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Blockadjustment Methods in Photographic Astrometry*

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Blockadjustment methods enable not only a rigorous treatment of large photogrammetric networks but have proved to be also an adequate tool for evaluating stellar positions from overlapping photographic plates. Extensive theoretical investigations have given a very favourable propagation of random errors as well for photogrammetric as for astrometric blocks. Compared with conventional solutions in both cases the accuracy is considerably improved and the number of control points or reference stars, respectively, can be reduced.

Theoretical calculations based on the inversion-method and simulation-technique provide accuracy models for studying the influence of single block parameters on the accuracy of the desired unknown star coordinates. The main results with special emphasis to astrometric applications are summarized.

The error propagation inside the block area mainly depends on the adopted relationship between measured and standard coordinates. A general mathematical model for performing the blockadjustment is discussed which takes into account different types of plate constant parameters. The general normal equations, resulting from a blockadjustment, are reduced to a sub-system with banded-bordered coefficient matrix containing only the plate constants as unknowns.

Based on a computer program, developed at the University of Stuttgart for the solution of linear equations with banded-bordered matrix, estimates for computing time and necessary external storage capacity are given.

Extensive astrometric data material is available the inherent accuracy of which could be fully exploited by a new measurement of the original plate material combined with a blockadjustment. The examples to be discussed here cover the most typical block patterns which arise with astrometric catalogue material. In each case a suitable ordering scheme of the plates is adopted to provide optimal band-structure of the normal equations.

1. The Vatican Zone of the Astrographic Catalogue

The Vatican zone covers the regions of a considerable number of young open clusters and associations. A remeasurement of the plates, which are still in good condition, combined with a blockadjustment will yield positions with a m.e. of $\sigma = 0\overset{''}{.}15$ (average epoch 1915). Taking new-epoch plates, proper motions with an accuracy of $\sigma = 0\overset{''}{.}004$/a can be obtained. The necessary computing time amounts to about 2 hours on a CDC 6600.

2. The AGK2 Plate Material

The fully inherent information of this extensive data material covering the northern hemisphere down to $\delta = -2\overset{\circ}{.}5$ with homogeneous limiting magnitude has not been used to establish the AGK2 catalogue.

A new measurement will provide positions for all stars with at least $m_{pg} = 12$ in the FK4 system, yielding an estimated accuracy of $\sigma = 0\overset{''}{.}14$.

A suitable reduction model including magnitude terms is established. Adopting an ordering scheme, similar to a zonal arrangement of the plates, the bandwidth can be minimized. A simultaneous blockadjustment of this whole hemisphere can be performed within about 3 hours computing time on a CDC 7600 or IBM 360/91 computer.

3. A Fourfold Coverage of the Whole Sphere

Results from theoretical accuracy investigations of the blockadjustment of a whole sphere are extrapolated to the case of a fourfold coverage of the whole sky. A reduction model, yielding optimal error propagation is established. The resulting computing effort will amount to 5 hours on a CDC 6600. According to current activities in the southern hemisphere, only a new coverage of the northern part of the sky would be necessary. The question of performing the necessary reference star positions, taking into account an extrapolation of the AGK2/3 data or new observations of high accuracy, is discussed. From a simultaneous blockadjustment of the whole sphere a positional accuracy of $\sigma = 0\overset{''}{.}10$ will be expected. The considered new fourfold coverage of the northern hemisphere as a part of the closed block combined with a new reduction of the AGK2 plates will provide proper motions with an accuracy of at least $\sigma = 0\overset{''}{.}005$/a for all stars with $m_{pg} \leqq 12$.

Key words: astrometry — photographic zone catalogue — blockadjustment — overlap technique — proper motion

1. Introduction

During the last decade a quite analogous evolution has taken place independently in the discipline of analytic photogrammetry and of photographic astrometry. The main common activities under consideration are the theoretical and practical aspects of a simultaneous adjustment of large photogrammetric blocks and the evaluation of stellar positions from overlapping photographic plates respectively. In the field of astrometry this reduction scheme, now known as "overlap reduction technique", has been generally introduced by H. Eichhorn (1960).

Closer theoretical investigations of these two methods, which will be denoted as blockadjustment in the following, have shown a very favourable propagation of random errors, both for photogrammetric blocks and astrometric blocks. Compared with conventional solution schemes where each plate is reduced as an individual unit, the accuracy of the resulting stellar positions or photogrammetric points is considerably improved and the number of control points or reference stars, respectively, can be reduced.

Photographic positional catalogues with a limiting magnitude of about $m_{\text{vis}} = 9$ covering the whole sphere with a homogeneous star density and a systematic positional accuracy of about $\sigma = 0\overset{''}{.}1$ are of primary importance for all purposes of positional astronomy and related fields, such as satellite-geodesy or time and latitude service. Of increasing value in the future will be an extension to still fainter limiting magnitudes, say $m_{\text{vis}} = 12$, with similar high accuracy to provide more representative samples for investigations concerning kinematical properties of different classes of galactic objects. Stellar positions of these fainter magnitudes could in addition be used as a second-order reference frame to provide independent optical positions of faint radio sources, where large magnitude intervals have to be covered. Although new photographic catalogue projects can be established, taking into account previously all necessary conditions for an optimal application of blockadjustment reduction, the evaluation of proper motions will still depend on the availability of old-epoch catalogue material. In recent years only a rather limited part of old-epoch data has been treated by the method of blockadjustment. There is considerably more material available, whose inherent accuracy could be fully exploited by new data-reduction using blockadjustment methods. For example in the case of the AGK2 catalogue and some of the zones of the Astrographic Catalogue the whole information contained in the original plate material has not been used for establishing the catalogues.

The present paper intends to summarize briefly some of the widely spread results concerning theoretical accuracy investigations with special emphasis on astrometric applications. Furthermore, suitable reduction models and the application to existing catalogue material, including estimates of computing effort, will be discussed.

2. Theoretical Investigation-Techniques Concerning the Accuracy Obtainable with Blockadjustment Methods

Theoretical investigations of this kind can be used to obtain accuracy models. Principally there are two possibilities for accomplishing such studies: the simulation technique and the inversion method. Both methods can be used to study the influence of single block-parameters on the accuracy of the desired unknowns which are the star coordinates or photogrammetric points. The main block-parameters are: blocksize and geometrical block structure, number and distribution of common stars in the overlapping area of the adjacent plates, number and type of the plate constants used in the mathematical relationship between measurements and unknowns, and finally number and distribution of reference stars in the whole block.

2.1 Simulation

After having adopted the block-parameters, the resulting normal equations (6) are solved with a suitable number of different sets of "observations" as right-hand sides. These observations are generated from theoretical error-free spherical coordinates which are then transformed back into error-free "measured" coordinates $x; y$. The $x; y$ coordinates are now superposed with random errors from a normal distribution with suitably adopted distribution parameters. Different weights or correlations between the observations can be included in a simple way. Each set of observations generated by this procedure provides a similar set of adjusted positions $\bar{\alpha}; \bar{\delta}$, after having solved the normal equations (6). The deviation of the $\bar{\alpha}; \bar{\delta}$ from the adopted error-free theoretical values gives an estimate for the standard deviations (m.e.) σ_α, σ_δ of the adjusted coordinates.

2.2 Inversion

From the adopted accuracy properties of the observations the associated covariance matrix C^{bb} can be constructed. The inversion of the resulting matrix N of the normal equations now provides the covariance matrix C^{xx} of the unknowns, the elements of which contain the theoretical variances of the spherical coordinates $\bar{\alpha}$; $\bar{\delta}$.

2.3 Discussion and Numerical Results

The simulation technique has the advantage that the accuracy properties of the observations can be varied without influencing the computing effort. In addition, no inversion of the normal equation matrix N is necessary. On the other hand there is the disadvantage that the resulting variances $\sigma_{\bar{\alpha}}^2, \sigma_{\bar{\delta}}^2$ depend on the arbitrarily adopted observations. Statistically significant therefore only mean values taken over many or all variances are.

The inverted matrix N^{-1} yields the proper values of the theoretical variances for each single coordinate $\bar{\alpha}$; $\bar{\delta}$, because N does not depend on the observations themselves. So the accuracy properties of adjusted blocks can be studied in detail. However, the inversion requires more computing time than a solution of the normal equations alone, as it is necessary for the simulation technique. In addition the computing effort is further increased if the observations are highly correlated.

In recent years a number of encouraging results have been obtained both in the programmetric field and the application to astrometric blocks. The main result is the very high and homogeneous accuracy of the adjusted coordinates in the whole block area if the known reference points are distributed with suitable density on the boundary of the block area. No additional reference points inside the block are necessary. A block containing $n = 200$ single units yields an average m.e. of all adjusted coordinates of $1.1\,\sigma_0$ and a maximum m.e. of $1.2\,\sigma_0$, where σ_0 denotes the mean error of unit weight. (Ackermann, 1967). A further enlargement of the block by up to $n = 10.000$ units yields a resulting maximum m.e. of only $1.5\,\sigma_0$, which appears in the central region of the block area. In addition it was found that both the average m.e. and the maximum m.e. only increase as $\log(n)$ (Ebner, 1970a). As these investigations are carried out in a plane, while all astrometric applications are a priori spherical problems, a direct extrapolation to astrometry should be made with some caution. However, a comparable example would be the blockadjustment of a limited sky region as occurs in the case of an association or cluster field. Using the simulation technique applicated to a cluster field, an improvement in accuracy of 20 percent has been found, compared with the classical astrometric treatment of each plate as an individual entity (de Vegt, 1967).

The case of a closed block where the whole celestial sphere is covered by a suitable net of partly overlapping photographs has been investigated in detail by Brown (1968) and Ebner (1969, 1970b) using the inversion method. Although both authors have assumed different arrangements of plates and number of unknown plate constants, a common result can be summarized as follows: assuming a mean error of unit weight $\sigma_0 = 2\,\mu$ for the measured rectangular coordinates on each plate, a final accuracy of 0."1 for the adjusted star positions can be reached with just a small number $n \ll 1000$ of reference stars for the whole block. Compared with an adjustment of a smaller region of the sphere or even a zonal pattern as it is realized in most existing astrometric plate material, the closed block will give the best possible results concerning error propagation inside the block, accuracy of the adjusted star positions and necessary minimum number of reference stars.

3. Choice of a Suitable Mathematical Model

3.1 Nonlinear Observation Equations

It seems to be most adequate for blockadjustment problems to use the formulation of indirect unconditional measurements in setting up the observation equations. For each measured coordinate a separate observation equation is set up. If the star i appears on plate k, the measured coordinates $x_{i,k}$, $y_{i,k}$ depend on the spherical coordinates $\bar{\alpha}_i$, $\bar{\delta}_i$ and on the plate constants $\bar{a}_k$, $\bar{b}_k$, $\bar{c}_k \ldots$ so one gets the following equations:

$$x_{i,k} + v x_{i,k} = f_1(\bar{\alpha}_i, \bar{\delta}_i, \bar{a}_k, \bar{b}_k, \bar{c}_k \ldots)$$

$$y_{i,k} + v y_{i,k} = f_2(\bar{\alpha}_i, \bar{\delta}_i, \bar{a}_k, \bar{b}_k, \bar{c}_k \ldots) \qquad (1)$$

where $v x_{i,k}$ and $v y_{i,k}$ are the residuals, which are minimized in the least squares adjustment. In principle arbitrary assumptions concerning the accuracy properties of the measured coordinates $x_{i,k}$ and $y_{i,k}$ are possible. In most cases, however, they will be assumed uncorrelated and of equal accuracy.

If the star i is a reference star, there will be the two additional observation equations:

$$\alpha_i + v\alpha_i = \bar{\alpha}_i$$
$$\delta_i + v\delta_i = \bar{\delta}_i$$

where α_i and δ_i are the known coordinates of the reference star i, $v\alpha_i$ and $v\delta_i$ are the corresponding residuals. As in the case of the measured rectangular star coordinates, the accuracy properties of the reference star coordinates can be arbitrarily chosen. In most cases α_i and δ_i will be assumed uncorrelated with different weights.

3.2 Plate Constants

In general one has to distinguish between two different types of plate constant parameters: individual parameters belonging only to one plate, and parameters which are common to several plates or groups of plates. As an example, terms which represent an existing optical distortion of the camera objective would be treated as common plate constants for all plates.

Concerning the adjustment method, the plate constants may be interpreted in two ways. Usually they are treated as free unknowns, as is done in the nonlinear observation equations (1). This standpoint is, of course, appropriate for parameters which can attain large values. On the other hand there will be parameters, the influence of which on the measured coordinates is only of the order of the mean error of unit weight so that a treatment as free unknowns, which theoretically may attain arbitrary large values seems to be unnecessary and dangerous in some respects because each additional unknown changes the error propagation for the worse. This has been clearly shown by Eichhorn and Williams (1963) in the case of classical plate reduction schemes.

In our case it will be more advantageous to treat these plate constants as free unknowns, but to introduce their expectation-values as additional observations with appropriate accuracy properties. If, for example, the plate constant $\bar{c}_k$ is treated in this way in addition to (1) and (2), the following observation equation has to be added:

$$c_k + vc_k = \bar{c}_k \tag{3}$$

The expectation-value c_k and its accuracy have to be estimated from investigations previous to the blockadjustment. In the case of a very uncertain previous knowledge of the exact value of c_k one has the possibility of starting with $c_k = 0$ and to assign only a small weight to the observation equation (3). After a first least squares solution, a correction to the assumed weights can be found from a more detailed study of the adjustment residuals. In practical applications affin- or tilt-terms would be unknowns which could be treated according to equation (3), because their influence on the measured coordinates will be small in most cases. As good estimates of the values which these unknowns could attain can be achieved in a special problem from a previous classical reduction of the plate material, which is always necessary to check the observational material for erroneous data, suitable weights can be assigned.

With suitable weight adopted, equation (3) provides the possibility of handling the unknown parameter $\bar{c}_k$ throughout its full range. If one assigns the weight ∞ (which corresponds numerically to $10^{10} \sim 10^{20}$) to c_k, the result of the adjustment will be exactly $\bar{c}_k = c_k$; in this case $\bar{c}_k$ is no longer a free unknown in the adjustment problem. On the other hand, if one assigns the weight 0 to c_k equation (3) does not affect the adjustment, i.e. c_k acts now as a free unknown. These two examples show the generality of this approach. As the knowledge of the accuracy properties of $\bar{c}_k$ is incorporated in the adjustment problem, an approach of this kind is more rigorous from the statistical standpoint than a treatment of $\bar{c}_k$ as free unknown without adding equation (3).

3.3 Linearisation of Observation Equations

As the blockadjustment is a nonlinear problem, a linearisation of the observation equations (1) is necessary. From a previous classical reduction of the single plates approximate values for the unknown spherical coordinates $\bar{\alpha}_i$, $\bar{\delta}_i$ and the plate constants $\bar{a}_k, \bar{b}_k, \bar{c}_k \ldots$ can be found, using rough data from existing catalogues so that the coefficients of the linearized system (4) can be evaluated. Practical experience has shown that, although the linearisation in principle requires an iterative treatment, in most cases a first solution of the corresponding normal equations (6) already provides the final adjustment result.

The complete system (1), (2), (3) of the observation equations contains as unknowns the corrections $\Delta\bar{\alpha}; \Delta\bar{\delta}$ to the spherical coordinates of all field and reference stars and correction $\Delta\bar{a}_k; \Delta\bar{b}_k; \Delta\bar{c}_k \ldots$ to the plate constants. The linearized system of observation equations then takes the form

$$\boldsymbol{v} = \boldsymbol{A}\boldsymbol{p} + \boldsymbol{B}\boldsymbol{k} - \boldsymbol{f} \tag{4}$$
$$\boldsymbol{f} = \boldsymbol{b} - \boldsymbol{a}_0 \tag{5}$$

with the following notation:

$\boldsymbol{b}$: Vector of observations

$\boldsymbol{v}$: Vector of residuals

$\boldsymbol{p}$: Vector of unknown positions (corrections $\Delta\bar{\alpha}$, $\Delta\bar{\delta}$)

$\boldsymbol{k}$: Vector of unknown plate constants (corrections $\Delta\bar{a}_k$, $\Delta\bar{b}_k$, $\Delta\bar{c}_k, \ldots$)

$\boldsymbol{A}$, $\boldsymbol{B}$: Matrices of coefficients

$\boldsymbol{a}_0$: Absolute vector

3.4 Reduction and Solution of Normal Equations

If $\boldsymbol{G}$ denotes the covariance matrix of all observations from (4), (5), the following system of normal equations results:

$$\begin{bmatrix} \boldsymbol{A}^T \boldsymbol{G}^{-1} \boldsymbol{A} & \boldsymbol{A}^T \boldsymbol{G}^{-1} \boldsymbol{B} \\ \boldsymbol{B}^T \boldsymbol{G}^{-1} \boldsymbol{A} & \boldsymbol{B}^T \boldsymbol{G}^{-1} \boldsymbol{B} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{p} \\ \boldsymbol{k} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}^T \boldsymbol{G}^{-1} \boldsymbol{f} \\ \boldsymbol{B}^T \boldsymbol{G}^{-1} \boldsymbol{f} \end{bmatrix} \qquad (6)$$

Because the total number of unknowns in (6) is usually very large and the system contains in practice many more unknown positions than plate constants, a reduction of the original normal equations (6) to a system which contains only the unknown plate constants is advisable. Using common matrix algebra, the resulting system of equations can be written as follows:

$$[\boldsymbol{B}^T \boldsymbol{G}^{-1} \boldsymbol{A} (\boldsymbol{A}^T \boldsymbol{G}^{-1} \boldsymbol{A})^{-1} \boldsymbol{A}^T \boldsymbol{G}^{-1} \boldsymbol{B} - \boldsymbol{B}^T \boldsymbol{G}^{-1} \boldsymbol{B}]\, \boldsymbol{k}$$
$$= [\boldsymbol{B}^T \boldsymbol{G}^{-1} \boldsymbol{A} (\boldsymbol{A}^T \boldsymbol{G}^{-1} \boldsymbol{A})^{-1} \boldsymbol{A}^T \boldsymbol{G}^{-1} - \boldsymbol{B}^T \boldsymbol{G}^{-1}]\, \boldsymbol{f} \qquad (7)$$

After having solved the reduced system (7), the known plate constants $\boldsymbol{k}$ can now be substituted in (6) to compute the desired star positions $\boldsymbol{p}$.

3.5 Numerical Solution and Estimation of Computing Effort

The matrix of the reduced normal equations according to (7) takes in general the form of a symmetric banded-bordered $n \cdot n$ matrix (Fig. 1) with bandwidth m and borderwidth l, where n denotes the number of unknown plate constants. The bandwidth depends not only on the adjustment model, i.e. number of unknown plate constant parameters per plate and the number of plates on which the same star appears, but in addition a strong dependence on the adopted plate-ordering scheme and the geometrical block structure itself in each special block under consideration exists. As the computing effort is mainly determined by the bandwidth, an optimal ordering-scheme has to be established in each practical case. (See section 4 of this paper.)

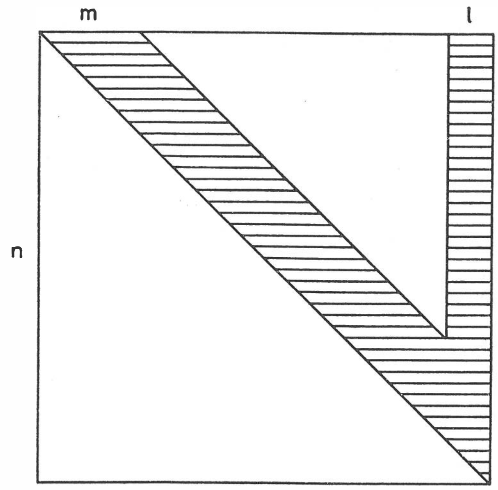

Fig. 1. Symmetric banded-bordered coefficient-matrix of reduced normal equations (elements below main-diagonal are suppressed)

The borderwidth depends directly on the number of unknowns common to all plates or groups of plates, but as previously on the ordering-scheme, too. The border allows one to accommodate unknowns in the adjustment-problem which would represent effects common to all plates, for example field distortions or magnitude-terms which may depend on the telescope optics. In each case, however, bandwidth and borderwidth should be as small as possible to guarantee optimal utilization of available storage capacity and computing time.

The following estimate of the computing effort necessary to solve a given linear system of normal equations with banded-bordered matrix is based on a computer program which has been developed at the Institute for Photogrammetry in Stuttgart by Klein and Ebner. The program uses a generalized Gauss-Cholesky direct solution scheme which is applied to submatrices of a given hypermatrix (Klein, 1971). The linear system of equations can be solved simultaneously with several right-hand sides without noteworthy increase of computing time. This facility is of special advantage for theoretical accuracy investigations with several simulated sets of observations as described in section 2. Selected columns of the inverse matrix can be computed by inserting the corresponding columns of the unit matrix as right-hand sides. The resulting computing time for the solution of blockadjustment problems with a banded-bordered $n \cdot n$ matrix can be estimated

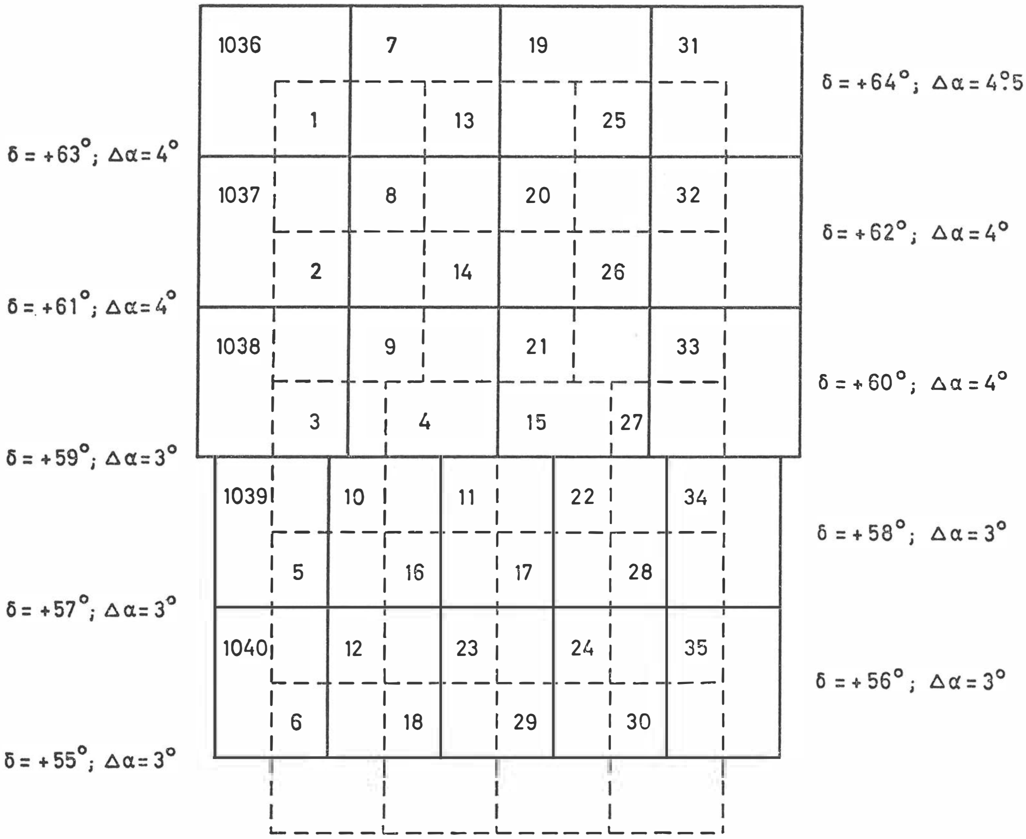

Fig. 2. Overlap pattern and ordering scheme for the Vatican zone of the Astrographic Catalogue

with an uncertainty of 10% from the following simple expressions:

$$n_{\text{mult}} \approx nm^2/2 + nm \tag{8}$$

$$t \approx 1.1 \cdot 10^{-9} \cdot n_{\text{mult}} \tag{9}$$

where n_{mult} is the number of multiplications during the solution process and t denotes the total computing time in hours on a Control Data Computer CDC 6600. The additional computing effort for establishing the reduced normal equations and the computation of the final positions after having solved the reduced system, can be neglected to the first order.

The non-vanishing elements of the coefficient matrix of the reduced system are stored in sub-matrix-form in an external storage (disk memory). The necessary storage capacity n_s (in full words) is given by the expression

$$n_s \approx n(m + l) \tag{10}$$

4. Examples for Suggested Blockadjustment Application to Various Astrometric Catalogue Material

In recent years only a few applications to astrometric data material have been made. With the exception of the Plejades catalogue (Eichhorn *et al.*, 1970) where all plates have been treated in a simultaneous adjustment, the other catalogue material such as the Hyderabad zone of the Astrographic Catalogue (Eichhorn and Gatewood, 1967) and the overlap approach to the AGK2/3 material (Lacroute and Valbousquet, 1970a), have been treated either partly by blockadjustment or else a division in zones of suitable extension has been performed.

The examples to be discussed now cover the most typical geometric patterns which arise with available plate material. In all cases the blockadjustment can be performed as one entity with realizable computing effort, even in the case of a closed sphere, by adopting a suitable ordering scheme.

4.1 Vatican Zone of the Astrographic Catalogue

This zone covers the area from 55° to 64° declination with 1040 plates, centered on the parallels corresponding to all full degrees of declination in the above interval (see Fig. 2). The plates were taken with a so-called Normal Astrograph, $f = 3437$ mm, producing a scale value of about 1′/mm in the focal plane, the approximate field being 130′ by 130′. Each plate contains 3 exposures with approximate durations of 6^m, 3^m, and 20^s. According to the original plan the second exposure should reach $m_{pg} = 11$. In practice, however, fainter magnitudes up to $m_{pg} = 14$ have often been reached (de Vegt, 1966).

The Vatican zone intersects the galactic equator over the longitude interval $100° < l^{II} < 145°$, so first epoch positions for a considerable number of young open clusters and associations could be made available. With an average epoch-difference of 65 years up to the present, proper motions of high accuracy could be obtained. Because the published rectangular catalogue-coordinates have been obtained using the Turner method for carrying out the measurements and in addition not all exposures of each star have been used, the inherent accuracy of the original plate-material has never been fully exploited, nor has the possibility of systematic errors depending on magnitude or colour been taken into account in the printed catalogue data. Recent experience with a smaller sample of the original plate material has shown that the emulsions are still in good condition for new measurements (de Vegt, 1971a).

Reduction Model and Estimation of Computing Effort

The following relationship between measurements x, y and standard-coordinates ξ, η is established

$$\xi = ax + by + c + \gamma m + \lambda mx + e(x^2 + y^2)x$$
$$\eta = a'x + b'y + c' + \gamma' m + \lambda' my + e(x^2 + y^2)y \quad (11)$$

The expressions (11) include the usual linear plate constants ($a; b; c; a'; b'; c'$), terms which compensate possible systematic influences depending on magnitude ($\gamma; \lambda; \gamma'; \lambda'$) and a parameter e, common to all plates, which could compensate a 3rd-order optical distortion as seems to be present according to Günther and Kox (1970). All three exposures on each plate will be treated as independent entities.

From the printed catalogue data of the Vatican zone a positional accuracy of $\sigma = 0\farcs32$ can be obtained, performing a classical reduction with the AGK2/3 catalogue as a reference frame (de Vegt, 1966; Dieckvoss, 1970). From a new measurement $\sigma = 0\farcs17$ has been obtained (de Vegt, 1971a). The performance of a blockadjustment will further improve the statistical significance of magnitude-terms and a final overall accuracy of $\sigma = 0\farcs15$ is estimated. Combining these positions with modern new-epoch plates, proper motions with an accuracy of at least $\sigma = 0\farcs004/a$ can be obtained.

Fig. 2 shows the optimal ordering scheme for the Vatican zone. The plates are ordered transverse to the zonal extension. Adopting 10 individual parameters for each exposure and 1 parameter common to all plates according to (11), the following numerical results for the matrix parameters $n; m; l$; of the reduced normal equations (1040 plates) are obtained:

$$n = 1040 \cdot 10 \cdot 3 = 31200 .$$

Because the connexion extends to 8 plates on the average, one gets for the bandwidth: $m \approx 8 \cdot 10 \cdot 3 = 240$. According to the ordering scheme the first 5 plates and the last 5 plates are connected, in addition 1 common parameter is adopted, therefore the borderwidth extends to $l = 5 \cdot 10 \cdot 3 + 1 = 151$. From (8), (9) a computing time of $t \approx 2.2$ hours is obtained. The required external storage capacity $n_s \approx 12 \cdot 10^6$ words, according to (10). These figures show that a blockadjustment of a zonal pattern can be easily performed.

4.2 The AGK2 *Plate Material*

The photographic plates, which will be considered here, were exposed at the observatories of Bonn (720 plates, declinations between +20° and −2°) and Bergedorf (1219 plates, declinations between +20° and +90°) with two similar zone-astrographs equipped with four-component objectives (corrected for the blue spectral region) and $f = 2060$ mm focal length producing an approximate scale value of 100″/mm. The useful plate area covers 5° × 5°. All plates were taken in a time interval of about 3 years centered around 1930.0.

In accordance with the proposals made by the zone commission of the Astronomische Gesellschaft (Bauschinger, 1927), each plate contains 2 exposures of 10^m and 3^m duration, but for establishing the AGK2 catalogue only the 10^m image was measured. Because the AGK2 is mainly a photographic repetition of the visually observed first zone catalogue of the Astronomische Gesellschaft (AGK1), the great majority of the measured stars has magnitudes up to about $m_{pg} = 9.5$. The photographic plates however go far beyond this limit. With the 10^m exposure all stars with at least $m_{pg} = 12$ were recorded, whereas the 3^m exposure does already include all AGK2 catalogue stars, and about $m_{pg} = 11$ has been reached.

The aforementioned situation confirms that the AGK2 plate material contains at least that information which it has been the aim of the Astrographic Catalogue project to obtain. Now, however, the AGK2 material covers the whole northern hemisphere with a homogeneous limiting magnitude, all plates being taken practically at the same epoch, the complete plate material being still immediately available and in excellent condition for a new measurement. Using the AGK2A catalogue (transformed to the FK4-system) for reference star positions, the expected positional accuracy from a new reduction will be $\sigma = 0\overset{''}{.}14$. Because of their mostly poorer quality and inhomogeneous limiting magnitude, a remeasurement of the AGK3 plates should not be considered. The epoch difference of 40 years up to now, combined with a new coverage of the northern hemisphere or the whole sky, as will be discussed in the following section, will provide proper motions in the FK4-system with an accuracy of at least $\sigma = 0\overset{''}{.}005/a$. Using a Galaxy-type automatic measuring machine the new measurement of all plates could be performed in about 1 year's time.

Reduction Model and Computing Effort

The large area covered by this blockadjustment requires a restriction of the number of unknown parameters to preserve favourable error propagation. It has therefore been decided not to introduce higher order terms in the adjustment. Necessary corrections due to imperfect knowledge of tangential point-coordinates or tilt are available from previous adjustments of each single plate. According to recent investigations by Lacroute and Valbousquet (1970b) and de Vegt (1971b), a magnitude equation may be present. The following relationship is therefore established for the blockadjustment

$$\xi = ax + by + c + \lambda m x$$
$$\eta = a'x + b'y + c' + \lambda' m y \qquad (12)$$

Both exposures will be treated as independent entities.

In order to get an optimal band-structure for the reduced normal equations, the plates should be arranged along zones as shown in Fig. 3. With two exposures and 8 plate-parameters for each exposure according to (12), the resulting bandwidth (the borderwidth is $l = 0$) for a zone containing n_z plates is:

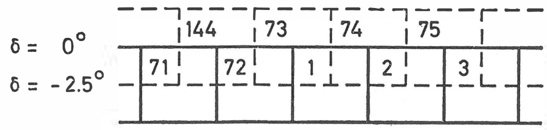

Fig. 3. Overlap pattern and ordering scheme for the AGK 2 plates

$$m = 2 \cdot 8 (n_z + 2) \qquad (13)$$

Because the bandwidth depends on the number of plates per zone, which is different in each zone, the relations (8) and (10) have to be generalized:

$$n_{\text{mult}} = \sum_1^n \frac{m^2}{2}\,; \quad (14) \qquad n_s = \sum_1^n m\,; \quad (15)$$

For a simultaneous adjustment of the 3878 exposures of the AGK2 (about 31.000 unknown plate constants) the following estimates are therefore obtained:

$$n_{\text{mult}} \approx 16 \cdot 10^9\,; \; t \approx 1.1 \cdot 16 \approx 18 \text{ hours}\,;$$
$$n_s \approx 31 \cdot 10^6\,;$$

A computing time of this order requires changing to a faster computer model. Taking into account the known conversion factors, the computing time is reduced to about 3 hours on a CDC 7600 or IBM 360/91. In addition the required larger external storage can be provided.

4.3 A Fourfold Coverage of the Whole Sphere

Optimal and homogeneous positional accuracy is expected from a blockadjustment reduction of a closed sphere. To take full advantage of the method, a generous overlapping of the photographic plates should be taken into account. A good compromise with respect to the necessary effort of taking and measuring the plates would be a fourfold overlap pattern, which consists of two superposed nets, each similar to the AGK2 plate arrangement (Fig. 3). The astrometric camera to be used could be similar to a zone-astrograph, as described in section 4.2, but should be corrected for the visual spectral region and fitted with a yellow filter to decrease the effect of atmospheric dispersion. Instead of taking double exposures, a coarse grating should be preferred to minimize magnitude effects.

Reduction Model and Computing Effort

According to the previously quoted theoretical investigations by Brown and Ebner (cf. section 2.3), the blockadjustment of a closed sphere will yield optimum results. But in addition, after having fixed the block structure, the resulting positional accuracy will depend strongly on the adopted reduction model (Ebner, 1969). We therefore propose to adopt a linear orthogonal relationship between measurements and standard coordinates

$$\xi = \quad ax + by + c$$

$$\eta = -bx + ay + c' \qquad (16)$$

which will provide a minimum number of unknown plate constants in the adjustment. This approach requires an orthogonal measurement of the rectangular star coordinates $x; y$ on each plate, furthermore the $x; y$ have to be corrected for all non-orthogonal terms due to differential refraction and aberration. The astrograph itself has to be monitored continuously for tilt and tangential point changes.

A positional accuracy of $\sigma = 0\overset{''}{.}1$ or even better will be expected from this blockadjustment. The computing effort for a closed block application can be estimated from the results of section 4.2, taking into consideration the following modifications:

closed block over both hemispheres
fourfold overlap pattern
4 plate constants
1 exposure per plate, taken with a coarse grating.

The plates are ordered in a zonal pattern according to 4.2. The resulting computing time is reduced to a quarter of the previous effort, and only half of the external storage capacity is required; this amounts to $t \approx 5^h$; $n_s \approx 16 \cdot 10^6$.

Concerning a realization of this project, current activities of the Cape Observatory (Clube, 1970) will already provide a fourfold coverage of the southern hemisphere. So new plates have to be taken only for the northern hemisphere. With both hemispheres having a certain region in common, possible systematic differences between both systems could be evaluated. As a final step the blockadjustment of the closed sphere could be performed.

Finally the question of a reference frame for the closed block application has to be discussed. While for the southern hemisphere this reference frame will be provided by the SRS program, the present situation is more difficult for the northern part.

Taking into consideration the AGK2/3 catalogue data as a possible reference frame, an extrapolation to, say 1975, will yield a m.e. of $\sigma = 0\overset{''}{.}28$ for one reference star position. To compensate for this considerable loss in accuracy, as compared for example with the AGK3R ($\sigma = 0\overset{''}{.}11$), the number of reference stars per plate has to be increased by at least a factor 3. This would give rise to a strong correlation of the new catalogue positions with the AGK2/3 data. Further on, existing local systematic deviations from the overall FK4-system which could amount to $0\overset{''}{.}08$ at 1975 in each individual small field of the sky, will be introduced in the new catalogue system (Dieckvoss, 1970). To be fully independent of previous data we therefore would prefer to have new observations of reference star positions with high accuracy ($\sigma = 0\overset{''}{.}1$).

However, the actual number of reference stars, necessary in a blockadjustment to obtain a prescribed accuracy of the final photographic positions, cannot be extrapolated without caution from the results quoted in section 2.3. Only in the simplest case of an orthogonal relationship as considered in the present section an extrapolation to astronomical block sizes may be justified. In this case a number of 1000 reference stars should be sufficient to provide the desired catalogue accuracy.

5. Conclusion

As has been discussed in the previous section extensive astrometric catalogue material is available, its fully inherent positional accuracy could now be evaluated using blockadjustment techniques combined with a new measurement of the original plate material. In addition a simultaneous adjustment of the whole sphere, where new plates have to be taken only for the northern part, will provide a homogeneous net of stellar positions. Combined with the remeasured AGK2 plates proper motions of high accuracy and homogeneity as a basis for kinematical studies of the galactic system will result for the northern hemisphere.

Blockadjustment will here be used under two mainly different aspects. In the case of old-epoch catalogue material most of the block parameters are prescribed and systematic errors depending on magnitude and colour have to be taken into account. The inclusion of all stars in common regions of adjacent plates will provide a larger range of those variables than can be provided by a classical reduction where only reference stars are available most of which being selected out of a small magnitude range. In contrast to the classical reduction scheme further on unknown parameters, common to all plates, can be introduced. The resulting, often complicated reduction model, will not provide optimal error propagation, this could be compensated by using all available reference stars. Application of block-adjustment in this case will strengthen the statistical significance of those terms, and a more homogeneous positional accuracy in the block area will be achieved.

For any new project optimal choice of block structure and parameters can be realized in accordance with the results of theoretical investigations from section 2. In this case the number of reference stars could be reduced as compared with a classical reduction scheme. This seems to be an important fact considering the current decreasing activities in meridian-circle work. However, the actual number of reference stars needed depends strongly on the adopted block structure and reduction model and cannot be extrapolated from available theoretical investigations with the exception of the simple case (section 4.3). For each special block situation under consideration, theoretical investigation techniques, as described in section 2, will provide all necessary information on a rigorous standpoint. This will be discussed in detail in a forthcoming paper.

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