*Combined Block Adjustment of APR Data and Independent Photogrammetric Models**

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Based on the computer program PAT-M for block adjustment by independent models, an extended system is presented for the combined simultaneous block adjustment of photogrammetric models with APR and/or statoscope data, including photogrammetric height measurements of shorelines of lakes, if available. The system is intended for small- and medium-scale topographic mapping. Because of possible application to very large blocks, is is expected to be of great economic importance.

The paper develops the basic mathematical formulas for the combined adjustment. In view of efficient and economic computation, some modifications are proposed, which the present programing of the system takes into account.

La compensation simultanée des données photogrammétriques et de certaines données auxilliaires appliquée à de très grands blocs de couples stéréoscopiques présente des avantages économiques très intéressants. On propose une méthode fondée sur le programme de calcul PAT-M qui permettra une telle compensation. Le système est conçu en vue de l'établissement de cartes topographiques à moyenne et à petite échelle. Les données auxilliaires pourront consister de mesures A.P.R., mesures au statoscope et de lectures photogrammétriques prises sur les bords de lacs. On dérive les formules qui permettent cette compensation simultanée. On envisage certaines modifications au programme existant afin de réaliser un calcul efficace et économique.

1. Introduction

In small-scale mapping, the specifications for horizontal accuracy are quite different from those for vertical accuracy. The same is true also for small-scale aerial triangulation.

The modern development of block-adjustment procedures — fully analytical or by independent models — permits simultaneous adjustment of large blocks, even of extremely large blocks consisting of up to several thousand models. With such blocks, the well-known favorable properties of horizontal accuracy can be fully exhausted; scarce horizontal control at the block perimeter is quite sufficient to meet specifications for small- and medium-scale topographic maps for areas covering up to 100,000 km² and more. This breakthrough seems to greatly simplify or to solve, in a way, the stringent problem of horizontal control for small-scale mapping.

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The situation, however, is very much different for vertical control. Block adjustment does not result in the saving of vertical control to the same degree as it does of horizontal control, not even when 60 per cent sidelap is used. For practical reasons, most of the small-scale flying is done with about 20 per cent sidelap which leaves the problem of vertical control most stringent. In particular 20-, 25- and 50-foot contouring require considerable density of control points. The provision of vertical control for large and remote areas, therefore, remains an economic and technical problem, even when barometric methods are applied, with or without the use of helicopters.

It seems that the use of auxiliary data, particularly from the statoscope, or airborne profile recorder (APR), offers the only practicable solution to the economic provision of the required vertical information. In fact, APR is used extensively in countries that have systematic mapping programs for large and remote areas [*EMR* 1968; *Lambert* 1970]. APR terrrain profiles are obtained together with the air-survey mission, or by separate low altitude APR flights. Preferably, the low-altitude APR profiles are flown along the lateral overlap areas of the photostrips, with additional cross profiles. In *Kure* [in print] the use of a network of crossing APR lines is reported.

The adjustment of APR profiles and their use for aerial triangulation are usually much simplified. The profiles, starting and closing over known areas, are adjusted by applying linear corrections. From the adjusted profiles, vertical control points are drawn for conventional strip and block adjustment. Such procedures seem to work sufficiently well for contouring at 50-foot and 100-foot intervals, or 25 m and 50 m, respectively. However, considerable difficulty is encountered in practical application owing to local instabilities of the isobaric surface, or other disturbances. The closing error of an APR profile, after Henry correction, often has little bearing on its accuracy. Little is said in publications about such phenomena, but they are well known to all who apply APR.

From the adjustment point of view, the conventional two-stage procedure of first adjusting APR data separately and then using them as vertical control makes poor use of the inherent accuracy of the combined system. Simultaneous adjustment of both APR data and photogrammetric block data is expected to overcome the drawbacks of the methods currently applied:

1) It will improve the overall absolute height accuracy of aerial triangulation to hopefully 2 m, perhaps even better, for very large areas with scarce and possibly no vertical control, provided that sufficient APR cross flights are utilized. The necessity of closing APR lines over known areas will be greatly reduced.
2) It will solve the problems that are due to local irregularities, or disturbances, of the isobaric surface. By proper weighting, the combined adjustment will follow, over short distances, the photogrammetric bridging with its high local accuracy, leaving large residuals for the APR data wherever the disturbances occur, thus locating them.

The combined adjustment will, in general, derive full benefit from the inherent capability of the method of least squares, combined with the power of electronic computation. Therefore, the combined processing of photogrammetric measurements (plate or model coordinates) with an APR network is anticipated to be highly economic and to meet high-accuracy expectations, even with virtually no vertical control in the area and without absolute closing of individual profiles.

This paper describes the approach to the development of a computer program for the combined adjustment of photogrammetric block data, APR data, and shore-line measurements of lakes. This joint APR block program is based on the recently developed Stuttgart computer programs for block adjustment, using the principle of independent models [*Ackermann, Ebner and Klein* 1970].

The mathematical approach to the combined adjustment is presented and some computational considerations are given. It is intended to report on the results obtained after completion and application of the program.

2. The basic program: block adjustment by independent models

2.1 The joint APR block program is based on the existing block adjustment program PAT-M for independent models [*Ackermann, Ebner and Klein* 1970]. A brief summary is given, for easier tie-in of the subsequent APR extension.
The relationship (similarity transformation) between a terrain point i and the associated measurement ij in model j is given by the nonlinear observation equations

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ij} = -\lambda_j R_j \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{ij} - \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix}_j + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i \qquad (1a)$$

$[x\ y\ z]^T_{ij}$ = vector of model coordinates of point i in model j

$[X\ Y\ Z]^T_i$ = vector of unknown terrain coordinates of point i

$[V_x\ V_y\ V_z]^T_{ij}$ = vector of corrections associated with the transformed point i in model j

$[X_o\ Y_o\ Z_o]^T_j$ = origin of model coordinate system j
λ_j = scale factor for model j
R_j = orthogonal rotation matrix for model j
} orientation parameters for model j 7 unknowns

For the orthogonal rotation matrix R_j a modified version of the Rodrigues-Caylay matrix is used

$$R = \frac{1}{k}\begin{bmatrix} 1 + \frac{1}{4}(a^2 - b^2 - c^2) & -c + \frac{1}{2}ab & b + \frac{1}{2}ac \\ c + \frac{1}{2}ab & 1 + \frac{1}{4}(-a^2 + b^2 - c^2) & -a + \frac{1}{2}bc \\ -b + \frac{1}{2}ac & a + \frac{1}{2}bc & 1 + \frac{1}{4}(-a^2 - b^2 + c^2) \end{bmatrix} \qquad (1b)$$

$k = 1 + \frac{1}{4}(a^2 + b^2 + c^2)$;

a, b, c are the 3 independent rotation parameters.

This approach implies seven unknown orientation parameters for each model and three unknown coordinates per terrain point.

The observations x_{ij}, y_{ij}, z_{ij} (model coordinates) can be weighted. In fact, the program allows the following weight-coefficient matrix to be introduced, scaled to terrain units

$$Q_{(ij)(ij)} = \left[\begin{array}{cc|c} Qxx & Qxy & 0 \\ Qyx & Qyy & 0 \\ \hline 0 & 0 & Qzz \end{array}\right] \qquad Q_{(ij)(kl)} = 0 \qquad (1c)$$

Thus the model points are treated as mutually uncorrelated. As it seems impracticable to weight the points individually, the program allows only one set of weight coefficients for model points according to (1c), and another set for perspective centers.

2.2 The coordinates of ground control points are not necessarily kept fixed through the adjustment. Therefore, the terrestrial coordinates of control points also are treated as observations which gives rise to the additional observational equations

$$\begin{bmatrix} V_x^c \\ V_y^c \\ V_z^c \end{bmatrix}_i = -\begin{bmatrix} X^c \\ Y^c \\ Z^c \end{bmatrix}_i + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i \tag{2}$$

Here X^c, Y^c, Z^c are the terrestrial coordinates of control point i to be corrected by $(V_x^c, V_y^c, V_z^c)_i$. X_i, Y_i, Z_i are the unknown (adjusted) coordinates of point i that tie equation 2 with (1a).

The terrestrial coordinates of control points are also weighted according to (1c). Separate weighting of individual control points is possible, however. Terrestrial control may be horizontal, vertical, or combined.

2.3 The observational equations 1a are nonlinear with respect to the orientation parameters. Therefore, linearization is applied, from intial O-approximations for the tilt parameters a, b, c giving

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{ij} = \begin{bmatrix} 0 & -z & y & -x \\ z & 0 & -x & -y \\ -y & x & 0 & -z \end{bmatrix}_{ij} \cdot \begin{bmatrix} da \\ db \\ dc \\ d\lambda \end{bmatrix}_j - \begin{bmatrix} dX_o \\ dY_o \\ dZ_o \end{bmatrix}_j + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i - \begin{bmatrix} x \\ y \\ z \end{bmatrix}_i \tag{3}$$

The symbols have the same meaning as in (1a); da, db, dc, $d\lambda$ represent increments to the parameters a, b, c of (1b); x, y, z are the model coordinates with which to start. During the successive iterations of the adjustment, they change their meaning to become model coordinates of the previous iteration.

2.4 The block-adjustment program, which is based on the approach 3, is given the name PAT-M7 (Program Aerial Triangulation, independent Models, 7-parameter transformations). Although this system has been prepared in all essentials, the actual programing has not been completed. Instead, the version PAT-M43 was chosen, which iterates sequential horizontal and vertical adjustments, applying 4-parameter and 3-parameter transformations, respectively. The choice was made because of computational economy, the computing time being improved by about a factor three.

The program version PAT-M43 determines the horizontal adjustment and the vertical adjustment separately, in successive steps. The basic formulas, replacing (3) and (2), are summarized below.

2.5 The horizontal block adjustment refers to the observational equations

$$\begin{bmatrix} V_x \\ V_y \end{bmatrix}_{ij} = -\begin{bmatrix} x & -y \\ y & x \end{bmatrix}_{ij} \begin{bmatrix} r \\ s \end{bmatrix}_j - \begin{bmatrix} X_o \\ Y_o \end{bmatrix}_j + \begin{bmatrix} X \\ Y \end{bmatrix}_i \qquad (4a)$$

$$\begin{bmatrix} V_x^c \\ V_y^c \end{bmatrix}_i = -\begin{bmatrix} X^c \\ Y^c \end{bmatrix}_i + \begin{bmatrix} X \\ Y \end{bmatrix}_i \qquad (4b)$$

The symbols *r, s* replace the conventional symbols *a, b* for the transformation parameters to avoid confusion with equations 1b and 3.

The observational equations 4a refer to photogrammetric measurements of model points, (4b) to terrestrial control. The weight coefficients for both types of observations are given in accordance with the planimetric part of (1c) and the specifications given under sections 2.1 and 2.2.

The perspective centers of the models are excluded from the horizontal adjustment because of their disturbing effects. The equations 4a and 4b are linear in the unknowns. Thus the horizontal block adjustment as such needs no linearization and no approximate values. If the models have been leveled previously to a sufficient degree, the horizontal adjustment can be applied separately, thus requiring no iterations.

2.6 The vertical block adjustment makes use of the following linearized observational equations

$$[V_z]_{ij} = [-y \quad x]_{ij} \begin{bmatrix} da \\ db \end{bmatrix}_j - [dZ_o]_j + [Z]_i - [z]_{ij} \qquad (5a)$$

$$\begin{bmatrix} V_x^{Pc} \\ V_y^{Pc} \\ V_z^{Pc} \end{bmatrix}_{ij} = \begin{bmatrix} 0 & -z \\ z & 0 \\ -y & x \end{bmatrix}_{ij} \cdot \begin{bmatrix} da \\ db \end{bmatrix}_j - \begin{bmatrix} 0 \\ 0 \\ dZ_o \end{bmatrix}_j + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i - \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{ij} \qquad (5b)$$

$$[V_z^c]_i = -[Z^c]_i + [Z]_i \qquad (5c)$$

The increments *da, db* refer to the same parameters *a, b* as used in (3) and (1b).
Equations 5a refer to model points
" 5b " " perspective centers (*PC*)
" 5c " " vertical control (*c*)
The weight coefficients associated with (5a-5c) are given according to (1c), respectively.

2.7 The program PAT-M43 always begins with the horizontal adjustment that does not require approximate values, proceeding from the given model coordinates and equations 4a, 4b to a straightforward least squares solution for the transformation parameters $(r, s, X_o, Y_o)_j$. Subsequently, the horizontal coordinates of each model are transformed accordingly, and the heights corrected for scale.

The model coordinates resulting from the previous step enter into the vertical adjustment according to (5a-5c). By a straightforward least squares procedure, the increments $(da, db, dZ_o)_j$ of the respective transformation parameters are solved and the model coordinates transformed by using the relevant terms of the rigorous three-dimensional similiarity transformations.

The plan-height sequence of adjustment is repeated until sufficient convergence is reached. Usually three steps are required at the end of which the final values for the unknown terrain coordinates are computed, together with the residual errors at tie and control points.

2.8 Great pains have been taken to make the actual computer program particularly efficient and general. Some of its features are mentioned briefly.

As in other sophisticated block-adjustment programs the computations proceed, at each iteration, from the model and control coordinates directly to partially reduced normal equations that contain only the unknown transformation parameters, the unknown coordinates having been eliminated. Thus the equations to be solved numerically are of size $4n$ or $3n$, respectively, n being the total number of models treated.

The partially reduced normal equations are solved directly by the solution program Hychol (*Hy*per *Chol*esky), which applies the Cholesky method operating, however, with submatrices three of which are in the core at a time. The program is adaptable to available core size and is not restricted by the number of equations to be solved.

In regular photo coverage of a block, the unknown transformation parameters can be ordered to form a band matrix of coefficients of the partially reduced normal equations. It is essential to keep the band as narrow as possible, as the computing time for solving such systems of equations will increase with the square of the band width. The PAT programs apply a highly automated procedure for minimizing the band width. The model numbers with which to start have to be read in at the beginning. From there on, the program searches for and arranges an optimum ordering, irrespective of the actual numbering of the models, or of their initial ordering.

It has been attempted to highly optimize the computer program for efficient computation especially also for large blocks and for a great many points per model. In addition, the program is self-sufficient and highly automated for the bookkeeping of data, including a great number of formal checks and error message printouts. In particular, arbitrary point numbering can be accepted, provided each terrain point has a unique number that is also given to all its associated model points. Search routines identify identical point numbers and establish ties between models and/or ground control.

The plan-height iterations of the block adjustment converge very rapidly. Usually not more than three steps are needed. Often the third step only confirms that two would have been sufficient. The total computing time required is about 1.5 sec per model (CDC 6600, system time), for standard blocks of several hundred models, with three plan-height iterations. This is the more noticeable as it includes all computations involved. Unlike most other block-adjustment programs, no preliminary computations are required; in particular, no preliminary strip or block formation or preliminary strip or block adjustment is applied.

The computer program is written in Fortran IV language. It is transferable, therefore, to other high-speed computers of suitable capacity (at least 64 K words core capacity, external storage) although, owing to peripheral equipment, the transfer of intricate programs meets with greater difficulties than is generally believed.

The authors consider the program a highly-optimized, general purpose program that is capable of adjusting virtually any block of independent models efficiently and economically. Within an extremely wide range, the program is practically not limited as to multiple overlap, block size, and numbers of points. It is rather the computing time that may set practical limits, depending on the type of computer used. Available experience from practical application confirms both the capability of the method of independent models and the performance of the program [*Akermann, Ebner and Klein* 1970].

3. *Adjustment of statoscope and APR data*

Before discussing the procedure that will be adopted to combine the adjustment of statoscope or APR data with the block adjustment by independent models, a general presentation of the basic relationships of such observational data seems to be expedient, bearing in mind an easy combination with the block-adjustment program as described in section 2.

3.1 APR data consist essentially of two different sets of continuously recorded measurements (Figure 1):

1) clearance s = distance measurement by radar from the momentary air station in vertical direction down to the terrain;
2) differential hypsometer measurement dz by statoscope, or hypsometer, of the deviation of the momentary air station from an unknown isobaric surface.

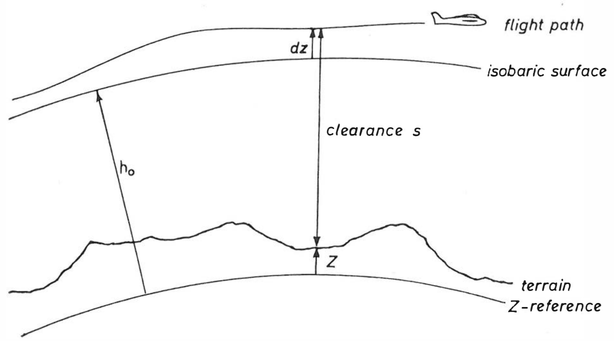

Fig. 1. Relationship between statoscope reading dz, clearance s, and terrain height Z.

At any instant, the difference of the two independent measurements dz and s is added to a constant h_0, which represents an approximate value of the flying altitude. By continuous recording of the resultant sum, an APR terrain profile is obtained, the heights of which read close to the true terrain heights.

$$Z^{APR} = h_o + dz^{stat} - s \tag{6}$$

Usually both the hypsometer readings and the terrain profile are recorded continuously on the APR chart. The moments of exposure of the air-survey camera are also recorded thereon. In addition a 35 mm spotting camera, the axis of which is aligned to the radar beam, takes photographs at short intervals. Thus a number of points of the APR terrain profile can be identified and transferred to the air-survey photographs to be also measured photogrammetrically.

If desired the clearances can be deduced from the APR chart by subtracting the hypsometer profile (and the constant h_o) from the terrain profile. From such clearances, however, only those that refer to the exposure stations of the air-survey photographs have photogrammetric significance.

3.2 If APR measurements have been taken simultaneously with aerial photography, the clearances can be used for scale information. An APR clearance s^{APR} referring to the exposure station of an aerial photograph can be compared with the same distance s^{mod} measured in the photogrammetric model. The ratio gives the local scale factor of the photogrammetric model for the APR-clearance:

$$\lambda^{APR} = s^{APR}/s^{mod} \tag{7}$$

λ^{APR} can be considered an estimate of the true scale factor λ of the photogrammetric model and hence be treated as a (derived) observation. By applying the law of propagation of errors to (7), treating both s^{APR} and s^{mod} as independent observations with known standard errors, a weight coefficient, or a weight, can easily be derived for λ^{APR}. Introducing the unknown scale factor λ_j of model j, the "observed" scale factor $\lambda_j{}^{APR}$ gives an observational equation

$$V_j^\lambda = \lambda_j - \lambda_j{}^{APR} \tag{8a}$$

or

$$V_j^\lambda = d\lambda_j - \lambda_j{}^{APR} + \lambda_j^o \tag{8b}$$

Here a minor problem exists as a model is associated with two air stations. It is sufficiently rigorous to always use the average of the two scale factors concerned and relate it as an independent observation to the scale of the model in question.

Equations 8a, 8b are of the same type as equations 2. Their processing as additional observational equations through the least squares adjustment would not raise any special problems. In particular, the structure of the partially reduced normal equations would not be affected.

It is an open question and remains to be investigated whether a common unknown constant $d\lambda$ should be added to take care of possible constant scale errors or index errors of the radar clearances.

The observational equations 8b could be combined directly with the approach 3 to the block adjustment. If combined with the program version PAT-M43, it seems admissible to utilize the scale information only with the horizontal block adjustment as specified by (4a). There, however, the scale factor λ does not appear explicitly, rather the parameters $r = \lambda \cdot \cos\kappa$ and $s = \lambda \cdot \sin\kappa$ are used, κ standing for the azimuthal rotation. Hence (8b) could be altered to

$$v_j^\lambda \cdot \cos \kappa_j = \lambda_j \cos \kappa_j - \lambda_j{}^{APR} \cos \kappa_j \tag{9a}$$

or

$$v_j^r = r_j - \lambda_j{}^{APR} \cos \kappa_j \tag{9b}$$

If κ is known within a few degrees, (9b) can be replaced by

$$v_j^r = r_j - \lambda_j{}^{APR} \tag{9c}$$

Even with large initial κ-rotations, however, it is generally permissible to use the observational equations 9c in combination with plan-height iterations. The approximation will always be sufficient for the second iteration.

3.3 Statoscope measurements are related to the heights (Z-coordinates) of exposure stations of the air-survey photographs. The statoscope data can be considered, in a way, as vertical control of limited accuracy and thus treated essentially according to observational equations 2 or 5c. However, the important distinction has to be observed that the isobaric surface to which the statoscope readings refer is not known. Therefore, additional unknown parameters are needed, for each strip separately, that describe the orientation and perhaps the shape of the isobaric reference along the flight line. It seems sufficient to take only constant and linear terms into account, leaving a quadratic term optional for long strips.

The original statoscope reading dz, updated by an arbitrary constant $\bar{h}_o$ (which may also be zero), is taken as an observational assessment of the height of the exposure station (perspective center PC) concerned

$$Z^{\text{stat}} = \bar{h}_o + dz^{\text{stat}} \tag{10a}$$

Combining (10a) with the unknown parameters for the isobaric reference gives rise, for an exposure station i in strip k, to the observational equation

$$(V_z)_{ik}^{stat} = -Z_{ik}^{stat} - (a_k + b_k t_{ik}) + Z_{ik}^{PC} \tag{10b}$$

The term $(a_k + b_k t_{ik})$ provides a constant shift and a tilt correction of the isobaric surface along the flight line k. The coefficient t_{ik} represents the distance of the exposure station i from an arbitrary beginning on the isobaric reference of run k. As this distance is not readily available, it is preferable and sufficient to use instead the elapsed time that can be read off the APR chart. Thus the tilt correction $b_k t_{ik}$ goes proportional with time instead of distance flown.

The statoscope readings can be treated as uncorrelated and given a constant weight that will depend on the flying height.

3.4 APR terrain profiles can be used as observational data for the combined adjustment by following essentially the same approach as for statoscope data in section 3.3. The recorded heights of selected points of an APR terrain profile (whether from a separate low altitude APR flight, possibly with laser APR, or from an APR recording obtained simultaneously with the air-survey photography) are treated as additional vertical control, with appropriate weighting. In addition, the orientation parameters for the isobaric surface must be considered. As in equation 10b, and considering (6), we can formulate the observational equation for the APR height of a point i, taken from an APR terrain profile of APR run k

$$(V_z)_{ik}^{APR} = -Z_{ik}^{APR} - (a_k + b_k t_{ik}) + Z_i \tag{11a}$$

The APR heights are treated as uncorrelated and given a constant weight, which is assessed by considering equation 6. The selected APR profile points should perferably be of equal precision. A rather irregular sequence resulting from omitting poor points is acceptable because of the combined adjustment.

The parameters a_k and b_k have the same meaning as in equation 10b. An actual APR run can be subdivided, in case necessary, into two or more computational runs k.

It can happen that APR heights refer directly to vertical control (for instance, identifiable water surfaces). In that case, the connection of (11a) to control is established by the identity of Z_i through the additional observational equation for the control value Z_i^c

$$(V_z^c)_i = -Z_i^c + Z_i \tag{11b}$$

The essential distinction of (11b) against the identical appearance of equation 5c is the fact that here the point i is not necessarily measured in a photogrammetric model. By (11a, 11b), APR profiles can be tied to vertical control outside the photo block.

4. *Height information from shorelines of lakes*

If arbitrary points on the shoreline of a lake are measured in one or several photogrammetric models, they can be used advantageously for the block adjustment, even if the absolute height of the water level is not known. There are several ways to use such lake measurements as additional information for improved leveling of the models. Here an approach is given that ties in directly with the system of independent models.

Let point i on the shoreline of lake l be measured in model j. The lake has the unknown height Z_l. As leveling of lakes contributes to the vertical adjustment only, we can refer to type 5a of observational equations, obtaining

$$[V_z]_{ijl} = [-y \quad x]_{ijl} \begin{bmatrix} da \\ db \end{bmatrix}_j - [dZ_o]_j + [Z]_l - [z]_{ijl} \tag{12a}$$

The triple indexing in (12a) is due to the different meaning of shoreline points with regard to planimetry and to heights. As to planimetry, they are just different points i, which have no particular function. Their heights, however, refer to the common height Z_l of the lake. Maintaining Z_l as one unknown per lake ensures the leveling and the tie effect of lakes by the adjustment. It does not matter, then, from the formal point of view, over how many models a lake stretches.

If the absolute height Z_l^c of the water level is known, the lake acts in addition as vertical control. This case is fully taken into account by using observational equations 5c for vertical control accordingly

$$[V_z^c]_l = -[Z^c]_l + [Z]_l \tag{12b}$$

The weighting of the observations in (12a, 12b) raises no problems, in principle. Whether shoreline points can be given the same weights in Z as ordinary model points is to be tested empirically.

5. *The program for the combined adjustment*

5.1 Although modern computers have considerably reduced the conventional considerations and rules about the allowable computational burden, such considerations still exist for large adjustment problems. For the time being, computer programs that are intended for large and very large block adjustments have to watch carefully the conditions of efficient and economic computation. The present problem of combining APR data and photogrammetric models in a grand block adjustment could be treated directly on the basis of the observational equations as derived in section 3 and 4, in particular of equations 9c, 10b, 11 and 12, in connection with program PAT-M43 as specified by the equations 4 and 5. Computational considerations suggest, however, that some modifications are applied to the effect that the rigor of the adjustment is sufficiently maintained, but that the computations are kept from growing beyond reasonable limits.

The combined adjustment program that is being developed takes considerations of computational economy into account, some of which are briefly commented upon here.

5.2 Amongst the rather rigorous and direct methods for three-dimensional block adjustment, the method of plan-height iterations by independent models has very favorable computational economy. Therefore, the combined adjustment program is based on the PAT-M43 version of block adjustment.

5.3 Theoretical and practical investigations have revealed the surprisingly good accuracy performance of horizontal block adjustment with perimeter control alone. It is anticipated that for blocks of up to 100,000 km^2, or more, horizontal perimeter control, spaced between about 25 km and 50 km, depending on specifications, will be sufficient to meet the requirements for 1:50,000 scale mapping. By comparison, the additional utilization of APR-data for scale information will have very little effect on the horizontal adjustment, in view of the poor accuracy of the clearings from high altitudes, and it would not result in a significant further reduction of the very few horizontal control points. In spite of the foundation by section 3.2, it is therefore not intended, for the time being, to apply observational equations of type 9c, that is, to extend the horizontal part of the block-adjustment program to incorprate APR scale information.

5.4 The adjustment of heights from APR terrain profiles is combined with the vertical part of the block-adjustment program. The extended vertical adjustment program uses directly observational equations 11a, 11b, in addition to the system 5a-5c.

Processing the combined observational equations 5 and 11 directly to partially reduced normal equations does not meet particular difficulties. The familiar band structure of the coefficient matrix for the orientation parameters of the models is not affected.

There appear, however, the additional orientation parameters (a_k, b_k) of the APR profiles, to be treated as unknowns. They are ordered in a separate group, at the end of the list of parameters, thus generating a system of partially reduced normal equations for all orientation parameters with a "banded-bordered" coefficient matrix. The bordering range will, however, not be very wide, the number of additional unknowns hardly exceeding 10^2.

The solution of the normal equations for the orientation parameters by the solution program Hychol does not raise special problems. The Hychol program is also highly efficient for solving large systems of normal equations with banded-bordered coefficient matrices.

The approach, as described, to the combined block adjustment with APR terrain profiles allows directly for arbitrary APR cross flights. The system automatically provides the interconnection of crossing lines via the photogrammetric models. The program is capable of handling, within the same adjustment, data from separate (low altitude) APR flights and APR data taken simultaneously with the aerial photography. Closing of APR lines on given heights, such as rivers and lakes that lie outside the photogrammetric block, is also included.

The APR-input data are supposed to have the Henry correction applied before entering into the combined adjustment.

5.5 Statoscope data are combined with the vertical block adjustment by applying the observational equations 10b together with 5a-5c. The statoscope equations 10b are, in fact, the same as equations 11a for heights from APR terrain profiles, with the restriction that they refer to perspective centers only. The block adjustment treats perspective centers the same as model points. Therefore, the combined APR block program of section 5.4 is also capable of handling statoscope data. However, slightly different weighting will be necessary.

The program could, in principle, process APR profiles and statoscope data from the same flight, neglecting their mutual correlation. It is suspected, however, that there will occur systematic disagreements between both groups of data, owing to problems of calibration that would require additional parameters. It is not intended, up to further empirical evidence, to incorporate such parameters into the program. Thus, for the time being, simultaneous adjustment of statoscope data and APR profile data, from the same APR flight, is excluded.

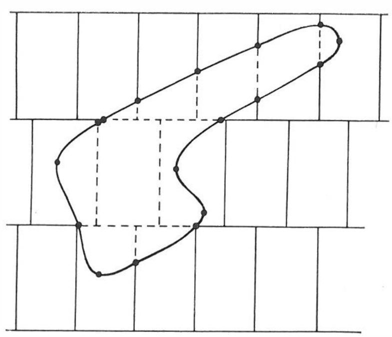

Fig. 2. Example of a lake stretching over many models.

5.6 The program will make use of the stabilizing effect of lakes for the height adjustment. The straightforward approach would combine the observational equations 10a and 10b with the system 5a-5c. However, this approach would severely affect the computational economy of the adjustment. Figure 2 shows that a lake can stretch over many models of a block. Because of the common height of Z_l of all points on the shoreline, all models concerned are directly interrelated which shows up in an accordingly increased band width of the partially reduced normal equations. The computing times for solving such systems of equations, therefore, could increase beyond acceptable limits.

Considering the various possibilities for matching this problem, we decided to subdivide a lake in a number of independent "sublakes" each of which is given a separate index l.

To each sublake l, the observational equations 12 are applied. The sublakes must be chosen small enough not to alter the band structure of the normal equations noticeably. For the time being, each model is associated with its own subunit of the lake. Later perhaps, larger overlapping units might be considered, or identity conditions imposed.

The separate sublakes are joined together by ordinary tie points. The ties can be strengthened by measuring additional tie points close to the shorelines. Thus a solution is obtained that satisfies the practical requirements and maintains computational economy without abandoning theoretical rigor very much.

6. Conclusion

The developed computer program for the combined block adjustment of independent models with statoscope or APR data, together with leveling conditions for lakes, is expected to be highly efficient and economic with very large blocks of possibly several thousand models and some ten thousands of points. Its accuracy capability is anticipated to allow great reduction of horizontal and vertical control, still meeting the accuracy standards for mapping.

Upon completion of the program, it is intended to report the test results obtained and further experience with the system.

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