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EXPERIENCE WITH APPLICATIONS OF BLOCK ADJUSTMENT FOR LARGE SCALE SURVEYS

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Abstract

Methods and results of practical applications of block adjustment by independent model cadastral surveys are described. The wider implications of the results are assessed in conjunction with current research trends in the adjustment of extremely large blocks for small scale mapping and in the use of the bundle adjustment method.

Introduction

Before discussing the subject of block adjustment and, in particular, recent experience with its application to large scale work, I want to recall the great tradition of numerical photogrammetry in this country and the important contributions which Great Britain has made to the development and application of block adjustment methods. It is, therefore, a great honour to be invited to address your renowned Society on such subjects.

I do not wish to attempt a review of the present situation of aerial triangulation or the success of numerical methods in general. I will report instead on the development and application of a system of programs for the adjustment of aerial triangulations at the University of Stuttgart where a group of photogrammetrists and mathematicians has concentrated, since 1968, on system development and computer programming in the field of numerical photogrammetry, giving priority to strip and block triangulation. The development has been carried out mainly by Dr. H. Ebner, H. Klein and H. Meixner on the one hand and Dr. K. Kraus, K. Ballein and R. Bettin, concentrating on cadastral applications, on the other. All of them are members of, or are associated with, the photogrammetric institute at the University of Stuttgart.

The System Development

Without reviewing the history, I want to mention briefly the *philosophy* and the scope of the system of programs as originally defined. It was intended to overcome the shortcomings and limitations of the state of aerial triangulation and, in particular, of the computer programs which were in use a few years ago. We believed in the great potential of block triangulation and were convinced that more generally applicable and more powerful computer programs of general scope were needed in order to bridge the gap between the advanced methods and every day practical application.

The essential specifications for the system which was to be developed can be summarised as follows:

(a) general applicability; virtually no limitations for conditions of overlap, number of points per model, number and distribution of control points (other than the minimum requirements);
(b) use of a generally available, problem oriented, language (Fortran IV);
(c) accommodation of large and very large blocks;
(d) rigorous least squares adjustment, using all the given data;
(e) use of large and fast computers if necessary;
(f) a high degree of automation and easy practical handling of the programs, with as little input specification as possible and preferably with no practical restrictions of any kind; and
(g) the programs should be highly efficient and competitive with approximate solutions.

The system of programs was designed to ultimately include several versions with *different mathematical models.* We started with adjustment programs based on the principle of independent models, simply for practical reasons. The instrumental and practical conditions for the application of the independent model method are general and the method is considered highly accurate and economic, notwithstanding the trend, which we also support, towards fully analytical aerial triangulation.

We started with a program for strip adjustment by independent models which served as a training experiment for a number of problems. Although it is in practical use with highly satisfactory results (computing times, including strip formations, are about 0·5 s system time per model with a CDC 6600) it will not be discussed here.

The first block adjustment program to be completed was the planimetric version of independent model triangulation which is also known as the Anblock method of block adjustment. It has been mainly applied to cadastral photogrammetry. It uses independent models which are sufficiently levelled so as not to affect planimetry owing to the remaining tilts. The planimetric block adjustment has the practical advantage that it involves a linear adjustment problem, requiring neither approximate values nor linearisation nor iterations. Therefore the computing times (about 0·5 s system time per model with a CDC 6600 and about 10 points per model) are considerably less than for the three dimensional block adjustment. Most of the examples which will be discussed later are planimetric block adjustments computed with this preliminary program.

The main line of development concentrated on the three dimensional block adjustment. However, the programming of the system which would apply directly the seven parameter transformation formulae (spatial similarity transformation) was left aside, for the time being, in favour of the highly efficient iteration between plan and height block adjustment. This program is called PAT-M-43 (*p*rogram *a*erial *t*riangulation with independent *m*odels, succession of *4* and *3* parameter transformations). It has only been operational since the end of 1971. It can be used either for three dimensional or for planimetric block adjustment. Very recently the system of programs has been extended to the fully analytical method (bundle adjustment by PAT-B). We have no practical experience with it as yet and so the examples to be discussed later will be restricted to the independent model method.

The *mathematical approach* to block adjustment by *independent models* requires, in principle, the simultaneous determination of a three dimensional similarity transformation of each model such that the identity conditions for tie points and for ground control points are taken into account. The general mathematical formulation for this adjustment problem is given by the following non-linear observational equations, referring to a point i measured in model j:

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ij} = -\lambda_j R_j \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{ij} - \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}_j + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i , \tag{1}$$

where

i = point number,

j = model number,

$[xyz]^{\mathrm{T}}_{ij}$ = vector of model co-ordinates of point i measured in model j (T = symbol for transposition),

$[XYZ]^{\mathrm{T}}_i$ = vector of terrain co-ordinates of point i (unknowns),

$[V_x\, V_y\, V_z]^{\mathrm{T}}_{ij}$ = vector of least squares residuals to the transformed point i of model j,

λ_j = scale factor of model j

R_j = 3×3 orthogonal matrix with independent parameters a, b, c

$[X_0\, Y_0\, Z_0]^{\mathrm{T}}_j$ = vector of shift parameters

(the last three: Transformation parameters of model j (seven unknowns))

For R_j we chose a modified form of the Rodrigues matrix:

$$R_j = \frac{1}{K} \begin{bmatrix} 1+\frac{1}{4}(a^2-b^2-c^2) & -c+\frac{1}{2}ab & b+\frac{1}{2}ac \\ c+\frac{1}{2}ab & 1+\frac{1}{4}(-a^2+b^2-c^2) & -a+\frac{1}{2}bc \\ -b+\frac{1}{2}ac & a+\frac{1}{2}bc & 1+\frac{1}{4}(-a^2-b^2+c^2) \end{bmatrix}_j ,$$

$$K = 1+\tfrac{1}{4}(a^2+b^2+c^2). \tag{2}$$

In the above mathematial approach to block adjustment by independent models, we have as unknowns

—the seven transformation parameters (λ, a, b, c, X_0, Y_0, Z_0), of each model j,

—the terrestrial co-ordinates $(X, Y, Z)_i$ of each point i.

Contrary to other existing systems the approach (1) is also maintained, in principle, for ground control points. Instead of giving the terrestrial values $(X, Y, Z)_i$ of a control point i in (1) known values, the given ground control co-ordinates are also treated as appropriately weighted observations which are linked with the unknowns of (1) by the additional observational equations

$$\begin{bmatrix} V_x^c \\ V_y^c \\ V_z^c \end{bmatrix}_i = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i - \begin{bmatrix} X^c \\ Y^c \\ Z^c \end{bmatrix}_i . \tag{3}$$

Here, the symbol c refers to terrestrial *c*ontrol. This approach also allows corrections to the terrestrial control co-ordinates, depending on the weights introduced. With weight ∞ (or 10^8 in the computer), the conventional case of strict adherence to control is obtained as a special case.

The approach (1) is used directly with the strip adjustment program and it would be used for the block program PAT-M-7 which has been left aside for the time being. The system (1) is non-linear in the unknowns and therefore it has to be linearised, starting from zero values and updated iteratively by repeated adjustments. As mentioned before, the program PAT-M-43, which is operational now, iterates between planimetric adjustment and height adjustment, performing within each step a complete least squares adjustment. The system has been described elsewhere (Ackermann, 1972b) with special consideration of the perspective centres and it will not be repeated here.

A general feature of all versions of our block adjustment program is, within each adjustment step, the direct generation of *partially reduced normal equations* which contain only the transformation parameters as unknowns, the unknown terrain co-ordinates of the points to be determined being eliminated. It is of basic concern that even such partially reduced systems of normal equations can be very large, running up to the order of magnitude of 10^4 equations or unknowns in practical cases. Therefore, the numerical solution of such systems of equations is a problem of central importance. After having experimented with iterative solutions and actually programmed the method of conjugate gradients in two cases, they were abandoned in favour of a direct solution method. The well known advantages of the iterative solution of systems of normal equations are, in our opinion, outweighed by some properties which can have undesired effects. The convergence depends strongly on the quality of the initial values and on the conditioning of the system; that is, on overlap and on the distribution of control. In poorly conditioned systems, very many iterations would be needed and, in addition, it is very difficult, in such cases, to find reliable criteria for stopping the iterative process. As a result it would be difficult to predict computing times. Consequently, H. Klein developed a highly efficient and general program for solving directly *large systems of linear equations* with symmetrical, positive definite, coefficient matrices. The program name, Hychol, refers to the use of submatrices as units for a Cholesky solution (*Hy*per-*Chol*esky). It can handle full matrices, but it is particularly suited for banded or banded/bordered matrices. Submatrices are brought in successively from the external storage (disc) and so there is no direct limitation of the total size of the system to be solved. Through the choice of the size t of such submatrices the program can be adapted to available core capacity of a computer, at the expense, of course, of computing time, or rather IO time in the main. For a given system of equations, the computing time required for solving with Hychol can be predicted very precisely (Ackermann *et al.*, 1972a).

TABLE I. Computing times (system time) for the solution of normal equations with Hychol on a CDC 6600

Number of unknowns	900	3600	19901
Half bandwidth	60	480	150
Number of right hand sides	1	1	25
Size t of submatrices ($t \times t$)	30	120	50
System time (s)	12	915	1236

(System time = CP time + K × IO time; K = used core memory/available core memory.)

The version of the program designated PAT-M-43 iterates between planimetric and height adjustments, performing within each step a complete least squares adjustment with four parameter transformations for planimetry and linearised three parameter transformations for heights, respectively, updating the model co-ordinates each time by applying the complete transformations. The efficiency of such a method depends very much on the total number of iterations required. The first computations with practical blocks have shown that the *convergence* of the method is surprisingly fast. Starting from zero tilts as initial approximations, three iteration steps are usually sufficient. Table II shows the rate of convergence of the plan and height iterations of the adjustments by listing the maximum co-ordinate differences between successive iteration steps. The figures refer to the first two spatial blocks which were adjusted recently. Block A contained 50 models of photo-scale 1 : 3400 and block B had 129 models of photo-scale 1 : 14 000. Counting a plan and height sequence as one step, the maximum alterations against the previous iteration were reduced to 6 mm (0 mm) in the terrain after three steps, whilst the first step changed the co-ordinates by up to 1758 m (10 229 m). The factor of convergence between iteration steps was about 100. It would have been safe to stop the computations after 2 steps. According to tests simulating poorer initial conditions, it is safe to assume a factor of convergence of at least 10. The fast convergence of the plan and height iterations is remarkable, in view of the fact that no preliminary transformations of the instrument co-ordinates of the independent models were applied.

TABLE II. Rates of convergence of plan and height iterations of three dimensional block adjustments with PAT-M-43

Block	Co-ordinate differences	*Iteration step* 1 *Plan* (*m*)	*Height* (*m*)	*Iteration step* 2 *Plan* (*m*)	*Height* (*m*)	*Iteration step* 3 *Plan* (*m*)	*Height* (*m*)
A	Δx_{max}	1501	8	0·78	0·18	0·005	0·001
	Δy_{max}	1758	23	0·93	0·19	0·006	0·001
	Δz_{max}	318	147	0·47	0·21	0·005	0·001
B	Δx_{max}	10 229	30	0·92	0·52	0·000	0·000
	Δy_{max}	7303	85	2·68	0·35	0·000	0·000
	Δz_{max}	389	527	0·78	0·65	0·000	0·000

It was hoped that the *computing times* required would be short enough to be economic, in spite of the high performance of the programs. The computing times depend to some extent on the size of a block and on the number of points involved apart, of course, from the type of computer used. The two blocks quoted in Table II (A with 50 models and 532 points measured; B with 129 models and 1354 points measured) had computing times (system time) with the CDC 6600 computer of 1·3 s per model for three iterations. For blocks of about 200 models the rule of thumb is about 2 s per model computing time for three iterations. The planimetric adjustment alone, with only one iteration step, requires from 0·4 s to 0·5 s per model.

The actual computing times compare very well with a previous semi-theoretical study (Ebner, 1971b) which also included the PAT-M-7 program version (three dimensional adjustment by independent models with seven parameter transformations) and PAT-B (fully analytical adjustment by "bundles") and which extended to very large blocks. It may be of interest to quote the computing times, which also refer to CDC 6600 system time and three iterations each.

Table III shows, firstly, that the computing times of different program versions differ considerably. The plan and height iteration is the most economic version and is practically the only version suitable for very large blocks. Secondly, the computing times per model do increase with the block size. However, the rate is very much less than the n^3 rule would suggest.

For cadastral applications we have often had the case of blocks of moderate size which nevertheless contain thousands of points. The empirical computing time for the planimetric adjustment of a block of 50 models with, for example, 5000 points is about (for CDC 6600 system time) 2 s per model or 0·02 s per point. All figures quoted here refer to one run of adjustment. Owing to gross data errors, normally three or four runs are necessary (Table V).

It is not possible to give here a complete description of the programs. Nevertheless a few *additional features* can be mentioned. Terrestrial control points can be given individual *weights*. The photogrammetric points are classified in two groups: model points and perspective centres. For each group the x, y and z co-ordinates can be weighted separately. The coefficient matrix of the (partially reduced) normal equations is a band matrix. Its *band width* depends on the ordering of the models. The computing time required for solving the system increases with the square of the band width. It is therefore essential to keep the band width as small as possible. The program provides a highly automated procedure for minimizing the band width, requiring as a start only some model numbers to be read in; from there on, the models are automatically ordered, according to their ties, irrespective of the model numbering.

TABLE III. Semi-theoretical comparison of computing times for block adjustment for CDC 6600 system time

Program	*Size of (~ square shaped) block* 200 *models*	1000 *models*	
PAT-M-43	1·5 s	3·6 s	per model
PAT-M-7	2·7 s	9·6 s	per model
PAT-B	4·2 s	17·4 s	per model

The program is intended to allow for *convenient handling*. Thus there is free numbering of models and points. In particular no special coding of point numbers is required with regard to the type of point or its function within the adjustment. The ties between models are established by search routines which identify identical point numbers. Similarly, control points are identified by comparing each point with the list of control points, which are collected in what is called the zero model. Also the input format of the data can be variable, the actual format being read in. Such features are necessary for the convenient and practical handling of block adjustments and aim at rather general applicability of the program.

The program is also general with regard to acceptable block size, the number of points involved, the kind of overlap and the number and distribution of control points. It can be adapted to the available core memory of a computer. However, the minimum *core capacity* should be 64 k words or 256 k bytes. The total program is made up of 40 k words. It is subdivided, however, into four parts, of which only one part is in the core at a given time, occupying a maximum of 12 k words.

In view of the wide range of intended application and the degree of sophistication involved, the program PAT-M-43 is intended to be used with rather large and fast computers. Our basic philosophy involves the use of the most powerful tools available in order to develop the full potential of aerial triangulation. This runs parallel with the fact that larger computers have cheaper operation per unit of computation.

THEORETICAL AND EXPERIMENTAL STUDIES

It should be mentioned very briefly that a number of theoretical and experimental studies have confirmed the great potential of block adjustment. They have helped to maintain the rather extreme specifications for the programs. Studies on the *planimetric accuracy* of blocks were of primary interest to us. The results have been published (Ackermann, 1966; Ackermann *et al.*, 1972a; Ebner, 1971a) and they can be summarised as follows:

(a) the planimetric accuracy is almost independent of the shape and size of a block, provided that its perimeter is well controlled;
(b) even with extremely large blocks of thousands of models the planimetric accuracy remains in the order of $1{\cdot}5\ \sigma_0$, σ_0 being the standard error of unit weight (i.e. the accuracy of model co-ordinates);
(c) consequently, the adjustment of blocks covering 100 000 km² or more seems feasible for small scale mapping. It could meet the planimetric specifications of small scale maps with extreme reduction of ground control;
(d) on the other hand, block adjustment can be used with very large photo-scales, promising accuracies of a few centimetres and with relatively little ground control;
(e) for the special case of strong ties (more than 20 tie points per model) block adjustment with perimeter control only is supposed to be even more accurate than fully controlled individual models.

Very recently the theoretical accuracy investigations have been extended to heights (Ebner, 1972). They will not be elaborated upon, as the subsequent contents of this paper will mainly deal with planimetry.

Large Scale Mapping Application of Planimetric Block Adjustment

The planimetric version of the block program was completed first. We have applied it (or, rather, preliminary versions of it) since 1970 in the field of photogrammetric *cadastral surveys* with great success. Although the adjustments refer to the particular circumstances of cadastral surveys in the Federal Republic of Germany the remarkable results display in general the accuracy which is obtainable with photogrammetric point determination.

The special circumstances can be listed briefly:

(a) all points are targetted in the field, with signals of size 0·1 m × 0·1 m up to 0·25 m × 0·25 m;
(b) there are many points involved, with up to several hundred points per model. Thus the favourable case of strong ties is predominant;
(c) a dense system of trigonometric or traverse points is usually available;
(d) the official accuracy requirements are very high, for distance accuracy in particular;
(e) rather large photo-scales are applied and in most cases these are 1 : 6000 or larger;
(f) measurements are mostly done with analogue instruments. Very recently stereocomparators have been used more often.

In the following summary, various results and some brief comments are presented.†

The first planimetric block which we adjusted concerned the cadastral survey of the Hermuthausen–Steinbach re-allotment in southern Germany. The relevant data are: area 1090 ha; photo-scale 1 : 6000; camera Wild RC 8; four strips running from north to south with a total of 32 models; measurements with a Zeiss stereoplanigraph C8 in 1968 (for single model restitution) by the state re-allotment authority; the total number of points was 4826, of which 860 were measured at least twice; there were 27 trigonometric points and 64 traverse points. The block adjustment was repeated several times with different control assumptions. The results are given in Table IV in which the results of another cadastral block (Ichenheim–Dundenheim) with very similar features are included (1 : 6000, 2000ha, 58 models, 6000 points, measured with C8).

† During the lecture, a number of slides were shown.

TABLE IV. Results of various planimetric adjustments of two cadastral blocks

Version	n_M	n_{pp}	n_{vp}	n'_{pp}	n'_{vp}	r	σ_0	$\bar{v}_{pp}$	$\bar{v}_{vp}$	v_{pp}^{max}	v_{vp}^{max}
Hermuthausen–Steinbach 0 single models	32	91	860	182	1720	(236)	(70 mm) (11·6 μm)	56 mm 9·3 μm	69 mm 11·5 μm	210 mm 35 μm	180 mm 30 μm
I all control points	32	90	855	179	1810	2140	48 mm 8·1 μm	73 mm 12·1 μm	29 mm 4·9 μm	300 mm 50 μm	180 mm 30 μm
II trigonometric points plus perimeter control	32	42	892	66	1907	2134	44 mm 7·3 μm	92 mm 15·3 μm	28 mm 4·7 μm	280 mm 47 μm	180 mm 30 μm
III trigonometric points only	32	27	898	45	1918	2002	41 mm 6·8 μm	82 mm 13·6 μm	27 mm 4·4 μm	200 mm 33 μm	190 mm 32 μm
IV perimeter points only	32	14	906	19	1941	1980	42 mm 7·1 μm	112 mm 18·7 μm	28 mm 4·6 μm	270 mm 45 μm	190 mm 32 μm
Ichenheim–Dundenheim II trigonometric points plus perimeter points	58	72	2284	107	5397	8208	60 mm 10·0 μm	86 mm 14·3 μm	48 mm 8·0 μm	230 mm 39 μm	260 mm 43 μm

n_M = number of models,
n_{pp} = number of control points,
n_{vp} = number of tie points,
n'_{pp} = number of measurements of control points,
n'_{vp} = number of measurements of tie points,
r = redundancy $= 2(n'_{pp}+n'_{vp})-4n_M-2n_{vp}$,
$\sigma_0 = \sqrt{([vv]/r)}$ = standard error of unit weight,
= accuracy of model coordinates (before adjustment),
$\bar{v}_{pp}$ = r.m.s. value of residuals at control points,
$\bar{v}_{vp}$ = r.m.s. value of residuals at tie points,
v_{pp}^{max}, v_{vp}^{max} = maximum residuals at control and tie points.

Table IV shows a surprising standard of accuracy as expressed by values for σ_0 of 10 μm and less. It is the more remarkable in that it refers to routine restitutions of the photogrammetric service of the Baden Württemberg re-allotment authority. The block adjustment gives better internal homogenity as compared with the single model restitution ($\bar{v}_{vv} = 69$ mm $\rightarrow$ 29 mm). This is one of the very few available cases which allows such a comparison. The residual errors at the tie points are extremely small, with average magnitudes of less than 30 mm or 5 μm. In fact, for Hermuthausen–Steinbach 50 per cent of all of nearly 4000 residuals had values of -10 mm, 0 or $+10$ mm. It is to be noted, however, that the residual errors at the ground control points are considerably larger than at the tie points. Thus the internal consistency of the adjustment block is much better than the fitting of the block to the control points.

The details of Table IV can be considered to be more or less representative of most other large scale block adjustments, of which quite a number have since been treated. Table V displays statistical data and confirms that most of our practical applications of block adjustment refer to large photo-scales, to pre-marked points and to strong ties. It is typical to have 300 points in a model, of which 150 are tie points. With the exception of two blocks from abroad, the values of σ_0, referred to negative scale, range between 6 μm and 13 μm, and in most cases do not exceed 10 μm. This confirms that the inherently high accuracy of photogrammetric point determination is effective in routine applications with conventional instruments. It should be pointed out that only two of the blocks of Table V were measured for research purposes. All the others are practical cases from various organisations. Because of gross errors, the block adjustments have usually to be repeated about three or four times. The two examples of Table V with 11 and 16 runs are exceptions, owing to special circumstances. The rate of gross errors at tie points is about 1 per cent and up to 2 per cent in some cases. Because of the strong ties, the gross errors can always be located easily.

A number of projects had enough ground control to allow some to be used as check points for separate runs. With check points the *absolute accuracy* of the adjusted blocks can be estimated with respect to the state co-ordinate system. Because of its expected accuracy, the case of planimetric ground control along the perimeter of the block (the interior of the block having no control) is of particular interest. Table VI gives the absolute accuracy of blocks, as estimated from check points. In all cases the points were targetted (control-, tie- and check-points).

The ratio $\sigma_{\text{check}}/\sigma_0$ ranges between 1·4 and 1·8 except for very large photo-scales with which the limited accuracy of the geodetic control and check surveys became noticeable. The results confirm in general the good accuracy properties of perimeter controlled planimetric blocks. It is also demonstrated that the absolute accuracies obtained are extremely good. Absolute co-ordinate accuracies in the order of 10 μm in the negative scale, for blocks of about 200 models, with perimeter control only, appear sensational to the practitioner, although they are to be expected theoretically.

Table VI contains a small test block of 1 : 1800 photo-scale. It is of scientific and of practical interest to study the accuracy behaviour of *very large photo-scales,* although such cases will remain exceptions. The results of the example of Table VI suggest that the accuracy rules for block adjustments remain essentially valid for very large photo-scales, provided the signalisation is appropriate. This has been confirmed by a recent investigation of Förstner and Gönnenwein (1972). The Böhmenkirch test area was flown with two different cameras, giving photo-scales of 1 : 1500 and 1 : 1000 and each covering the test area with one strip (five and nine

TABLE V. Statistical data of planimetric block adjustment by independent models

Type of project	Photo-scale	Instrument	No. of models	No. of control points	No. of unknown points	No. of tie points	No. of runs	No. of gross errors at control points	No. of gross errors at tie points	σ_0 mm	σ_0 μm
Re-allotment	1 : 6000	C8	32	42	4800	892	—	—	—	44 mm =	7·3 μm
	1 : 6000	C8	58	72	6000	2284	4	0	20	60	10·0
	1 : 4300	Planimat	54	65	3178	1548	4	0	23	45	10·4
	1 : 4300	C8	30	47	2709	1075	4	0	32	48	11·1
	1 : 6000	C8	29	49	2674	1050	3	0	51	56	9·3†
	1 : 4300	Planimat	42	62	4586	1768	5	1	41	48	11·1
	1 : 4000	Planimat	46	19	2925	1750	11	2	47	41	10·2
	1 : 10 000	C8	33	19	3791	406	3	—	—	82	8·2
Cadastral survey	1 : 7500	A7	170	32	1065	950	5	0	29	57	7·6
	1 : 5000	C8	14	18	656	480	2	0	1	39	7·8†
	1 : 5000	C8	12	17	2121	296	4	0	5	45	9·0
	1 : 4200	C8	6	34	493	107	16	9	50	51	12·1
	1 : 4000	C8	17	37	2502	540	2	0	7	29	7·2
	1 : 3600	Planimat	9	61	419	226				38	10·6
	1 : 3600	PSK	9	62	392	213				20	5·6
	1 : 6000	PSK	3	60	384	39				38	6·3
	1 : 1800	PSK	6	21	125	55				17	9·4
	1 : 3400	C8	50	17	244	177				55	16·2
	1 : 7500	A7	4	12	173	160				55	7·3
Vineyards re-allotment	1 : 6000	C8	3	8	932	168	1	0	0	60	10·0
	1 : 6000	C8	5	11	3811	1490	4	0	32	70	11·7
	1 : 6000	C8	3	9	1845	576	3	1	18	78	13·0
	1 : 6500	C8	3	13	1702	530	2	0	5	37	5·7
	1 : 28 000	PSK	200	40	1400	900				200	7·2
	1 : 14 000	A8	129	36	442	366				280	20·0‡
	1 : 84 000	Stecometer	243	54	1363	825				2·46 m	29·0‡

† Double overlap. ‡ Tie points marked artificially.

models, respectively). The absolute accuracies of check points obtained from well controlled strip adjustment by independent models based on comparator measurements are listed in Table VII.

TABLE VI. Absolute co-ordinate accuracy σ_{check} of planimetric blocks of independent models

Photo-scale	*Instru-ment*	*No. of Models*	*No. of control points*	*No. of check points*	σ_0 *mm*	σ_0 *µm*	σ_{check} *mm*	σ_{check} *µm*	$\frac{\sigma_{\text{check}}}{\sigma_0}$	*Control*
1 : 6000	C8	32	42	48	44	7·3	80	13·3	1·8	Scattered
1 : 7500	A7	170	32	14	57	7·6	80	10·6	1·4	Perimeter
1 : 3600	PSK	9	62	392	20	5·5	39	10·8	1·9	Scattered
1 : 1800	PSK	6	21	125	17	9·4	33	18·3	1·9	Scattered
1 : 10 000	C8	33	13	6	81	8·1	120	12·0	1·5	Perimeter
1 : 28 000	PSK	200	40	500	200	7·1	350	12·5	1·25	Perimeter
1 : 28 000	PSK	32	16	80	190	6·8	280	10·0	1·5	Perimeter

TABLE VII. Results from the large scale Böhmenkirch test

Camera	*Photo-scale*	*Number of check points*	*after strip adjustment* σ_x^{check}	σ_y^{check}
$f = 150$ mm	1 : 1500	117	12 mm	13 mm
	1 : 1000	89	15 mm	13 mm
$f = 300$ mm	1 : 1500	111	11 mm	14 mm
	1 : 1000	78	12 mm	13 mm

Almost all of the practical blocks which we have adjusted up to now were measured with analogue plotting instruments (C8, Planimat, A7 and A8) and the results were highly satisfactory. It would be of great interest to compare them with results based on stereocomparator measurements. For one experimental block we have such a comparison. Details of the example photo-scale are: 1 : 3600, nine models, about 250 tie points (650 measurements of tie-points) and 30 planimetric control points. The photographs were measured twice, using a Zeiss (Oberkochen) PSK stereocomparator from the data of which model co-ordinates were computed, and using a Zeiss (Oberkochen) Planimat in which case independent models were measured directly. With both sets of model co-ordinates, planimetric block adjustments were carried out by the method of independent models. The values for σ_0 (standard error of unit weight = standard error of model co-ordinates) of both block adjustments compare as follows:

$$\text{PSK}: \sigma_0 = 18 \text{ mm} = 5{\cdot}1\ \mu\text{m},$$
$$\text{Planimat}: \sigma_0 = 38 \text{ mm} = 10{\cdot}6\ \mu\text{m}.$$

Note: The processing of PSK measurements by the method of independent models is different from the fully analytical block adjustment.

Thus there is strong evidence that the model co-ordinates which originate from comparator measurements are considerably more accurate (by about a factor 2) than model co-ordinates from analogue restitution instruments. The absolute comparison of both adjusted blocks with the results of the geodetic cadastral surveys gave r.m.s. values for the co-ordinate differences of

$\sigma_{\text{check}} = 41$ mm $= 11{\cdot}4\ \mu$m for the block observed by PSK.
$\sigma_{\text{check}} = 51$ mm $= 14{\cdot}2\ \mu$m for the block observed by Planimat.

The factor 2 between the basic accuracy of PSK and Planimat model co-ordinates is not displayed any more in the absolute comparison. The obvious reason is the additional effect of the limited accuracy of the field survey and of the trigonometric and traverse net.

Very often, after block adjustment the residual errors at the control points are considerably larger than at tie points. This effect is mainly due to tensions in the geodetic system and to systematic photogrammetric errors. For cadastral surveys this effect disturbs the relative accuracy. A subsequent treatment of the adjusted blocks can rectify it by applying a *least squares interpolation* (Kraus and Mikhail, 1972). The interpolation filters out systematic deformations of a block and reduces the residual errors at control points. It is applied regularly and successfully for photogrammetric cadastral surveys of high precision, when the internal photogrammetric accuracy of a block is superior to that of the local geodetic net.

Another adjustment program may finally be mentioned which also refers to the practice of photogrammetric cadastral surveys in Germany. In order to check the photogrammetric (or other) determination of cadastral points quite a number of distances are measured in the field by tape. We use such distances, not only for checking purposes, but also for a final combined adjustment of the photogrammetric block co-ordinates and the additional terrestrial measurements (Ackermann *et al.*, 1972a). For a re-allotment project, for example, there may be several thousands of distances and points to be adjusted together. As a result, the distances as computed from finally adjusted co-ordinates agree with the directly measured distances with r.m.s. differences of 10 mm to 20 mm. Although the method is applied, at present, to cadastral surveys, it is of general importance as a first example of the joint adjustment of photogrammetric and geodetic data.

Additional Developments and Summary

So far, the results shown in this paper have referred to the practical application of the method of block adjustment by independent models to large scale, high precision, planimetric point determination for cadastral purposes. This application has been highly successful and has been accepted by the cadastral authorities. Of course our investigations and program developments have not been restricted to the special case of cadastral surveys. I will briefly point to other developments.

Aerial triangulation has its challenge in the field of small scale mapping. Both theoretical expectations and the high performance of the PAT-M-43 program for block adjustment suggest the practical use of extremely *large blocks*. Blocks of up to several thousand models are feasible and are expected to be highly economical. We expect very soon, in co-operation with the Department of Energy, Mines and Resources in Ottawa, Canada, to adjust a block of 2000 models, covering an area of about 100 000 km^2 (Gauthier *et al.*, 1972). With the aim of increasing the efficiency and accuracy of small scale block adjustment further, with respect to heights in particular, we are preparing an extension to the PAT-M-43 program for the simultaneous adjustment of airborne profile recorder (APR) or statoscope data and photogrammetric blocks (Ackermann, *et al.*, 1972). This program is expected to give height accuracy of 2 m or better for large blocks, with virtually no height control within the blocks.

Another research program can be mentioned here; it deals with an experimental and statistical investigation into the accuracy of strips and blocks. The photogrammetric institute of the University of Stuttgart is the pilot centre for such investigations of the Oberschwaben test field of OEEPE (the European Organisation for Experimental Photogrammetric Research). The purpose is a statistical investigation into the validity of theoretical accuracy models. In the course of the investigation, which is well on its way, hundreds of strip and block adjustments are to be carried out, in order to test the influence of various parameters, such as block size, control distribution, overlap and method of adjustment, on accuracy. According to the results which are available at present, very interesting conclusions can be expected which, in general, confirm the high accuracy capability of aerial triangulation. The results will be published in due course.

In connexion with the Oberschwaben experiments, a preliminary program for the fully analytical method of block adjustment ("bundle" method) has been developed (Meixner, 1972). The fully analytical method is the most accurate method of aerial triangulation. It is expected that its application will increase in practice. Its accuracy potential is such that new applications of photogrammetry seem feasible, such as the photogrammetric breakdown of geodetic triangulation nets.

We intend to apply the bundle program to practical cadastral surveys and other high accuracy projects. On the theoretical and experimental side we want to compare the analytical method with the independent model method. The first results obtained seem to indicate, somewhat surprisingly, that the bundle method is rather sensitive to systematic image errors and may not reach its theoretically predicted accuracy (Ebner, 1972), unless special measures are taken.

During the past two years, considerable experience has been gained from practical applications of the computer programs for strip and block adjustment at the University of Stuttgart. Although most of the applications up to now refer to cadastral surveys, experience suggests more general conclusions. In the first instance, we consider that the basic philosophy of our system of programs has been strongly confirmed. In particular, the generality of the approach, the absence of severe limitations and the high degree of optimisation of the programs have become most important as too many practical blocks are non-standard in one way or another.

The computing times have been reduced to be truly economic, even for very large blocks. With fast computers the costs of block adjustments are moderate or negligible. The expectations of accuracy of blocks have been very high from the beginning. They have even been surpassed. Only three or four years ago, it would have been considered sensational to consistently reach the 10 μm level with practical routine work. With analytical methods, now even the 5 μm level is approached.

In concluding this paper, it can only be emphasised how powerful a tool numerical photogrammetry has become through sophisticated and general computer programs. There is no doubt that photogrammetry is extending and intensifying its application in survey and geodetic fields. It is hoped that surveyors and photogrammetrists become increasingly aware of the really great accuracy and economic potential of photogrammetric point determination and that these factors will be utilised. The consistent application, at all scales, of numerical methods in photogrammetry is to be highly recommended.

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Résumé

L'auteur décrit les méthodes et les résultats de la compensation des blocs par modèles indépendants dans les levés cadastraux. En outre on discute les applications des résultats à la compensation des blocs extrèmement grands au service des levés à petite echelle et à la compensation par gerbes.

Zusammenfassung

Beschreibung der Verfahren und Ergebnisse praktischer Anwendungen von Blockausgleichungen mit unabhängigen Modellen für Katastermessungen. Einschätzung der aus den Ergebnissen resultierenden Schlussfolgerungen im Zusammenhang mit den gegenwärtigen Forschungstendenzen bei der Ausgleichung extrem grosser Blöcke für die Herstellung kleinmassstäbiger Karten und bei der Anwendung der Methode der Ausgleichung mit Strahlenbündeln.

DISCUSSION

Mr. Smith (*Chairman*) thanked Professor Ackermann and invited questions and comments from the audience.

Mr. Proctor: You did mention early in your paper that you were talking mainly about independent models. However, I do not think that you intended to imply that the method of adjustment was necessarily different. After all, the block adjustment that you were carrying out could have been equally applicable to models which had been observed on a stereocomparator or in an analogue instrument.

Professor Ackermann: It is true that the adjustment by independent models can operate with models which originate either from analogue instruments or from comparator measurements. The plate co-ordinates would have to be processed to model co-ordinates by analytical model formation. After that the computed models would go into the block adjustment as independent units.

Professor Thompson: Can you say why you did this? You did mention one experiment with a stereocomparator in which you did what you have just said. Can you say why you did not use the "bundle" method?

Professor Ackermann: Yes, because we hadn't a stereocomparator available.

Professor Thompson: Yes, but why not? Why did you prefer this method in the first place?

Professor Ackermann: I preferred this method in the first place because the practical conditions for applying it were much more readily available than for fully analytical methods. There are not many comparators in practical use and certainly were not some years ago. Therefore we concentrated our efforts on the independent model method at first. The method is more general as everybody can use it. Even those who work with comparators may use it in case they lack a program for the direct bundle adjustment.

The second reason for first concentrating on the independent model method was the expectation that the method would be very accurate. It is still an open question as to the extent to which the fully analytical adjustment would be more accurate. We have always expected the analytical method to be somewhat more accurate. Therefore we have always intended, as stage two of our system, to develop a fully analytical block adjustment program. That program has not been completed as yet, but it did run for the first time a few weeks ago. It will be of great interest to see its results compared with the independent model adjustments. For the time being our original expectations have been confirmed that the independent model method gives really outstanding accuracy and can successfully and economically be applied in practice.

Professor Thompson: When you use this independent model method, do you assume that the obtained model co-ordinates are independent? Do you apply any correlation matrix?

Professor Ackermann: No, in general we do not. We can in fact introduce a correlation matrix, correlating the x and y co-ordinates of a point, separately, for model points, perspective centres and ground control points. We have, however, no possibility of correlating different points. This is clearly an approximation, but obviously it works well enough.

Professor Thompson: From what experience I have had, it does not make much difference to the result.

Professor Ackermann: Yes, that is my experience as well. There are theoretical considerations which support this experience. Therefore we feel justified in abandoning complete rigour which would impose a heavy burden on the adjustment computations.

Professor Thompson: I wonder, Mr. Chairman, if I could say a word about Rodrigues. The story is a small personal tragedy. I did "invent" the matrix, in the sense of obtaining it without help, but subsequently discovered that it had been published by Rodrigues in 1840. Rodrigues was a French mathematician on the staff of the Polytechnique, I think. He developed his expression before the invention of matrices, by taking Euler's formula which gave the effect of a rotation in terms of the three direction cosines of the rotation axis and the magnitude of the rotation and eliminating one of the four parameters by making use of the relation between the three direction cosines.

The Rodrigues matrix thus consists of nine elements each of which is a function of three *independent* parameters in which, of course, lies its value. My only contribution is that of having introduced a neglected formula into photogrammetry.

Chairman: Thank you, Professor Thompson. Let's hope Rodrigues is listening!

Mr. Fereday: I would like to know Professor Ackermann's views on whether he thinks it is best to keep the perspective centre fixed or to float it in the adjustment of the overlap joins.

Professor Ackermann: This is a question which has not been really investigated as yet. Consequently, I can only give my preliminary views until we know better. I do not think that it makes much difference whether we join the common perspective centres of adjacent models rigidly or not. We have evidence from a theoretical study (by Dr. Mohl) that keeping the perspective centres fixed will give slightly inferior results. The explanation for this is probably the inadequate simplicity of the mathematical model. Perspective centres left floating can take up some additional systematic errors or model deformations which otherwise cannot be accommodated. Our adjustment program is prepared to work with either assumption by allowing arbitrary weights for the perspective centres. High weights can constrain the perspective centres up to the point where they are kept fixed.

Mr. Warren: The results which you have achieved from the gradual reduction of control dealt almost exclusively with planimetry. Have you studied the effects of reductions in height control, and do you expect the same kind of results?

Professor Ackermann: I am afraid we have not yet done many three dimensional block adjustments, because the program has been working for only about three months. We have done, however, quite a number of spatial strip adjustments. Regarding the height accuracy of blocks, there are a number of theoretical studies known and very recently Dr. Ebner has completed a very extensive investigation. With 20 per cent lateral overlap the classical arrangement of height control in chains across the block is still the most effective, completed perhaps by additional vertical perimeter control. It is not advisable, except for considerable loss of accuracy, to use scattered control instead of chains inside a block.

Mr. Warren: So the advantages that you are talking about from the reductions to control mainly apply to cadastral work.

Professor Ackermann: No, they apply to planimetry, in general.

Mr. Warren: Which is cadastral really, because in all other applications you really need the height control.

Professor Ackermann: Horizontal and vertical accuracy in a block are virtually independent. It is true that in cadastral applications, only the planimetry is of primary concern. In small scale photogrammetry, however, both planimetry and heights are of importance. Here the extreme reduction of planimetric ground control which is possible by block methods is effective separately. In addition the heights require special and independent attention. It is rather unfortunate that relaxation of height control is not possible to the same degree as it is in planimetry. There is not much one can do about height control except fly 60 per cent lateral overlap or use APR. In fact we consider APR a most effective method of height determination for small scale mapping which requires only a minimum of height control. We are working in Stuttgart on the extension of our block program for the simultaneous adjustment of APR data together with the block adjustment. I feel this could solve the height problem of small scale mapping almost completely, provided the terrain is suitable for APR.

Professor Thompson: You gave an example of the iteration procedure between planimetry and height. To what extent are the heights influenced by the plan positions and *vice versa*?

Professor Ackermann: May I refer to the example which I showed? It demonstrates the interrelations between separate plan and height adjustments. At the beginning the models are tilted which causes errors in the planimetric adjustment depending on the tilts and on the height differences of the ground. *Vice versa*, the height adjustment is affected by incorrect scales of the models. With large height differences of the ground the planimetric adjustment requires precise levelling and the height adjustment requires precise scaling of the models. However, with flat ground the interactions are weak. The example showed that normally two iterations are sufficient. If good approximations are available, one adjustment for planimetry and one adjustment for height will be sufficient. There are some adjustment programs in practical use which make considerable efforts to obtain, first of all, good approximations by preliminary computations. After that one final planimetric adjustment and one final height adjustment is sufficient.

Mr. Mott: Do I understand that, if you have APR running simultaneously, you could dispense with height control except on planimetry?

Professor Ackermann: Yes, I think so, provided that the APR flying is done appropriately. It means that in addition to the standard APR lines together or along with the air survey photographic strips, a number of APR cross flights are required. The spacing of the APR cross flights would be somewhere between 5 and 15 models. Then virtually no height control points will be needed inside the block area. It is even possible to pick up the APR reference heights from outside the block area.

Mr. Mott: Over what distance?

Professor Ackermann: Mr. Mott, I am sure you know more about this subject than I do, as I have no direct APR experience of my own. It seems possible to pick up APR heights from distances up to perhaps 300 km, depending on the accuracy which is finally wanted as, for instance, for 50 m or 20 m contour intervals.

Major Fagan: I would like to ask a question which is more commercial than technical. It seems that a number of people have used this program and I wonder what kind of facilities you offer. Do you, for example, do the computations on behalf of any organisation or do you sell copies of the program?

Professor Ackermann: We have applied our block program for tests, for pilot projects and for practical routine adjustments, in particular for cadastral surveys in Germany. We have rendered this service because we wanted to gain practical experience and because it was essential to demonstrate the successful application under practical conditions. Also similar programs on powerful computers were not available in the area. However, our computing service is most likely to be reduced in the near future as soon as our state governments become equipped with adequate computers, for which we shall make the program available.

Chairman: It only remains for me to bring this meeting to a close. It is not often that we have a lecturer from the other side of the English Channel. I suppose that by 1973 the European Economic Community will give such an occasion a different impact, but just for the moment you are still a foreign guest lecturer, Professor Ackermann. We are delighted to have had you with us for a most interesting lecture and for a discussion period which has certainly lasted longer than usual. That is a good indication of the interest which we have found in what you had to say. Let us thank Professor Ackermann for a most interesting discussion.