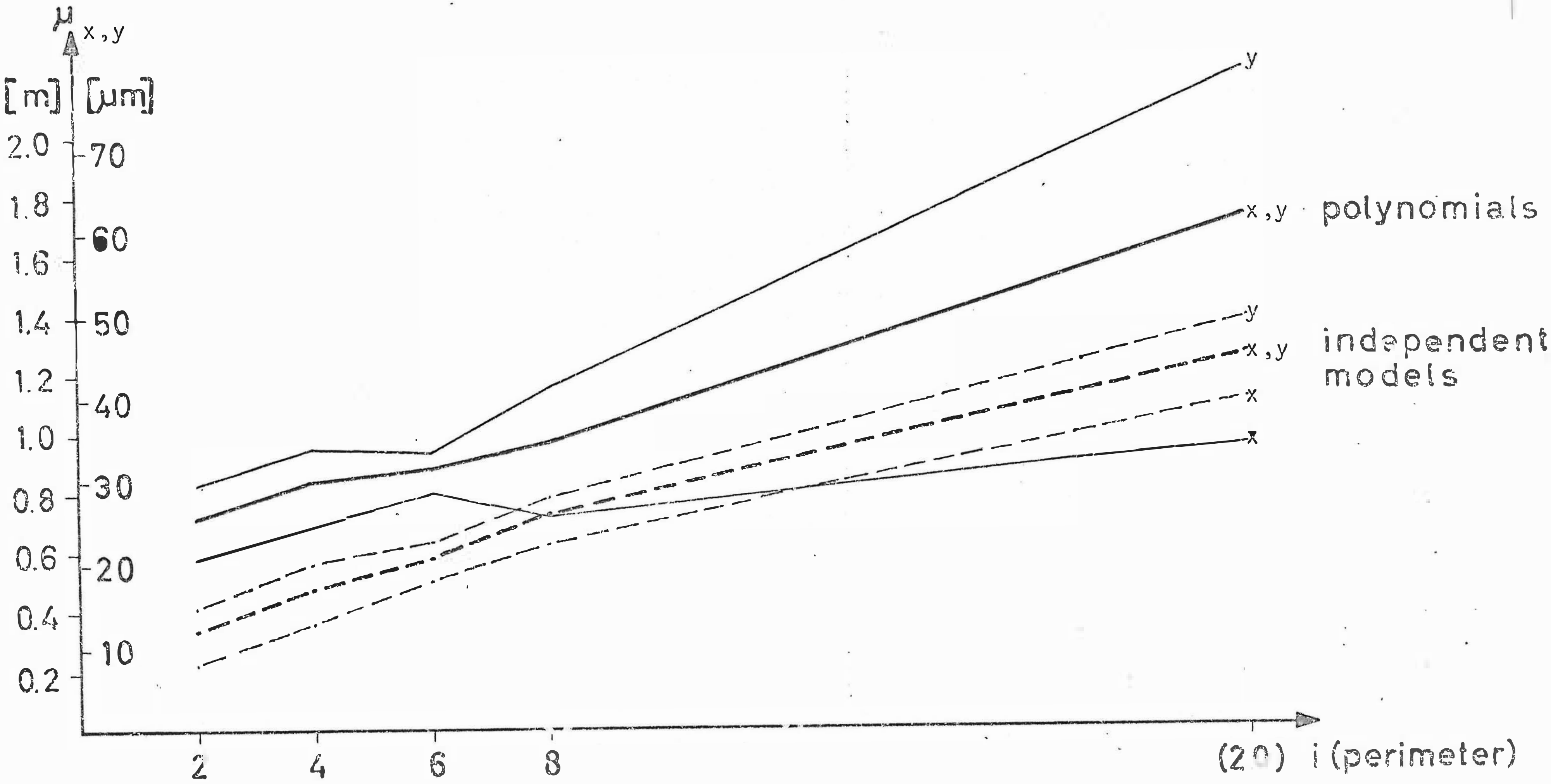
[3] Meixner, H.: A universal computer program for analytical aerotriangulation. Presented paper of Commission III, 12th Congress of the International Society of Photogrammetry, Ottawa. AVN, 281 - 289, 1972.

[4] Bauer, H. and Müller, J.: Height Accuracy of Blocks and Bundle Adjustment with Additional Parameters. Presented paper of Commission III, 12th Congress of the International Society of Photogrammetry, Ottawa 1972.



Control distributions for 20 % side lap

Figure 1



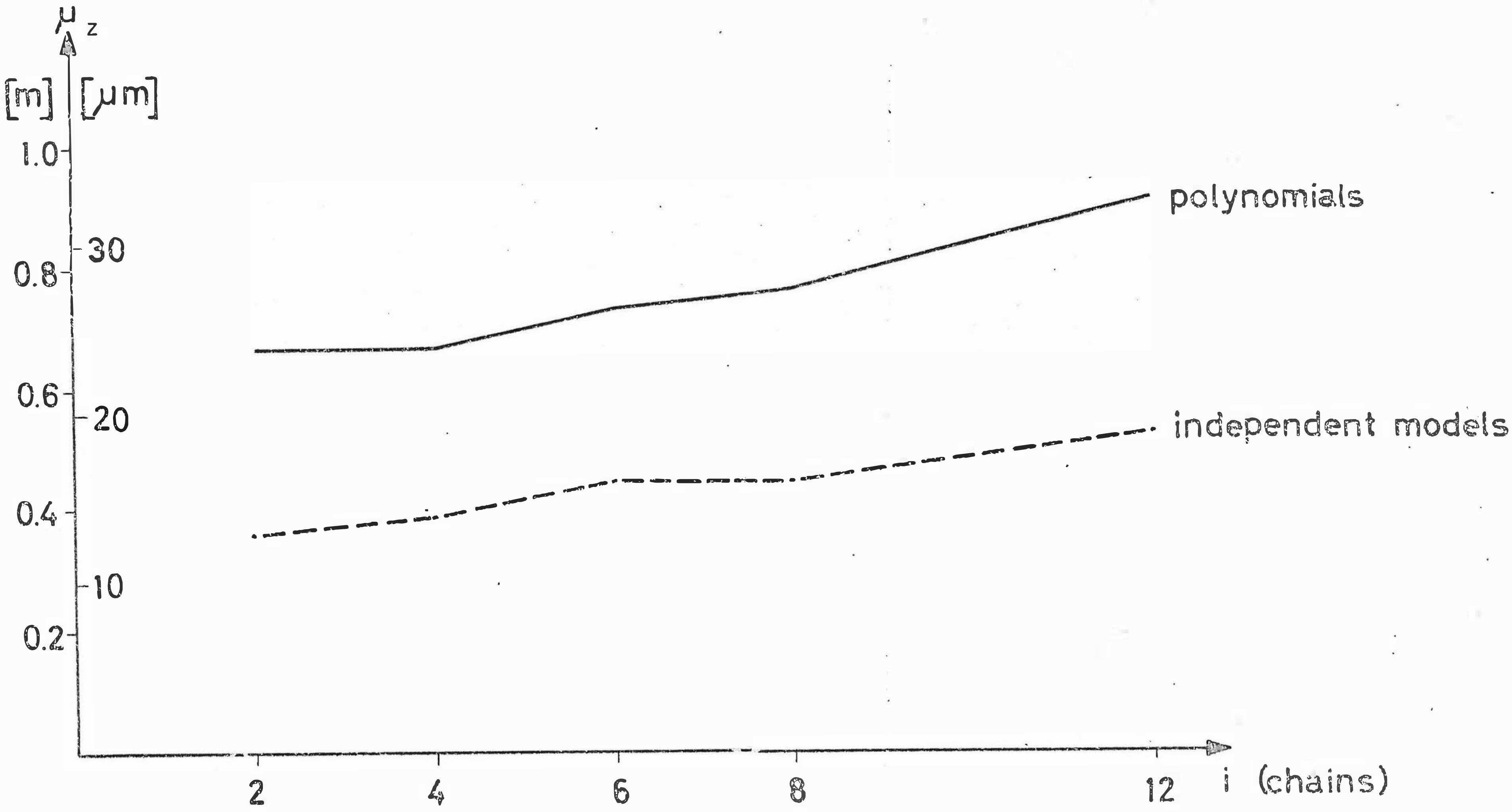
$\mu_{x,y}$ = RMS of the x and y differences at check points

Block Frankfurt: wide angle, 20 % side lap

Empirical planimetric accuracy after block adjustment

Comparison of polynomials and independent models

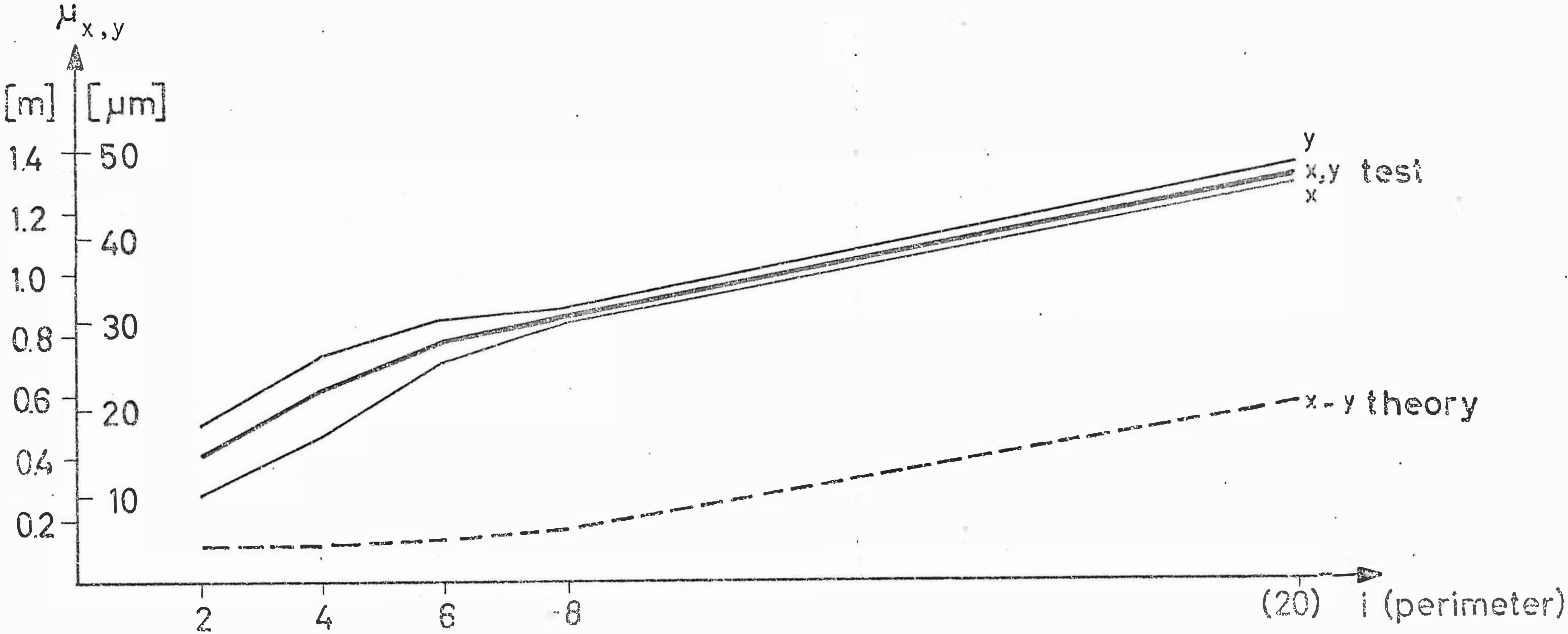
Figure 2



μ_z = RMS of the z differences at check points

Block Frankfurt: wide angle, 20 % side lap
Empirical height accuracy after block adjustment
Comparison of polynomials and independent models

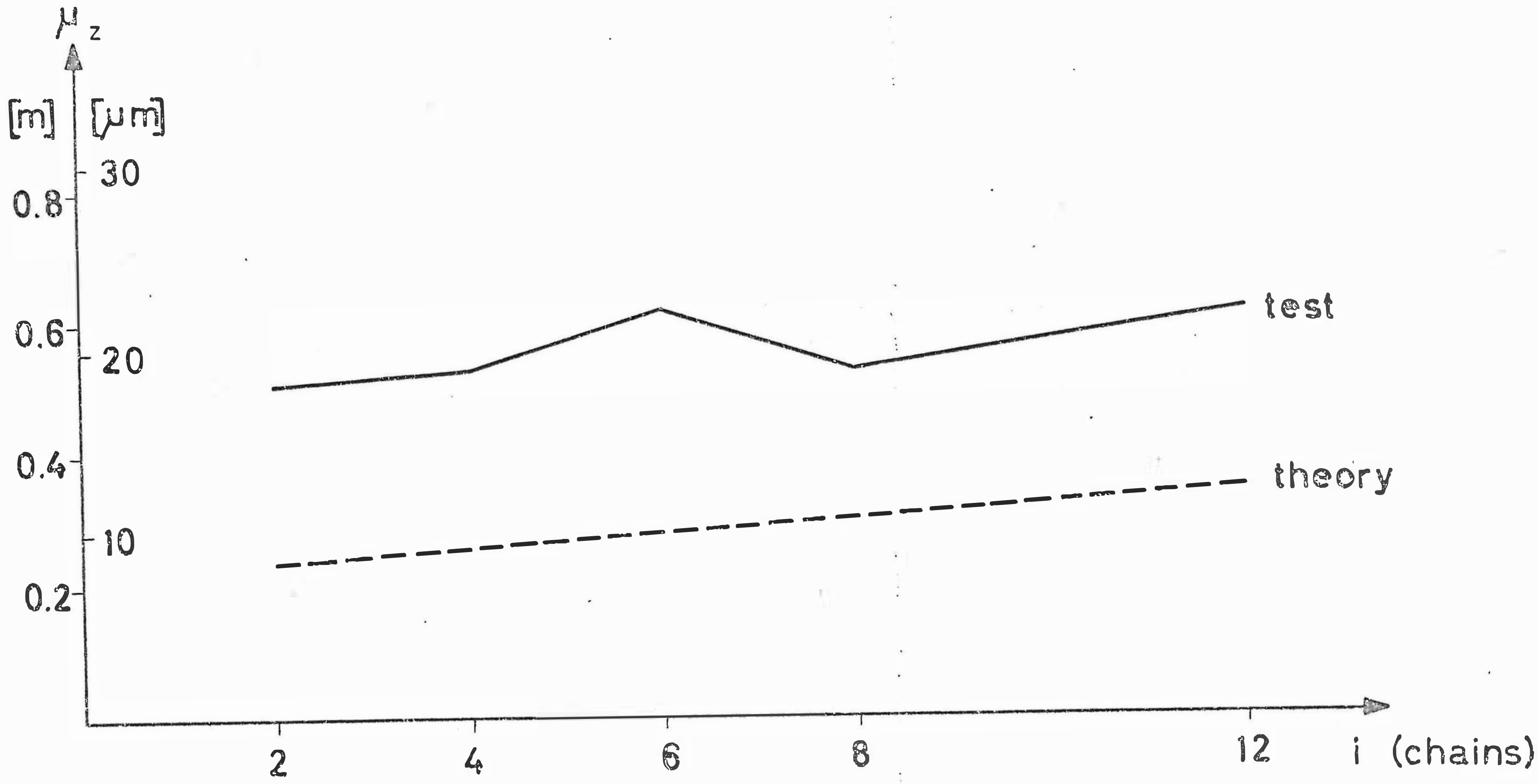
Figure 3



$\mu_{x,y}$ = RMS of the x and y differences at check points

Block Frankfurt: wide angle, 20 % side lap
Planimetric accuracy after bundle block adjustment
Comparison of theory and empirical test

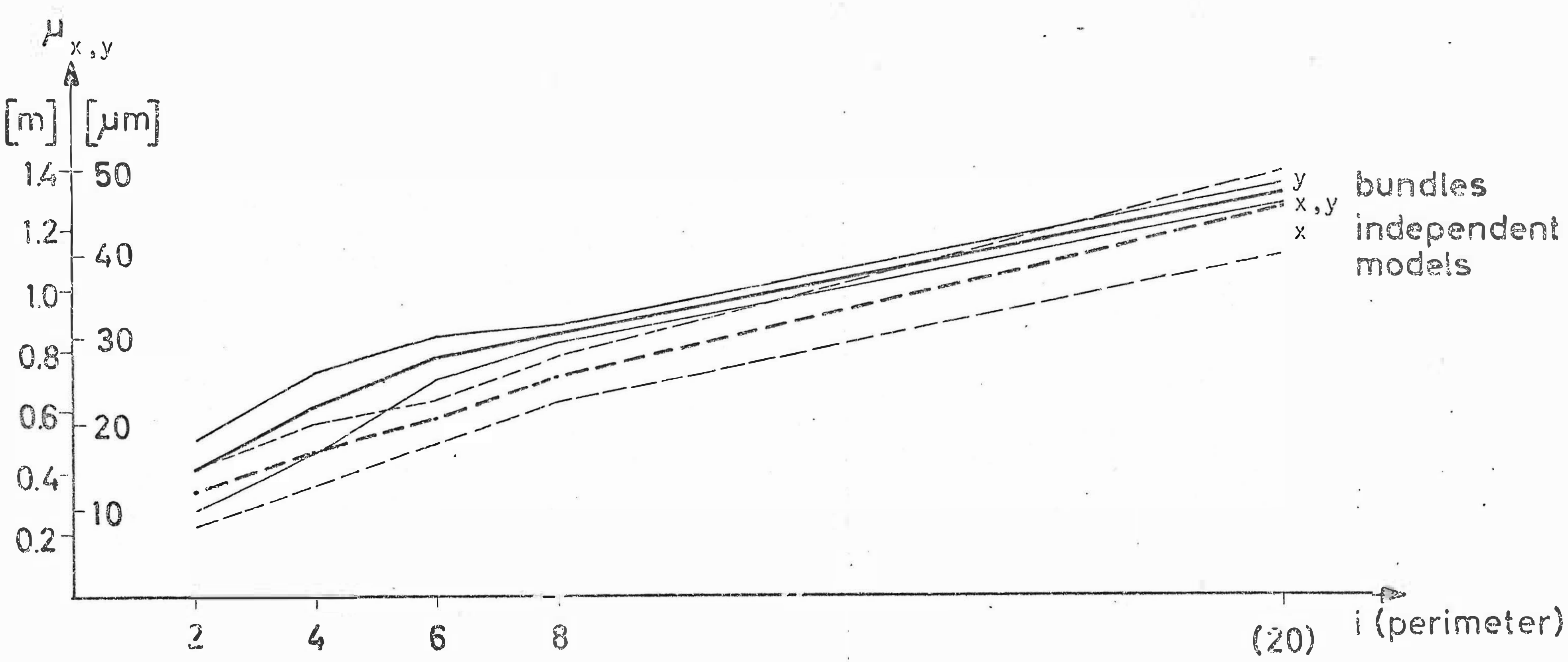
Figure 4



μ_z = RMS of the z differences at check points

Block Frankfurt: wide angle, 20 % side lap
Height accuracy after bundle block adjustment
Comparison of theory and empirical test

Figure 5



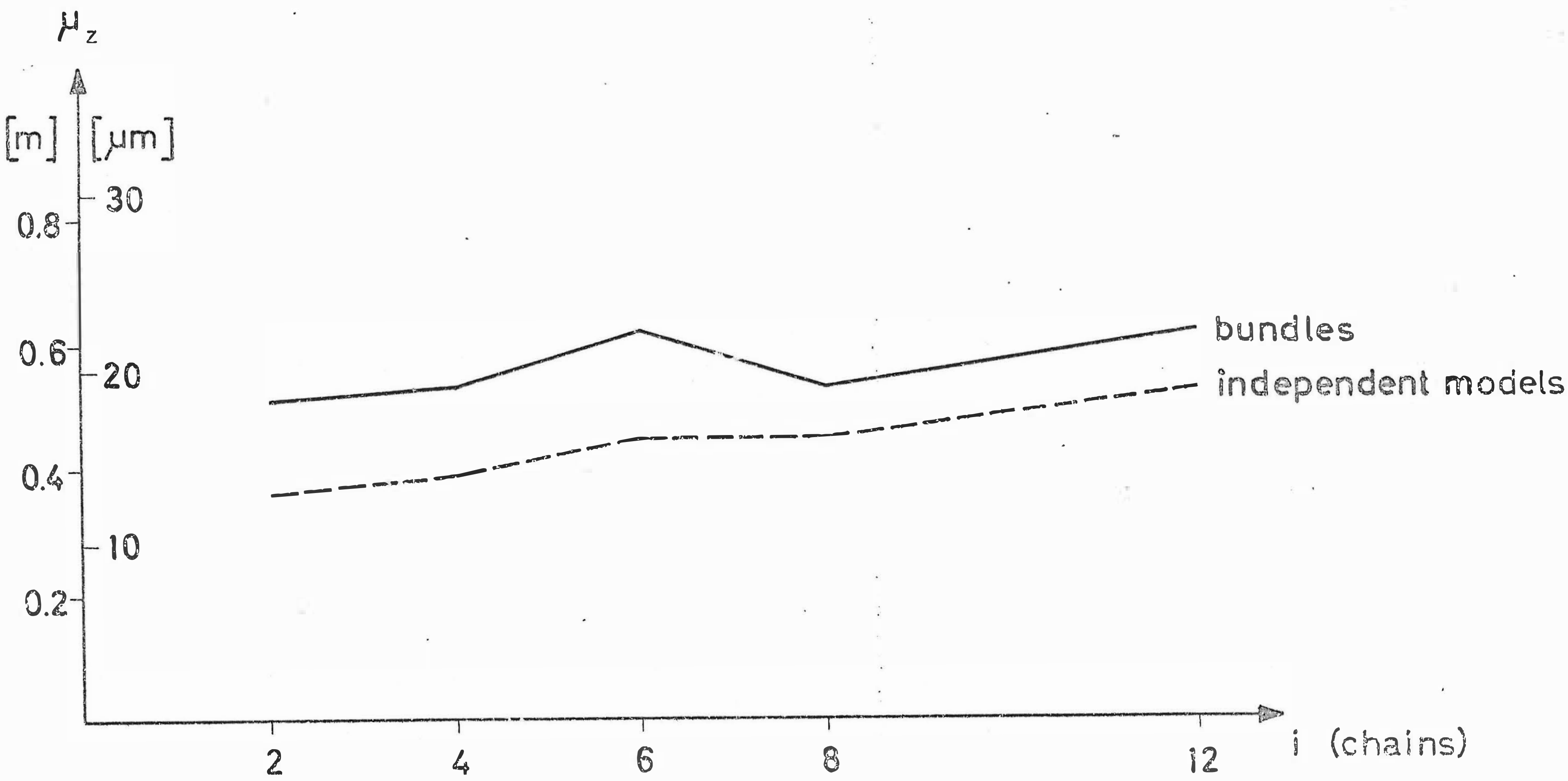
$\mu_{x,y}$ = RMS of the x and y differences at check points

Block Frankfurt: wide angle, 20 % side lap

Empirical planimetric accuracy after block adjustment

Comparison of bundles and independent models

Figure 6



μ_z = RMS of the z differences at check points

Block Frankfurt: wide angle, 20 % side lap
Empirical height accuracy after block adjustment
Comparison of bundles and independent models

Figure 7

Version	control points		check points		μ [m]		μ [m]		μ [μm]	
	x, y	z	x, y	z	x	y	x, y	z	x, y	z
1	40 i=2	117 i=2	472	334	0.590	0.832	0.721	0.668	26	24
2	20 i=4	75 i=4	492	376	0.704	0.958	0.840	0.674	30	24
3	14 i=6	53 i=6	498	398	0.812	0.936	0.876	0.740	31	26
4	10 i=8	42 i=8	502	409	0.728	1.163	0.970	0.769	35	27
5	4 (i=20)	31 i=12	508	420	0.969	2.241	1.727	0.919	62	33

i planimetry: perimeter control points in a distance of i base lengths

i height: chains of control points in a distance of i base lengths

μ = RMS of the coordinate differences at check points

Block Frankfurt: wide angle, 20 % side lap

Empirical accuracy after polynomial block adjustment

Table 1

Version	control points		check points		accuracy ratio		accuracy ratio	
	x, y	z	x, y	z	x	y	x, y	z
1	40 i=2	117 i=2	472	334	2.54	1.99	2.13	1.85
2	20 i=4	75 i=4	492	376	1.91	1.69	1.75	1.74
3	14 i=6	53 i=6	498	398	1.61	1.45	1.51	1.66
4	10 i=8	42 i=8	502	409	1.16	1.47	1.35	1.71
5	4 (i=20)	31 i=12	508	420	0.86	1.60	1.36	1.73

i planimetry: perimeter control points in a distance of i base lengths
i height: chains of control points in a distance of i base lengths

Block Frankfurt: wide angle, 20 % side lap
Comparison of polynomials and independent models

Table 2

Version	control points		check points		μ [m]		μ [m]		σ_0 [μm]	μ [μm]	
	x, y	z	x, y	z	x	y	x, y	z		x, y	z
1	40 i=2	117 i=2	469	330	0.275	0.512	0.411	0.510	5.7	15	18
2	20 i=4	75 i=4	489	372	0.473	0.732	0.616	0.534	5.0	22	19
3	14 i=6	53 i=6	495	394	0.714	0.845	0.782	0.622	4.7	28	22
4	10 i=8	42 i=8	499	405	0.836	0.880	0.858	0.533	4.3	31	19
5	4 (i=20)	31 i=12	505	416	1.285	1.351	1.318	0.623	4.0	47	22

i planimetry: perimeter control points in a distance of i base lengths

i height: chains of control points in a distance of i base lengths

μ = RMS of the coordinate differences at check points

Block Frankfurt: wide angle, 20 % side lap
Empirical accuracy after bundle block adjustment

Table 3

Theoretical accuracy after bundle block adjustment

wide angle 20 % side lap

Planimetry: perimeter control points in a distance of i base lengths

$\sigma_{x\ mean} \sim \sigma_{y\ mean} \sim (0.7 + 0.011 i^2)\ \sigma_o$

(block size 10 strips, 21 photos each)

Height: chains of control points in a distance of i base lengths

$\sigma_{z\ mean} \sim (1.2 + 0.16 i)\ \sigma_o$

(independent of block size)

Table 4

Version	control points		check points		accuracy ratio		accuracy ratio	
	x, y	z	x, y	z	x	y	x, y	z
1	40 i=2	117 i=2	469	330	1.19	1.23	1.22	1.41
2	20 i=4	75 i=4	489	372	1.29	1.29	1.29	1.38
3	14 i=6	53 i=6	495	394	1.41	1.31	1.35	1.40
4	10 i=8	42 i=8	499	405	1.33	1.11	1.20	1.18
5	4 (i=20)	31 i=12	505	416	1.14	0.97	1.04	1.17

i planimetry: perimeter control points in a distance of i base lengths

i height: chains of control points in a distance of i base lengths

Block Frankfurt: wide angle, 20 % side lap

Comparison of bundles and independent models

Table 5